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Optimization of FFR for LTE Uplink Systems

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Abstract — **Fractional Frequency Reuse (FFR) can be deployed in OFDMA systems to preserve the cell capacity while mitigating the inter-cell interference (ICI) for cell-edge users. In the uplink of such systems, power control can also be used to control ICI. The problem of determining the joint optimal power control settings and the optimal configuration of the FFR algorithm is of a great interest to the operators deploying LTE systems and their evolutions. In this study, we apply the fluid model to reuse factor 3 and use the result to find the optimum FFR parameters (distance to switch between reuse 1 and 3 and the bandwidth allocation between the two reuse plans) and the optimum power control path-loss compensation factor while maximizing the average cell throughput. The result shows that FFR performs better than ordinary reuse plans in the uplink with power control for cell edge and cell centre but lower on the cell average.**

Keywords — **LTE, Uplink; Fractional Frequency Reuse, Fluid Model, Power Control; Compensation Factor; FFR Optimization**

I. NOMENCLATURE

BS – Base Station
PC –Power Control
CLPC – Closed Loop Power Control
FPC – Fractional Power Control
OLPC – Open Loop Power Control
PL – Path Loss
PUSCH – Physical Uplink Shared Channel
RB – Resource Blocks
SINR – Signal-to-Interference-Plus-Noise-Ratio
SNR – Signal-to-Noise-Ratio
UE – User Equipment
FFR – Fractional Frequency Reuse
ICI– Inter Cell Interference
ICIC – Inter Cell Interference Coordination
eNode-B – evolved Node B

II. INTRODUCTION

In LTE systems, Power Control and Fractional Frequency Reuse can be designed to be effective in controlling inter-cell interference at the cell edge. It was shown in [1] that an optimized FFR algorithm in the downlink of an LTE system performs better than N=1 at cell edge and better than N=3 near cell centre. In this paper, the FFR and Power Control parameters are optimized jointly to maximize the cell throughput. In our approach, the cell area is partitioned into two regions [2]: an inner region with N=1 and an outer region with N=3 as clarified in literature [5].

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The power control system for the shared data channel in the uplink of LTE is governed by the following equation (in dBm),

$$\mathbf{P_{PUSCH}} = \min\{\mathbf{P_{max}},\ \mathbf{10log\ (M)} + \mathbf{P_o} + \alpha\mathbf{L} + \mathbf{\Delta_{TF}} + \mathbf{c}\} \quad (1)$$

where $\mathbf{P_{max}}$ is the maximum power that can be transmitted by a User Equipment, **M** is the number of RBs assigned to a UE, P_o is the received power target at eNode-B, α is the path loss compensation factor, **L** is the downlink path loss

between the UE and eNode-B, $\mathbf{\Delta_{TF}}$ is the correction factor that depends on the format of data, and **c** is a closed loop command sent by eNode-B.

In this paper, we ignore $\mathbf{P_{max}}$, $\mathbf{\Delta_{TF}}$, and **c** for simlicity; and since **M** will cancel out, it can be any value, we will assume M=1. All to be able to study α, the common parameter between *open loop* and *closed loop*. In this case, the transmit power P_{Tx} (in dBm) can be written as

$$\mathbf{P_{Tx}} = \mathbf{P_o} + \alpha\mathbf{L} \quad (2)$$

And the received power P_{Rx} at eNode-B can be written as

$$\mathbf{P_{Rx}} = \mathbf{P_o} + (\alpha-1)\mathbf{L} \quad (3)$$

As α increases the received power at eNode-B increases, resulting in a higher inter-cell interference, so α must be optimized. This simplified model for the power control allows us to develop the model analytically for the signal-to-interference ratio (SIR) and cell throughput.

In regard to power control and as referenced from results from [3] and [4] the closed loop power control with FPC can improve performance significantly. But the authors only use the values of 0.7, 0.8 and 1 for α. But there is no single study except [2] that studied all the values for α. And there are almost no studies that combine FPC and FFR together to answer the question proposed by the study herein.
Our study investigates the combined effect of FFR and power control on inter-cell interference in the uplink. Specifically, the problem that we want to solve is the maximization of the system throughput as a function of f_1, r_o and α, where f_1 is the fraction of total bandwidth allocated to reuse 1 and r_o is the distance in meters at which FFR switches from reuse of 1 to reuse of 3.

III. SIR AND THROUGHPUT MODELS

The fluid model introduced in [3] will be used to derive the SIR for N=1 and N=3 operations. For N=1, it was shown in [3] that

$$SIR_1 = \gamma_1 = \frac{r^{-\eta(1-\alpha)}}{2\pi\theta_{UE}\sum_1^{\infty}(2nR)^{\alpha\eta+2-\eta}E_n(\alpha,\eta)} \quad (4)$$

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Where Θ_{UE} is a uniform UE density which equals in our case $\frac{2}{3\sqrt{3}R^2}$, η is the propagation exponent, R is the cell radius, and $E_n(\alpha,\eta)$ is defined as

$$E_n(\alpha,\eta) = \int_0^{\frac{1}{2n}} y^{\alpha\eta}[(1-y)^{1-\eta} + (1+y)^{1-\eta}]\,dy \qquad (5)$$

where n is the tier number. One key note about the fluid model is that it assumes that the cells are circular. The justification for that is that hexagon models are aproximations of the real world and circles apraoch reality more closley. Another assumtion made in the fluid model is presuming that the users in the discrete cells are a continuous fluid. The rationalization for this assumtion is, we can make this assumtion in any physical system where the value is discrete and the tally is huge; like electric charges for example in capacitive plates. Or another example is when atoms are analyzed they are analyzed in terms of density to study their macroscopic properties in thermodynamic systems. These systems are treated as if they are a cloud with a certain density. This methodology offers outcomes with enough accuracy to permit investigating and comprehending the real world problems experienced in many situations.

To derive the SIR model for N=3 we first derive the received power of user u located in cell b=0 as in figure 2. We consider

$$\frac{P_{Rx}}{P_{Tx}} = F \qquad (6)$$

Where F is the free-space path gain factor which is equal to the inverse of the path loss L, Using (2) and (3) in (6) and denoting by $F_{u,0}$ the path gain (inverse of path loss) between user u and eNode-B 0, the received power from user u in the reference cell can be written as

$$P_u = P_o F_{u,0}^{1-\alpha} \qquad (7)$$

The received power from the interfering cells is the sum of single-user powers in B co-cells. For the case of a single user interfering with user u from each cell, the received power from a single cell is

$$P_{Rx} = P_{Tx_{u,b}}(F_{r_{u,0}} F_{t_{u,0}}) = P_{Tx_{u,b}} F_{n,0} \qquad (8)$$

Since $P_{Tx_{u,b}} = P_o {F_{u,b}}^{-\alpha}$, (8) becomes

$$P_{Rx} = I_{c,b} = P_o {F_{u,b}}^{-\alpha} F_{n,0} \qquad (9)$$

Finally, the total interference experienced in eNode-B 0 from all cells can be written as

$$E_c = \sum_{b=1}^{B-1} P_o F_{u,b}^{-\alpha} F_{u,0} \qquad (10)$$

And the exact SIR can be written as

$$SIR = \frac{P_o F^{1-\alpha}}{\sum_{b=1}^{B-1} P_o F_{u,b}^{-\alpha} F_{u,0}} \qquad (11)$$

We assume that the received signal follows the inverse η–th power law as an approximation, i.e.

$$F_{u,b}(r) = Ar^{-\eta} \qquad (12)$$

where A is the propagation intercept which equals to 0dB at 1 meter and r is the distance between u and b.

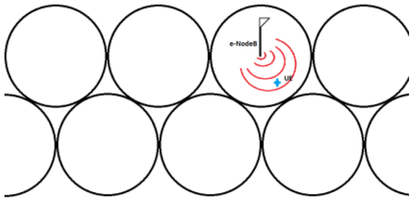

Figure 2. The cell geometry, with circle representation used.

The model becomes even more difficult than equation 11 to deal with so, Consider the diagram in Figure 3, For N=3, assuming a continuous instead of discrete distribution of users, E_c can be aproximated by

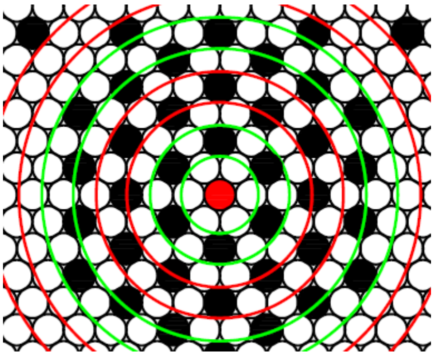

Figure 3. An N=3 omni network of several tiers drawn to scale.

$$E_c = \int_{R_c}^{R_{nw}} \int_0^{2\pi} \Theta_{UE3} F_{u,b}^{-\alpha} F_{u,0}\, r\,dr\,d\theta \qquad (13)$$

Where $R_{nw} = (2N_c+1)R$, where N_c is the number of tiers. We can see from the diagram in figure 3 that for the first tier of sites $[1+2[(\sqrt{3}-1)]R < r < (1+2\sqrt{3})R$ and the for the second tier $[1+2(2\sqrt{3}-1)]R < r < (1+4\sqrt{3})R$ and for the nth tier $[1+2(n\sqrt{3}-1)]R < r < (1+2n\sqrt{3})R$. Therefore, a UE located in the nth tier is located in the sub-regions

$$r \in [\,[1+2(n\sqrt{3}-1)]R\,;\, 2n\sqrt{3}R]$$
$$\text{or } r \in [\,2n\sqrt{3}R\,;\,(1+2n\sqrt{3})R] \qquad (14)$$

And using equation 12, 13 and 14 the interference from the nth tier can be written as

$$E_{n,c3} = 2\pi \int_{(1+2(n\sqrt{3}-1))R}^{(2n\sqrt{3})R} \Theta_{UE3} P_o A^{-\alpha} \left((2n\sqrt{3})R - r\right)^{\alpha\eta} A r^{-\eta} r\,dr$$

$$+2\pi \int_{(2n\sqrt{3})R}^{(1+2n\sqrt{3})R} \Theta_{UE3} P_o A^{-\alpha} \left(-r + (2n\sqrt{3})R\right)^{\alpha\eta} A r^{-\eta} r\,dr \qquad (15)$$

where $E_{n,c3}$ is the values of interference from all cells in the nth tier and $\Theta_{UE3} = \frac{\Theta_{UE}}{3}$. Denoting by $y = 1 - \frac{r}{(2n\sqrt{3})R}$ for the first part and $y = \frac{r}{(2n\sqrt{3})R} - 1$ for the second part, we obtain

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$$E_{n,c3} = 2\pi\theta_{UE3}P_oA^{1-\alpha}((2n\sqrt{3})R)^{\alpha\eta+2-\eta}\int_0^{\frac{1}{2\sqrt{3}n}} y^{\alpha\eta}[(1-y)^{1-\eta} + (1+y)^{1-\eta}]\,dy$$
(16)

Further, denoting

$$E_{n3}(\alpha,\eta) = \int_0^{\frac{1}{2\sqrt{3}n}} y^{\alpha\eta}[(1-y)^{1-\eta} + (1+y)^{1-\eta}]\,dy \quad (17)$$

we have,

$$E_{n,c3} = 2\pi\Theta_{UE}P_oA^{1-\alpha}((2n\sqrt{3})R)^{\alpha\eta+2-\eta}E_{n3}(\alpha,\eta) \quad (18)$$

$$E_{c3} = \sum_{n=1}^{Nc} E_{n,c3} \quad (19)$$

Finally, using equations 10,11,17 and 18 we obtain the SIR for the N=3

$$SIR_3 = \gamma_3 = \frac{r^{-\eta(1-\alpha)}}{2\pi\Theta_{UE3}\sum_1^{\infty}(2n\sqrt{3}R)^{\alpha\eta+2-\eta}E_{n3}(\alpha,\eta)} \quad (20)$$

And by normalizing equation 4 and 20 by denoting $\rho = \frac{r}{R}$ and assuming $\Theta_{UE} = \frac{2}{3\sqrt{3}R^2}$ for equations 4 and 20 we get

$$SIR_1 = \gamma_1 = \frac{\rho^{-\eta(1-\alpha)}}{\frac{4\pi}{3\sqrt{3}}\sum_1^{\infty}(2n)^{\alpha\eta+2-\eta}E_n(\alpha,\eta)} \quad (21)$$

And

$$SIR_3 = \gamma_3 = \frac{\rho^{-\eta(1-\alpha)}}{\frac{4\pi}{9\sqrt{3}}\sum_1^{\infty}(2\sqrt{3}n)^{\alpha\eta+2-\eta}E_{n3}(\alpha,\eta)} \quad (22)$$

The spectral effeciencies associated with the three reuse plans at distance r from e-NB can now be written as

$$C_1(r,\alpha) = log_2[1 + min(\gamma_0, \gamma_1(r,\alpha))] \quad (23)$$

$$C_3(r,\alpha) = \frac{1}{3}log_2[1 + min(\gamma_0, \gamma_3(r,\alpha))] \quad (24)$$

$$C_{FFR}(r,\alpha) = \begin{cases} f_1C_1(r,\alpha), & \varepsilon < r \le r_o \\ (1-f_1)C_3(r,\alpha), & r_o < r \le R_c \end{cases} \quad (25)$$

Where ε is the minimum distance between e-UE and e-NB, r_o is distance from e-NB that defines the RF coverage for N=1 users, f_1 is the fraction of bandwidth allocated to the N=1 region, and γ_0 is the SINR threshold at which the user throughput saturates. For 16 QAM in the uplink, throughput saturates at around $\gamma_0 = 20$dB [8].

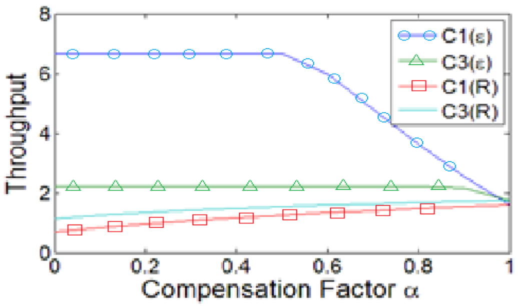

Figure 4. Power control (N=3) compared to no power control (N=1)

And we define the cell average throughput to be

$$\underset{\alpha,f_1,r_o}{C_{av}} = \frac{2}{1-\rho(\varepsilon)^2}\left(\int_\varepsilon^{r_o} C_{FFR}(r,\alpha)\,\rho d\rho + \int_{r_o}^{R} C_{FFR}(r,\alpha)\,\rho d\rho\right) \quad (26)$$

IV. JOINT OPTIMIZATION OF FFR AND PC ALGORITHMS

The metric we propose to maximize is the average cell throughput subject to cell-edge throughput C_0. That is,

$$\max_{\alpha,f_1,r_o}\left\{\frac{2}{1-\rho(\varepsilon)^2}\left(\int_\varepsilon^{r_o} C_{FFR}(r,\alpha)\,\rho(r)d\rho + \int_{r_o}^{R} C_{FFR}(r,\alpha)\,\rho(r)d\rho\right)\right\} \quad (27)$$

Subject to

$$C_{FFR}(R,\alpha) = C_0 \quad (28)$$

And

$$f_{1,min} \le f_1 \le f_{1,max} \quad (29)$$

The constraints on f_1 in (29), ensures that FFR offers as better throughput performance as possible. We can see from figure 4 that: 1) N=3 with or without power control is better than pure N=1 at cell edge so we conclude that FFR with or without PC comes in between in performance. And 2) N=3 with power control is less in performance than pure N=1 at cell centre, so we conclude that the fact that FFR alone comes in between in performance is guaranteed. We may write the first conclusion as:

$$C_3(R,\alpha) > (1-f_1)C_3(R,\alpha) > C_1(R,0) \quad (30)$$

Or

$$f_{1,max}(\alpha) = 1 - \frac{C_1(R,0)}{C_3(R,\alpha)} = 1 - 3\frac{log_2[1+\gamma_1(R,0)]}{log_2[1+\gamma_3(R,\alpha)]} \quad (31)$$

Note that γ_0 was dropped from (31) since $\gamma < \gamma_0$ at cell edge. The second conclusion yields $f_{1,min}$ and can be written as

$$C_1(\varepsilon,0) > f_1C_1(\varepsilon,0) > C_3(\varepsilon,\alpha) \quad (32)$$

Or

$$f_{1,min}(\alpha) = \frac{C_3(\varepsilon,0)}{C_1(\varepsilon,\alpha)} = \frac{1}{3}\frac{log_2[1+min(\gamma_0,\gamma_3(\varepsilon,0))]}{log_2[1+min(\gamma_0,\gamma_1(\varepsilon,\alpha))}} \quad (35)$$

The above equation reduces to $f_{1,min} = \frac{1}{3}$ for almost all values of the compensation factor due to the saturation effect near cell centre.

Keeping this in mind, equation (26) is plotted and repeated for α = 0, 0.3 and 0.7, respectively

TABLE I. CONSTRAINTS ON f_1 AS A FUNCTION OF α

α	f_{1min}	f_{1max}
0	1/3	0.38
0.1	1/3	0.44
0.2	1/3	0.48
0.3	1/3	0.51
0.4	1/3	0.53
0.5	1/3	0.55
0.6	1/3	0.56
0.7	1/3	0.58
0.8	1/3	0.59
0.9	1/3	0.60
1	0.26	0.61

Table I gives the range for f_1 that satisfies the constraints in (25). We choose $C_0 = 0.71$b/s/Hz and we get a set of values for f_1, those values are the values that will be used in this study. Equation 27 becomes dependent on only r_o and α so we can produce a plot for each α with its corresponding f_1, and from those plots we can find optimal α. So after graphing the eleven plots, for η = 3.5[8] and ε = 0.1R. We found that the optimal α is equal 0.2 with a corresponding f_1 equals to 0.47 and r_{opt} is found to be 0.767R, defined by $r_{opt} = R * \rho_{opt}$ (see figure 8).

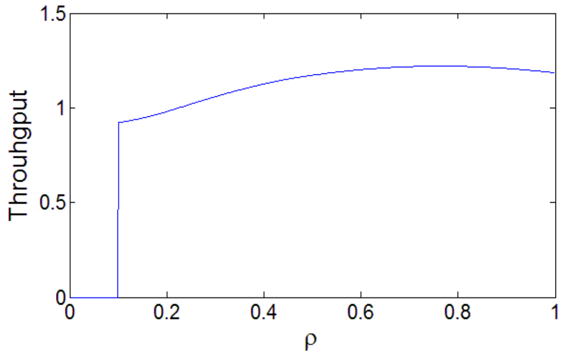

Figure 8. Optimum r_o found at optimal f_1 and α, where r_{opt} is equal to ρ_{opt}*R.

V. CONCLUSIONS

In this paper, we used the fluid geometry model to optimize the parameters of the power control and FFR algorithms for the uplink of an LTE system. We added two constraints on the optimization of the system to ensure superior throughput performances at cell edge and near cell centre. The numerical analysis provided optimum values of α=0.2 and r_o=0.7691R and f_1=0.471 which means a gain in user capacity of 94.2% compared to N=3, and with a cell average throughput of 1.22 b/s/Hz. The throughput gains approximately of 57.4% compared to full compensation (α =1), and a throughput gain of 75.2% is achieved when using these values compared to no power control and no FFR at cell edge (r=R f_1=1, α=0) and less than the case of no power control and no FFR (r_o=R f_1=1, α=0) in terms of cell average by 45.9%.

REFERENCES

[1] A. Simonsson and A. Furuskar, “Uplink Power Control in LTE, Overview and Performance,” in *IEEE Vehicular Technology Conference VTC*, 2008.

[2] C. U. Castellanos, D. L. Villa, C. Rosa, K. I. Pedersen, F. Calabresse, P. Michaelsen and J. Michel, “Performance of Uplink Fractional Power Control In UTRAN LTE,” in *IEEE Vehicular Technology Conference VTC*, Singapore, 2008.

[3] W. Xiao, R. Ratasuk, A. Ghosh, R. Love, Y. Sun and R. Nory, “Uplink Power Control, Intereference Coordination and Resource Allocation for 3 GPP E-UTRA,” in *IEEE Vehicular Technology Conference VTC*, 2006.

[4] M. Amer, "Optimal Configuration of Fractional Frequency Reuse System for LTE Cellular Networks," in *2012 IEEE 76th Vehicular Technology Conference*, Quebec, 2012.

[5] F. Khan, LTE for 4G Mobile Broadband, Cambridge: Cambridge University Press, 2009.

[6] M. Coupechoux, “How to Set the Fractional Power Control Compensation Factor in LTE ?,” in *Sarnoff Symposium*, Princeton, 2011.

[7] B. Muhammad and A. Mohammed, “Performance Evaluation of Uplink Closed Loop Power Control for LTE System,” in *Vehicular Technology Conference VTC*, Taipei, 2010.

[8] A. Goldsmith, Wireless Communications, Cambridge: Cambridge University Press, 2005, 2005.

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