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Covariance Based Selection of Risk and Return Factors for Portfolio Optimization

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Abstract— **In this paper, we analysed the main inputs of the portfolio optimization: Risk measures and return. Although these parameters are compelling and comprehensive from theoretical perspective, the calculation method and reliability of these parameters are still questionable. In this paper, return and risk parameters are analysed based on the S&P 500 historical data from the beginning of 2000 to the end of 2009. We analysed correlation of risk and return vectors from 2000 to 2009 on a yearly basis. Our findings are as follows. First, historical returns provide no information for future returns, which was expected. Second, historical variances and historical value-at-risk measures provide insight for the change in the future risk level of individual securities.**

Index Terms — **Modern portfolio theory, parameter selection, risk measures, value-at-risk**

I. Introduction

One of the frequent questions in finance is how to measure risk and return for portfolio optimization, performance evaluation and asset allocation. There is an ongoing debate on the appropriate risk measures both from traders' perspective and regulators' perspective. In this paper, we will focus on the portfolio optimization process from traders' point of view. Our main goal is to evaluate the applicability of historical risk factors and return data to portfolio optimization problem based on the S&P 500 historical daily data between 2000 and 2009. We believe that 10-year duration for this analysis would provide us significant results because it includes economic downturns as well as upturns. Evaluation of covariance matrix, another crucial input of portfolio optimization problem, is out of scope of the paper. Also, analysing risk and return factors based on multistage portfolio optimization is not in scope of the paper.

One of the main differences of this paper is that it doesn't aim to forecast risk and return of specific securities. It treats historical yearly risk and return measures of equities as a column vector and it attempts to understand whether changes of risk and return column vectors are correlated or not. This is an important difference because if the risk and return measures of securities are positively correlated, portfolio optimization decision may still help us to outperform the market even if the individual variances can't be forecasted in a reliable manner. As a simple example for illustration of this concept is given below:

Assume that variance of security A is 5 and variance of security B is 2 for the previous year. We define variance vector as $[5\ 2]^T$ for the previous year. In portfolio optimization problems we incorporate next years' variance vector into the optimization problem. What we propose is that, even if we can't forecast next periods' variances for each individual security, there is a high probability that A will be riskier than B in next year given that yearly variance vector is positively correlated with the subsequent years' variance vector. In this case, portfolio optimization will still be efficient even if the future periods variances are unknown. As illustrated in this example, if we look at the correlations between years and analyse finding, we can form more efficient portfolio based on the out-of-sample data, which is important for traders.

The rest of the paper is organized as follows: Section 2 summarizes the literature on portfolio optimization and risk factors. Section 3 provides an overview on the widely used risk factors. Section 4 introduces the method for evaluating risk factors from portfolio optimization perspective. Application of the method and the analysis of results are provided in Section 5. Section 6 presents the conclusions and implications of this paper.

II. Literature Review

The Portfolio Theory, also known as Modern Portfolio Theory, was first developed by Harry Markowitz. He had introduced the theory in [1] and [2]. In 1990, along with Merton Miller and William Sharpe, he won the Nobel Prize in Economic Sciences for the Theory. Markowitz formulated the portfolio problem as a choice of the mean and variance of a portfolio of assets.

As summarized in [3], in its simplest form, Modern Portfolio Theory provides a framework to construct and select portfolios based on the expected performance of the investments and the risk appetite of investor. Modern Portfolio Theory, also commonly referred to as mean-variance analysis, introduced a whole new terminology, which has become the norm in the area of investment management. Considering and diversifying the risk are one of the most important steps in making investment decisions. Prior to the modern portfolio theory there wasn't any quantitative method that helped us to understand the

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diversification effect. Although there was an adage suggesting not putting all of your eggs in one basket, there was no method in practice to do this in a reliable and quantifiable manner. Currently, incorporation of this diversification effect by running a portfolio optimization problem into investment decisions is not widely used in investment community. Accordingly, [4] states that risk-return optimization is used primarily by more quantitatively oriented firms since risk-return optimization is very sensitive to changes in inputs. There are various reasons for this preference that we will analyse in the subsequent paragraphs.

Reference [5] shows empirically that in-sample performance of mean-variance optimization method is superior to other methods, while out-of-sample performance is lower compared to other methods in terms of Sharpe ratio. The difference in performance is caused by the instability of portfolio optimization inputs. As no trader would get bonuses based on the in-sample performance, poor out-of-sample performance could be one of the drivers for not using mean-variance portfolio optimization.

Considering only mean return and variance of return of a portfolio is, of course, a simplification relative to including additional moments that might more completely describe the distribution of returns of portfolio. Reference [6] offers alternative portfolio theories that included more moments such as skewness. Nevertheless, mean-variance theory has remained the cornerstone of modern portfolio theory despite alternatives.

There are also numerous risk measures proposed to be included as an input to the portfolio optimization problem. We'll explore these measures with the literature review of these factors in the subsequent section.

III. Main Risk Factors

We can divide risk measures into two categories: two-sided and downside risk measures. While two-sided risk measures are calculated based on the up and down movements of asset, downside risk measures cover only down movements. Standard deviation, variance, mean-absolute deviation are among the most known two-sided risk measures. Semi-variance, value-at-risk (VaR), conditional value-at-risk, in other words expected shortfall, are among the most known downside risk measures. Two-sided measures and downside risk measures provide equivalent information about the distribution of returns if the returns are symmetrical, which is not a common case. Two-sided measures penalize the up and down movements equally whereas in real world up movements are more preferable and shouldn't be penalized. Two-sided risk measures may not provide intuitive insights regarding the risk of the portfolio if the returns are not normally distributed. However, as mentioned in [7], downside risk measures are generally harder to compute and aggregate in the portfolio context even they may seem more intuitive from risk management point of view. Also, downside risk measures use only a portion of the original data – maybe even just the tail of the empirical distribution – and this may lead to the increase of estimation errors.

One of the most popular risk measures from the regulatory perspective is value-at-risk measure. Till Guldimann can be viewed as creator of the term value at risk, he developed the term while he was head of global research at JP Morgan in the late 1980s. The bank decided that "value risks" were more important than "earnings risk", paving the way for VaR. At that time, there was much concern about managing the risks of derivatives properly. The Group of Thirty (G-30), which had a representative from J.P. Morgan, provided a venue for discussing best risk management practices. The term found its way through G-30 report published in July 1993. Apparently, this was first widely publicized appearance of the term value at risk. Value-at-risk is also explained in detail in [8] with its applications areas and back-testing methods.

Value-at-Risk can be defined as the maximum loss that might occur within a certain horizon and a certain confidence interval. Based on the definition, value-at-risk measure has two parameters: confidence interval and horizon. Horizon is depending on investment duration and liquidity of the security. For more illiquid securities, longer horizon is used. In this paper, we will calculate Value-at-Risk for one year horizon assuming that investment horizon is one year.

An additional risk measure, expected shortfall has been proposed to address these disadvantages. Expected shortfall is the average loss given the loss amount exceeded specified confidence interval. Expected shortfall is additive and it gives insights regarding the nature of tail. Basel Committee on Banking Supervision also proposed a revision paper to its current market risk framework that is based on Value-at-Risk in [9]. Although it is not finalized yet, usage of expected shortfall may gain momentum with regulatory guidance in near future.

It is advocated that downside risk measures are more intuitive and yet none of the downside risk measures provide the complete picture of risk. For this reason, methods for blending several risk measures to one common risk measure are proposed. In this regard, [10] argues that the combined risk measures address weaknesses of individual risk measures.

After the global financial crisis of 2007-2008 new risk measures are proposed to estimate dependency between the institutions, particularly financial institutions. These new risk measures fall into category of systemic risk measures. Systemic risk is any set of circumstances that threatens the stability of or public confidence in the financial system, as defined in broadly manner. Systemic risk has two main elements: shocks and propagation mechanisms. There are two types of shocks: idiosyncratic and systematic. Idiosyncratic shocks are caused by change of the price of a single institution while systemic shocks are caused by the co-movements of prices of multiple institutions at the same time. Propagation is the transmission of the shock to the other markets, institutions and sectors, thus the economy. Propagations may occur through two main channels: (i) domino effects (ii) imperfect information. References [11] and [12] provide a detailed summary of the proposed systemic risk measures. Since these risk measures are in early stage of their development and some of these measures

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are applied only to financial institutions they are not in the scope of this paper.

IV. Method

In this paper, the reliability of risk factors and historical measures are explored in detail from the portfolio optimization perspective. We assume that a certain trader wants to run a portfolio analysis to allocate his funds to S&P 500 equities for one-year horizon. Typical portfolio optimization process requires three types of inputs: return, individual risk measures and interaction level of individual risk measures such as covariance. These measures are generally calculated based on the historical data. In this aspect, historical return and risk data should reflect future return and risk data to have a profitable portfolio optimization decision. This paper mainly focuses on determining the extent of validity of this assumption. In order to test this assumption we will use QuantQuote free daily data for S&P 500. Date range for this analysis is from the beginning of 2000 to the end of 2009. This analysis requires the daily price data to be complete for each individual security. There are 435 securities that have complete price data. We will do analysis from a traders' perspective that allocate his portfolio based on the portfolio optimization steps below:

1) *At each years beginning, he calculates risk and return measures, covariance matrices based on the previous year data.*

2) *He runs portfolio optimization process based on the input data calculated at Step 1.*

3) *When the year ends, he recalculates the same risk and return measures for the past year and reallocates his portfolio.*

Although this strategy may not be a complete real life example, it could help us to understand similar strategies that rely on historical data. The strategy mentioned above could be successful if the past data can predict future returns and risk measures. We will do an empirical analysis that tests the validity of this assumption. Analysis involves 2 steps of calculation:

1) *Calculating mean return, median return, variance, skewness, kurtosis, VaR(%99, 1 year), VaR(%95, 1 year), VaR (%90, 1 year), VaR (%75, 1 year), P (%25 percentile, 1 year) P10 (%10 percentile, 1 year), P5 (5% percentile, 1 year), P1 (%1 percentile, 1 year) for each security and each year. A sample of calculation of these measures are provided in appendix. Percentile measure will help us to analyse the effect of historical return spikes over the future returns. This will lead to the change of sign in the correlation matrix. We will have a two dimensional matrix with 435 rows for each securities and 130 columns. 130 (13*10) columns include 13 risk factors calculated for each year in the sample data.*

2) *Calculating correlations between each column. We will have two-dimensional matrix with 130 columns and 130 rows.*

This calculated correlation matrix would reveal empirical insights about risk and return factors. When the correlation is close to zero between consecutive years for the same factor, it will help us to conclude that there is no predictive power of the historical data for such risk factor. Based on the correlation matrix results, we can analyse both the relationship between factors over time as well as relationship between past values of the risk factor to the future values of the risk factor. If past data and future data were uncorrelated for a specific factor it would be nonsense to use the risk factor for portfolio optimization purposes.

V. Application

We calculate risk and return measures based on the formulas below for each security and each year:

$P(i, j, k)$: Percentile at level %i for security j in year k.

$X_{j,k}$: Daily return of security j in year k.

$$i = prob\left[X_{j,k} \geq P(i,j,k)\right] = \int_{P(i,j,k)}^{+\infty} f(x_{j,k})dx \qquad (1)$$

We calculate $P(i, j, k)$ for:

$i \in (0.01, 0.05, 0.10, 0.25, 0.75, 0.90, 0.95, 0.99)$

$j \in (1, 2, ..., 425)$

$k \in (2000, 2001, ... 2009)$

All value-at-risk and percentile measures are computed according to eq. (1).

Empirical distributions are used for $f(X_{j,k})$ based on the historical data provided. Based on eq. (1) we calculated VaR(%99, 1 year), VaR(%95, 1 year), VaR (%90, 1 year), VaR (%75, 1 year), P (%25 percentile, 1 year) P10 (%10 percentile, 1 year), P5 (5% percentile, 1 year), P1 (%1 percentile, 1 year), median return for each security and each year.

Mean return, variance, skewness and kurtosis are calculated in the same manner for each security and year based on the empirical data. Based on these calculations, we can form a two-dimensional matrix of risk and return measures for each years where columns are risk and return measures and rows are securities as demonstrated below:

$$X_k = \begin{pmatrix} \alpha_{1,1} & \cdots & \alpha_{13,1} \\ \vdots & \ddots & \vdots \\ \alpha_{1,425} & \cdots & \alpha_{13,425} \end{pmatrix} \qquad (2)$$

$k \in (2000, 2001, ... 2009)$

We appended two-dimensional matrixes for calculating correlation of risk and return measures between risk and return factors calculated based on the each years data as shown below:

$$A = (X_{2000} \quad X_{2001} \quad \ldots \quad X_{2009}) \qquad (3)$$

Matrix **A** has 425 rows for each equity and 130 columns for risk and return factors calculated for each year. Columns of Matrix **A** are given in the below:

$$A = (Mean_{2000} \quad Median_{2000} \quad \ldots \quad VaR(0.99)_{2009}) \; (4)$$

$Mean_{2000}$ column consists of mean daily return of 425 equities for year 2000. We calculate a correlation matrix

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based on columns of matrix **A**. Correlation matrix has 130 columns and 130 rows.

Results of the wide correlation matrix are very wide that the full version can't be shared in the appendix. However, a sample of the covariance matrix for the years 2000, 2001, 2000-2001 will be shared in the appendix. This wide correlation matrix includes the statistics below:

1) Correlation of risk and return factors in the same year: This kind of statistics doesn't help us a lot for portfolio optimization perspective. Traders need to understand the future behaviour of risk and return factors based on the historical data. So, the correlation risk and return factors in the same year provide only information regarding the current year.

2) Correlation of the same risks or return factor between years: These statistics help us to understand whether the risk factors of securities move in the same direction over time. If they move in the same direction, we can reliably use such risk factors.

3) Correlation of different risk and returns factors between years: These statistics help us to reveal interaction between different risk and return factors over time. For example, if there was a positive correlation between past year variance and current year return, we could conclude that more risky securities could generate higher returns on average.

Based on the analysis of correlation matrix, summary of findings based on the results are shared below:

1) Historical yearly mean return almost has no predictive power for future mean return. For 5 years we see weak negative correlation and for 4 years we calculate weak positive correlation. Average correlation for each consecutive year is -0.06.

2) Historical kurtosis may provide limited insight for future kurtosis measures. There is weak positive correlation between past kurtosis and future kurtosis for each consecutive year analysed. Average correlation for each consecutive year is 0.27.

3) Historical skewness has no predictive power over future return and historical return has no predictive power over future skewness.

4) Historical percentile measures have strong predictive power over next years' percentile measures for the same level of percentile. We see the same decay behaviour when the percentile decreases as we approach to the tail of the return distribution. For P10, correlation between past year and current year is 0.80, while the correlation decreases to 0.63 for P1.

VI. Conclusion

In this article, we evaluated inputs of portfolio optimization strategies based on the historical data. The success of the strategy relies on the assumption that past behaviour and future behaviour are statistically similar. Otherwise, portfolio optimization strategy based on the historical data may underperform various portfolio optimization strategies.

To explore the relationship between historical and future data, we calculated a correlation matrix on the following variables: equity and level of yearly risk and return factors. The results imply that past return data does not provide information regarding future return behaviour.

Thus, we can conclude that a portfolio optimization strategy relying on the past historical return data could overinvest to recent winners and underinvest to recent losers. As a result, this approach may lead to suboptimal portfolio allocation results. On the other hand, past value-at-risk measures and variance measure could provide strong insight regarding future risk behaviour.

Another finding of this paper is that adding skewness and kurtosis as an input to the portfolio optimization may not improve the performance of the portfolio although it makes sense from theoretical perspective.

Running a similar analysis for covariance between securities over time for each year and testing whether covariance matrix stays constant over time may improve this empirical analysis further. Similarly, expected shortfall and systemic risk measures could be incorporated into the analysis. Based on the results of these empirical tests, new factors could also be added to portfolio optimization process as an input. Also, conducting a similar test in an emerging market equity data could provide insights regarding portfolio optimization process. Another room for improvement is to include a longer time horizon in the analysis such as more than 30 years. Final improvement regarding the analysis is to calculate correlations based on 5-year or 10-year periods instead of yearly periods to eliminate the cyclicality effects. We may find correlation between return vectors over time within longer periods. It will be worth to investigate this with a longer horizon.

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APPENDIX

For the tables below, we will use the abbreviations below:

- Mean_2000: Mean returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- Median_2000: Median of returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- Skew_2000: Skewness during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- Kurt_2000: Kurtosis during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- Var_2000: Variance during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- Q3_2000: Value-at-Risk (%75, 1 year) during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- P95_2000: Value-at-Risk (%95, 1 year) percentile returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- P90_2000: Value-at-Risk (%90, 1 year) percentile returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- P99_2000: Value-at-Risk (%99, 1 year) percentile returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- P5_2000: %5 percentile returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- P1_2000: %1percentile returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.
- P10_2000: %10 percentile returns during 2000 year for each 435 S&P500 Equities included in the empirical analysis.

Similar abbreviations apply for 2001 year.

Table I: Correlation Between Risk and Return Factors during 2000

Correlation Matrix/2000 Year	**Mean_2000**	**Skew_2000**	**Q3_2000**	**Var_2000**	**P5_2000**	**P1_2000**	**Kurt_2000**	**Median_2000**	**P25_2000**	**P99_2000**	**P95_2000**	**P90_2000**	**P10_2000**
Mean_2000	1.00	0.21	0.22	-0.06	0.03	0.02	-0.12	0.66	0.23	0.16	0.14	0.14	0.07
Skew_2000	0.21	1.00	-0.03	-0.03	0.05	0.17	0.07	-0.13	-0.05	0.18	0.06	0.02	0.03
Q3_2000	0.22	-0.03	1.00	-0.89	-0.88	-0.81	0.03	0.36	-0.79	0.81	0.90	0.93	-0.89
Var_2000	-0.06	-0.03	-0.89	1.00	0.95	0.91	0.20	-0.26	0.84	-0.92	-0.95	-0.94	0.93
P5_2000	0.03	0.05	-0.88	0.95	1.00	0.89	0.05	-0.22	0.85	-0.85	-0.92	-0.92	0.95
P1_2000	0.02	0.17	-0.81	0.91	0.89	1.00	0.23	-0.24	0.75	-0.79	-0.85	-0.85	0.86
Kurt_2000	-0.12	0.07	0.03	0.20	0.05	0.23	1.00	-0.14	-0.14	-0.20	-0.05	0.00	-0.02
Median_2000	0.66	-0.13	0.36	-0.26	-0.22	-0.24	-0.14	1.00	0.00	0.24	0.25	0.26	-0.20
P25_2000	0.23	-0.05	-0.79	0.84	0.85	0.75	-0.14	0.00	1.00	-0.74	-0.82	-0.84	0.90
P99_2000	0.16	0.18	0.81	-0.92	-0.85	-0.79	-0.20	0.24	-0.74	1.00	0.89	0.86	-0.83
P95_2000	0.14	0.06	0.90	-0.95	-0.92	-0.85	-0.05	0.25	-0.82	0.89	1.00	0.97	-0.91
P90_2000	0.14	0.02	0.93	-0.94	-0.92	-0.85	0.00	0.26	-0.84	0.86	0.97	1.00	-0.92
P10_2000	0.07	0.03	-0.89	0.93	0.95	0.86	-0.02	-0.20	0.90	-0.83	-0.91	-0.92	1.00

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Table II: Correlation Between Risk and Return Factors during 2000 and 2001

_Correlation Matrix/Between 2000 and 2001	Mean_2001	Skew_2001	Q3_2001	Var_2001	P5_2001	P1_2001	Kurt_2001	Median_2001	P25_2001	P99_2001	P95_2001	P90_2001	P10_2001
Mean_2000	-0.32	-0.21	0.00	-0.04	-0.08	-0.10	-0.14	-0.01	-0.06	-0.02	-0.02	-0.02	-0.07
Skew_2000	-0.05	0.03	-0.01	-0.03	-0.02	-0.01	-0.07	-0.08	-0.01	0.06	0.02	0.02	-0.01
Q3_2000	-0.12	-0.40	0.78	-0.78	-0.79	-0.77	0.09	0.24	-0.79	0.69	0.78	0.77	-0.80
Var_2000	0.12	0.40	-0.78	0.81	0.83	0.82	-0.07	-0.21	0.80	-0.75	-0.81	-0.80	0.82
P5_2000	0.10	0.38	-0.79	0.81	0.81	0.79	-0.09	-0.22	0.80	-0.74	-0.81	-0.80	0.82
P1_2000	0.12	0.37	-0.71	0.73	0.74	0.73	-0.09	-0.21	0.73	-0.67	-0.73	-0.72	0.74
Kurt_2000	0.22	0.06	0.09	0.01	0.03	0.08	0.17	0.08	-0.02	-0.01	0.04	0.06	0.00
Median_2000	-0.24	-0.26	0.16	-0.19	-0.23	-0.24	-0.07	0.04	-0.19	0.11	0.14	0.15	-0.21
P25_2000	-0.08	0.27	-0.75	0.74	0.72	0.69	-0.14	-0.25	0.73	-0.70	-0.78	-0.77	0.73
P99_2000	-0.15	-0.41	0.72	-0.77	-0.78	-0.77	0.01	0.19	-0.75	0.70	0.75	0.74	-0.77
P95_2000	-0.12	-0.40	0.78	-0.80	-0.81	-0.80	0.08	0.21	-0.80	0.74	0.79	0.79	-0.81
P90_2000	-0.11	-0.40	0.79	-0.80	-0.82	-0.80	0.10	0.21	-0.81	0.74	0.81	0.80	-0.82
P10_2000	0.05	0.35	-0.80	0.81	0.81	0.78	-0.10	-0.23	0.80	-0.74	-0.82	-0.82	0.82

Table III: Correlation Between Risk and Return Factors during 2001

Correlation Matrix/2001 Year	Mean_2001	Skew_2001	Q3_2001	Var_2001	P5_2001	P1_2001	Kurt_2001	Median_2001	P25_2001	P99_2001	P95_2001	P90_2001	P10_2001
Mean_2001	1.00	0.32	0.09	0.06	0.17	0.17	-0.01	0.48	0.19	0.04	0.02	0.04	0.15
Skew_2001	0.32	1.00	-0.42	0.39	0.46	0.54	-0.16	-0.24	0.37	-0.20	-0.36	-0.38	0.43
Q3_2001	0.09	-0.42	1.00	-0.93	-0.91	-0.88	0.10	0.40	-0.89	0.82	0.93	0.95	-0.92
Var_2001	0.06	0.39	-0.93	1.00	0.96	0.95	0.09	-0.29	0.93	-0.91	-0.96	-0.96	0.96
P5_2001	0.17	0.46	-0.91	0.96	1.00	0.95	-0.02	-0.27	0.93	-0.85	-0.93	-0.94	0.98
P1_2001	0.17	0.54	-0.88	0.95	0.95	1.00	0.07	-0.27	0.89	-0.83	-0.90	-0.90	0.93
Kurt_2001	-0.01	-0.16	0.10	0.09	-0.02	0.07	1.00	-0.02	-0.11	-0.11	0.06	0.08	-0.06
Median_2001	0.48	-0.24	0.40	-0.29	-0.27	-0.27	-0.02	1.00	-0.17	0.25	0.29	0.32	-0.27
P25_2001	0.19	0.37	-0.89	0.93	0.93	0.89	-0.11	-0.17	1.00	-0.83	-0.92	-0.92	0.95
P99_2001	0.04	-0.20	0.82	-0.91	-0.85	-0.83	-0.11	0.25	-0.83	1.00	0.89	0.88	-0.86
P95_2001	0.02	-0.36	0.93	-0.96	-0.93	-0.90	0.06	0.29	-0.92	0.89	1.00	0.98	-0.94
P90_2001	0.04	-0.38	0.95	-0.96	-0.94	-0.90	0.08	0.32	-0.92	0.88	0.98	1.00	-0.94
P10_2001	0.15	0.43	-0.92	0.96	0.98	0.93	-0.06	-0.27	0.95	-0.86	-0.94	-0.94	1.00

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