Optimization of FFR for LTE Uplink Systems

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Abstract — Fractional Frequency Reuse (FFR) can be deployed in OFDMA systems to preserve the cell capacity while mitigating the inter-cell interference (ICI) for cell-edge users. In the uplink of such systems, power control can also be used to control ICI. The problem of determining the joint optimal power control settings and the optimal configuration of the FFR algorithm is of great interest to the operators deploying LTE systems and their evolutions. In this study, we apply the fluid model to reuse factor 3 and use the result to find the optimum FFR parameters (distance to switch between reuse 1 and 3 and the bandwidth allocation between the two reuse plans) and the optimum power control path-loss compensation factor while maximizing the average cell throughput. The result shows that FFR performs better than ordinary reuse plans in the uplink with power control for cell edge and cell centre but lower on the cell average.

Keywords — LTE, Uplink; Fractional Frequency Reuse, Fluid Model, Power Control; Compensation Factor; FFR Optimization

I. NOMENCLATURE

BS – Base Station
PC – Power Control
CLPC – Closed Loop Power Control
FPC – Fractional Power Control
OLPC – Open Loop Power Control
PL – Path Loss
PUSCH – Physical Uplink Shared Channel
RB – Resource Blocks
SINR – Signal-to-Interference-Plus-Noise-Ratio
SNR – Signal-to-Noise-Ratio
UE – User Equipment
FFR – Fractional Frequency Reuse
ICI – Inter Cell Interference
ICIC – Inter Cell Interference Coordination
eNode-B – evolved Node B

II. INTRODUCTION

In LTE systems, Power Control and Fractional Frequency Reuse can be designed to be effective in controlling inter-cell interference at the cell edge. It was shown in [1] that an optimized FFR algorithm in the downlink of an LTE system performs better than N=1 at cell edge and better than N=3 near cell centre. In this paper, the FFR and Power Control parameters are optimized jointly to maximize the cell throughput. In our approach, the cell area is partitioned into two regions [2]: an inner region with N=1 and an outer region with N=3 as clarified in literature [5].

The power control system for the shared data channel in the uplink of LTE is governed by the following equation (in dBm),

\[ P_{\text{pusch}} = \min\{P_{\text{max}}, 10\log (M) + P_0 + aL + \Delta_{\text{RF}} + c\} \quad (1) \]

where \( P_{\text{max}} \) is the maximum power that can be transmitted by a User Equipment, \( M \) is the number of RBs assigned to a UE, \( P_0 \) is the received power target at eNode-B, \( a \) is the path loss compensation factor, \( L \) is the downlink path loss between the UE and eNode-B, \( \Delta_{\text{RF}} \) is the correction factor that depends on the format of data, and \( c \) is a closed loop command sent by eNode-B.

In this paper, we ignore \( P_{\text{max}}, \Delta_{\text{RF}}, \) and \( c \) for simplicity; and since \( M \) will cancel out, it can be any value, we will assume \( M=1 \). All to be able to study \( a \), the common parameter between open loop and closed loop. In this case, the transmit power \( P_{\text{tx}} \) (in dBm) can be written as

\[ P_{\text{tx}} = P_0 + aL \quad (2) \]

And the received power \( P_{\text{rx}} \) at eNode-B can be written as

\[ P_{\text{rx}} = P_0 + (a-1)L \quad (3) \]

As \( a \) increases the received power at eNode-B increases, resulting in a higher inter-cell interference, so \( a \) must be optimized. This simplified model for the power control allows us to develop the model analytically for the signal-to-interference ratio (SIR) and cell throughput.

In regard to power control and as referenced from results from [3] and [4] the closed loop power control with FPC can improve performance significantly. But the authors only use the values of 0.7, 0.8 and 1 for \( \alpha \). But there is no single study except [2] that studied all the values for \( \alpha \). And there are almost no studies that combine FPC and FFR together to answer the question proposed by the study herein.

Our study investigates the combined effect of FFR and power control on inter-cell interference in the uplink. Specifically, the problem that we want to solve is the maximization of the system throughput as a function of \( \alpha, \tau_1, \) and \( \alpha, \) where \( \tau_1 \) is the fraction of total bandwidth allocated to reuse 1 and \( \tau_0 \) is the distance in meters at which FFR switches from reuse 1 to reuse of 3.

III. SIR AND THROUGHPUT MODELS

The fluid model introduced in [3] will be used to derive the SIR for N=1 and N=3 operations. For N=1, it was shown in [3] that

\[ SIR_1 = \gamma_1 = \frac{r^{-\eta(1-\alpha)}}{2\pi\eta\Delta_{\text{UE}}^{\eta} \sum_{k=1}^{\infty} (2\eta\Delta_{\text{UE}}^{\eta})^{2-\eta} \Delta_{\text{En}}(\alpha, \eta)} \quad (4) \]
Where $\theta_{\text{UE}}$ is a uniform UE density which equals in our case $\frac{2}{3\pi R^2}$, $\eta$ is the propagation exponent, $R$ is the cell radius, and $E_u(\alpha, \eta)$ is defined as

$$E_u(\alpha, \eta) = \int_0^\infty y^{-\eta}[(1-y)^{1-\eta} + (1+y)^{1-\eta}] \, dy$$

(5)

where $n$ is the tier number. One key note about the fluid model is that it assumes that the cells are circular. The justification for that is that hexagon models are approximations of the real world and circles approach reality more closely. Another assumption made in the fluid model is that the users in the discrete cells are a continuous fluid. The rationalization for this assumption is, we can make this assumption in any physical system where the value is discrete and the tally is huge; like electric charges for example in capacitive plates. Or another example is when atoms are analyzed they are analyzed in terms of density to study their macroscopic properties in thermodynamic systems. These systems are treated as if they are a cloud with a certain density. This methodology offers outcomes with enough accuracy to permit investigating and comprehending the real world problems experienced in many situations.

To derive the SIR model for $N=3$ we first derive the received power of user $u$ located in cell $b=0$ as in figure 2. We consider

$$\frac{P_{Rx}}{PRx} = \frac{F}{R}$$

(6)

where $F$ is the free-space path gain factor which is equal to the inverse of the path loss $L$. Using (2) and (3) in (6) and denoting by $F_{u,0}$ the path gain (inverse of path loss) between user $u$ and eNode-B 0, the received power from user $u$ in the reference cell can be written as

$$P_u = P_R F_{u,0}^{1-\alpha}$$

(7)

The received power from the interfering cells is the sum of single-user powers in B co-cells. For the case of a single user interfering with user $u$ from each cell, the received power from a single cell is

$$P_{Rx} = P_{Rx,u}(F_{u,0} F_{u,b}^{-\alpha}) = P_{Rx,u} F_{u,0}$$

(8)

Since $P_{Rx,u} = P_o F_{u,b}^{-\alpha}$, (8) becomes

$$P_{Rx} = I_{c,b} = P_o F_{u,b}^{-\alpha} F_{u,0}$$

(9)

Finally, the total interference experienced in eNode-B 0 from all cells can be written as

$$E_c = \sum_{b=1}^{N-1} P_o F_{u,b}^{-\alpha} F_{u,0}$$

(10)

And the exact SIR can be written as

$$SIR = \frac{P_o F_{u,0}^{1-\alpha}}{\sum_{b=1}^{N-1} P_o F_{u,b}^{-\alpha} F_{u,0}}$$

(11)

We assume that the received signal follows the inverse $\eta$–th power law as an approximation, i.e.

$$F_{u,b}(r) = A r^{-\eta}$$

(12)

where $A$ is the propagation intercept which equals to 0dB at 1 meter and $r$ is the distance between $u$ and $b$.

The model becomes even more difficult than equation 11 to deal with so, Consider the diagram in Figure 3. For $N=3$, assuming a continuous instead of discrete distribution of users, $E_c$ can be approximated by

$$E_c = \int_{R_c}^{R_m} \int_0^{2\pi} \theta_{\text{UE3}} P_o A^{-\alpha} \left( (2n\sqrt{3})R - r \right)^{\alpha} Ar^{-\eta} r dr d\theta$$

(13)

Where $R_m = (2N_\text{c}+1)R$, where $N_\text{c}$ is the number of tiers. We can see from the diagram in figure 3 that for the first tier of sites $[1+2(\sqrt{3} - 1)]R < r < (1+2\sqrt{3})R$ and for the second tier $[1+2(2\sqrt{3} - 1)]R < r < (1+4\sqrt{3})R$ and for the $n$th tier $[1+2(n\sqrt{3} - 1)]R < r < (1+2n\sqrt{3})R$. Therefore, a UE located in the $n$th tier is located in the sub-regions

$$r \in [1+2(n\sqrt{3} - 1)]R : 2n\sqrt{3}R]$$

or $r \in [2n\sqrt{3}R : (1+2n\sqrt{3})R]$ (14)

And using equation 12, 13 and 14 the interference from the $n$th tier can be written as

$$E_{n,c} = \int_{1+2(n\sqrt{3} - 1)}^{2n\sqrt{3}} R \theta_{\text{UE3}} P_o A^{-\alpha} \left( (2n\sqrt{3})R - r \right)^{\alpha} Ar^{-\eta} r dr$$

(15)

where $E_{n,c}$ is the values of interference from all cells in the $n$th tier and $\theta_{\text{UE3}} = \frac{\theta_{\text{UE}}}{3}$. Denoting by $y = \frac{r}{2n\sqrt{3}}$ for the first part and $y = \frac{r}{(2n\sqrt{3})R}$ for the second part, we obtain
Further, denoting

\[ E_{n3}(\alpha, \eta) = \int_{0}^{1+y} y^\eta [(1 - y)^{1-\eta} + (1 + y)^{1-\eta}] dy \]  

we have,

\[ E_{n3}(\alpha, \eta) = 2\pi \Theta_{UE} \rho A^1 - \eta \left((2n\sqrt{3}R)^{\alpha+2-\eta}\right) E_{n3}(\alpha, \eta) \]  

Finally, using equations 10, 11, 17 and 18 we obtain the SIR for the N=3

\[ SIR_3 = \gamma_3 = \frac{\rho^{-\eta(1-\alpha)}}{2\pi \Theta_{UE} \sum_{n=1}^{\infty} (2n\sqrt{3}R)^{\alpha+2-\eta} E_{n3}(\alpha, \eta)} \]  

And by normalizing equation 4 and 20 by denoting \( \rho = \frac{\sum}{R} \) for equations 4 and 20 we get

\[ SIR_1 = \gamma_1 = \frac{\rho^{-\eta(1-\alpha)}}{2\pi \Theta_{UE} \sum_{n=1}^{\infty} (2n\sqrt{3}R)^{\alpha+2-\eta} E_{n3}(\alpha, \eta)} \]  

And

\[ SIR_3 = \gamma_3 = \frac{\rho^{-\eta(1-\alpha)}}{2\pi \Theta_{UE} \sum_{n=1}^{\infty} (2n\sqrt{3}R)^{\alpha+2-\eta} E_{n3}(\alpha, \eta)} \]  

The spectral efficiencies associated with the three reuse plans at distance \( r \) from e-NB can now be written as

\[ C_1(r, \alpha) = log_2\left[1 + min(\gamma_0' \gamma_1'(r, \alpha))\right] \]  

\[ C_3(r, \alpha) = \frac{1}{3} log_2\left[1 + min(\gamma_0' \gamma_1'(r, \alpha))\right] \]  

\[ C_{FFR}(r, \alpha) = \begin{cases} f_1 C_1(r, \alpha), & r \leq r_0 \vspace{5pt} \cr \left(1-f_1\right) C_3(r, \alpha), & r_0 < r \leq R_c \end{cases} \]  

Where \( \epsilon \) is the minimum distance between e-UE and e-NB, \( r_0 \) is distance from e-NB that defines the RF coverage for N=1 users, \( f_1 \) is the fraction of bandwidth allocated to the N=1 region, and \( \gamma_0 \) is the SINR threshold at which the user throughput saturates. For 16 QAM in the uplink, throughput saturates at around \( \gamma_0 = 20\text{dB} \) [8].

Figure 4. Power control (N=3) compared to no power control (N=1)

And we define the cell average throughput to be

\[ \frac{C_{ave}}{\alpha f_1 \rho} = \frac{2}{\text{SNR}_\text{thr}} \left( \int_0^\infty C_{FFR}(r, \alpha) \rho d\rho + \int_0^\infty C_{FFR}(r, \alpha) \rho d\rho \right) \]  

IV. JOINT OPTIMIZATION OF FFR AND PC ALGORITHMS

The metric we propose to maximize is the average cell throughput subject to cell-edge throughput \( \text{C}_0 \). That is,

\[ \max_{f_1, \rho} \left\{ \frac{2}{\text{SNR}_\text{thr}} \left( \int_0^\infty C_{FFR}(r, \alpha) \rho(r) d\rho + \int_0^\infty C_{FFR}(r, \alpha) \rho(r) d\rho \right) \right\} \]

Subject to

\[ C_{FFR}(R, \alpha) = C_0 \]  

And

\[ f_1, \min \leq f_1 \leq f_1, \max \]  

The constraints on \( f_1 \) in (29), ensures that FFR offers as better throughput performance as possible. We can see from figure 4 that: 1) N=3 with or without power control is better than pure N=1 at cell edge so we conclude that FFR with or without PC comes in between in performance. And 2) N=3 with power control is less in performance than pure N=1 at cell centre, so we conclude that the fact that FFR alone comes in between in performance is guaranteed. We may write the first conclusion as:

\[ C_3(R, \alpha) > (1 - f_1) C_3(R, \alpha) > C_1(R, 0) \]  

Or

\[ f_{1, \min}(\alpha) = 1 - \frac{C_1(0, 0)}{C_3(\alpha, \alpha)} = 1 - \frac{3 \log_2[1 + \min(\gamma_0 \gamma_1(0, 0))]}{\log_2[1 + \min(\gamma_0 \gamma_1(\alpha, \alpha))]} \]

Note that \( \gamma_0 \) was dropped from (31) since \( \gamma < \gamma_0 \) at cell edge. The second conclusion yields \( f_{1, \min}(\alpha) \) and can be written as

\[ f_{1, \min}(\alpha) = \frac{C_3(\alpha, 0)}{C_1(0, 0)} = \frac{1}{3} \frac{\log_2[1 + \min(\gamma_0 \gamma_1(0, 0))]}{\log_2[1 + \min(\gamma_0 \gamma_1(\alpha, \alpha))]} \]

The above equation reduces to \( f_{1, \min} = \frac{1}{3} \) for almost all values of the compensation factor due to the saturation effect near cell centre.

Table I. Constraints on \( f_1 \) as a function of \( \alpha \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{1, \min} )</td>
<td>0.39</td>
<td>0.44</td>
<td>0.48</td>
<td>0.51</td>
<td>0.53</td>
<td>0.55</td>
<td>0.56</td>
<td>0.58</td>
<td>0.59</td>
<td>0.60</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table I gives the range for \( f_1 \) that satisfies the constraints in (25). We choose \( \text{C}_0 = 0.71\text{b/s/Hz} \) and we get a set of values for \( f_1 \), those values are the values that will be used in this study. Equation 27 becomes dependent on only \( r_0 \) and \( \alpha \) so we
can produce a plot for each $\alpha$ with its corresponding $f_1$, and from those plots we can find optimal $\alpha$. So after graphing the eleven plots, for $\eta = 3.5$ and $\varepsilon = 0.1R$. We found that the optimal $\alpha$ is equal 0.2 with a corresponding $f_1$ equals to 0.47 and $r_{opt}$ is found to be 0.767R, defined by $r_{opt} = R \cdot \rho_{opt}$ (see figure 8).

![Figure 8](image)

Figure 8. Optimum $r_{0}$ found at optimal $f_{1}$ and $\alpha$, where $r_{opt}$ is equal to $\rho_{opt} \cdot R$.

V. CONCLUSIONS

In this paper, we used the fluid geometry model to optimize the parameters of the power control and FFR algorithms for the uplink of an LTE system. We added two constraints on the optimization of the system to ensure superior throughput performances at cell edge and near cell centre. The numerical analysis provided optimum values of $\alpha=0.2$ and $r_{0}=0.7691R$ and $f_{1}=0.471$ which means a gain in user capacity of 94.2% compared to $N=3$, and with a cell average throughput of 1.22 b/s/Hz. The throughput gains approximately of 57.4% compared to full compensation ($\alpha=1$), and a throughput gain of 75.2% is achieved when using these values compared to no power control and no FFR at cell edge ($r=R \cdot f_{1}=1$, $\alpha=0$) and less than the case of no power control and no FFR ($r_{0}=R \cdot f_{1}=1$, $\alpha=0$) in terms of cell average by 45.9%.

REFERENCES


