# Review of the Hybrid Finite-Discrete Element Method (FDEM)

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*Abstract*—**The hybrid finite-discrete element method (FDEM) has become a highly popular means of simulating the failure process in natural and engineered materials due to its ability to model the transition from continuum to discontinuum by means of fracture and fragmentation processes. In this article the progress and status of FDEM application in research is reviewed systematically from the following four aspects: the fundamentals of theory and methodology, fracture and contact models, graphical user interface, and applications. In addition, the authors point out the primary research directions in which FDEM is being applied to good effect.**

*Index Terms*—**Finite element method, Discrete element method, Finite-discrete element method**

## I. INTRODUCTION

ENGINEERING materials are usually considered to form a continuum when modeling their mechanical behavior. This convention arises from the earliest needs of engineering design for load bearing applications, from civil construction to machine components. Therefore, the continuum methods are most commonly used, including the boundary-element methods (BEM), the finite element methods (FEM), and the finite-difference methods (FDM). However, explicit representation of the onset and development of fracture, crack propagation under monotonic and cyclic loading, and the progression towards catastrophic failure are very important aspects of modern engineering design, particularly in view of the importance of such approaches as damage-tolerant design that has found widespread use in the aerospace industry. Incorporating fracture in the classical continuum numerical methods is not straightforward, mainly because of the scale difference between object size and fracture features, and the continuum assumptions that lie at the very core of the computational structure. The limitations of continuum methods motivated the development of the discrete element methods (DEMs) [1-8].

By combining DEM with FEM, through each discrete element being sub-divided into finite elements, the combined finite-discrete element method (FDEM) was proposed by Munjiza [9-10]. The advantage of this approach lies in the fact that deformability can be well represented by finite elements, whilst discontinuities such as cracks can be explicitly described by discrete elements. Thus FDEM can be used to simulate the both continuous and discontinuous mechanical behavior, and capture the entire loading and crack path and the gradual degradation process of materials that undergo progressive fracture.

The distinguishing feature of FDEM is its ability to simulate the transition from continuum to discontinuum that is the most crucial aspect of the fracture and fragmentation processes [11]. The key aspects of the analysis procedure in FDEM include contact detection, precise description of the interaction and friction between discrete elements, elastic deformation of the finite elements, and crack creation and propagation within and between finite elements.

The past decades have witnessed many developments in the following aspects of FDEM: the fundamental theory and method; fracture and contact mechanics modelling; graphical user interface development and flexibility; and demonstration of the method's power for specific applications. In this article these aspects of research progress are reviewed, and the outlook is summarized.

## II. PROGRESSES IN FDEM

### A. *Fundamental Theory and Method*

In the past decades important progress has been made in the fundamental theory and methodology.

Firstly, a most significant facet of FDEM capability is an excellent contact detection algorithm that allows efficient identification of all pairs of nodes in contact, and, conversely, the removal from the list of contacting pairs of those couples of nodes that previously were considered to be in close contact, but have moved sufficiently far apart to be removed from the list. In the original FDEM code, the so-called "No Binary Search" (NBS) contact detection algorithm [9, 12] was implemented (Munjiza, [9-10]). The NBS algorithm remains the most efficient and highly developed contact detection algorithms till now, since unlike any algorithm based on binary for which the total CPU time for detecting all contacting couples scales as $N \ln N$ with the number of discrete entities $N$, the Munjiza-NBS algorithm is a linear function $\sim N$ of the total number of discrete elements. In the following years, many researchers focused their efforts on the development of this basic idea based on the original codes. For example, a prescriptive procedure to arrive at a combination of input parameters for the newly developed Y-Geo FDEM code was developed designed to be used in

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Manuscript received April 2nd, 2016; revised April 20, 2016. This work was partially supported by EU FP7 project iSTRESS (604646) as well as by EPSRC via grants EP/I020691, EP/G004676 and EP/H003215.

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combination with laboratory-scale simulations that provided data for the material to be simulated. Method developments were responsible for further progress towards obtaining reliable and consistent results using FDEM approach [13]. A hierarchical multi-scale approach was proposed to simulate the mechanical behavior of granular materials [14]. A rigorous procedure for hierarchical coupling of FEM and DEM was developed and employed by Guo and Zhao [14]. In this approach, FEM was first used to for geometric discretization of the entire macroscopic domain geometry into an FEM mesh. Following this, a DEM assembly was embedded at each Gauss integration point. Each such instance retained memory of its loading history and served as the representative volume element (RVE). A coupled FDEM approach was devised by Azadi et al. [15] to enable the prediction of effective thermal properties of paint (pigmented coating layers). Coating layers of relevant thicknesses comparable with a few pigment particle diameters were considered, and their effective thermal conductivity evaluated in view of the application of the model to simulating drying processes. The influence of particle size and size distribution, the morphology of pigment particles, and the porosity of coating on the drying behavior was considered. This allowed the quantification of the effects of pigment and binder properties on the overall thermal conductivity of the coating layers containing pores.

To simulate the rock fracturing process under the propellant hydraulic pressure, Yan et al. devised a coupled hydro-mechanical approach that used FDEM to construct a numerical model of hydro-fracturing that may lead to convoluted geometries of fracture. An algorithm was proposed for dynamic updating of the description of hydraulic fracture network that maintained and updated a record of element and node connectivity within arbitrarily complex networks of blocks and cracks between them. On the basis of these algorithmic developments the authors thus put forward a new combined FDEM numerical code (Y-flow) for the simulation of hydraulic fracturing [16].

Lisjak et al. proposed a new acoustic emissions modeling technique based on the combined FDEM, a numerical tool for simulating material failure by means of explicitly modeling crack initiation and propagation in the modeling domain. The effect of radiated seismic energy, stress wave propagation and the explicit time integration scheme of the solver can be directly captured in the modeling. On the basis of the monitoring of internal variables close to propagating cracks [17], a newly developed algorithm is presented in FDEM model to extract the quasi-dynamic seismic information.

FDEM-DFN (discrete fracture network) modeling approach was put forward with the purpose of studying the formation of a failure surfaces at open pit slopes [18]. This approach was also used to study the step-path failure development due to block caving to address one of the most challenging tasks in mining rock engineering, namely, the interaction between block cave mining and a large overlying open pit. A multi-scaling method was proposed based on FEM-DEM coupling to simulate the shearing between two parallel plates with entrained solid particles [19]. This problem is of fundamental interest from the point of view of contact deformation, as well as three-body tribology, and has obvious application in seismology. FDEM code called Y-Geo was developed for geomechanics applications by Mahabadi et al. [20] on the basis of the original FDEM algorithm. The features of Y-Geo include: (1) the incoproration of the Mohr-Coulomb failure criterion; (2) a dissipative impact model; (3) the mapping function of materials according to exact representation of heterogeneous models; (4) a quasi-static friction law; (5) the shear strength criterion for rock joint; (6) the initialization routine for the in situ stress; and (7) a tool to incorporate material heterogeneity and transverse isotropy [21]. Since the cracks extend along the cell boundary in FDEM, the propagation of crack has mesh-dependency problems. In order to obtain a good approximation of the crack propagation shape, dense initial mesh is needed. To tackle this problem, an adaptive FDEM method with local dynamic unit splitting was proposed with the purpose of overcoming the mesh-dependency of crack morphology [22]. In order to improve the computational efficiency of FDEM, a set of strategies for parallelizing the original serial algorithms was proposed by Yan et al. [22]. This involved determining the hot zones of the serial program, then parallelizing the hot zones as far as possible, and using as many private variables as possible to circumvent competition.

### B. Fracture and Contact Models

A strain-based cohesive crack model has been proposed in the original FDEM code [9, 11]. In this model, the typical tensional stress-strain curve of rock material be segmented into two sections: (a) the strain hardening before reaching the peak deformation (the constitutive law in FDEM); (b) the strain softening (the stress decreases according to an increase in strain). In order to model the interaction between the geosynthetic sheet and the particles of soil, a contact law specifically related to the friction behaviour of the composite soil-geosynthetic was proposed by Villard et al. [23]. The tangential contact force implementation of 2D FEM/DEM was verified by the analytical solutions for ‘‘block sliding’’ [24] in order to address sliding friction in FEM and rolling friction in DEM. Based on original FDEM coupling analysis method, a new model for blast simulation is implemented by Yan et al. [25]. The model considers increasment of the volume occupied by the gas and stable decrease in gas pressure with the expansion of cracks due to the blast gas embedding, and also takes into account the action of gas on the fissures linked at the explosion chamber. The limitation is overcome in the existing blasting model of FDEM, where the pressure is only applied on the surface rock of blasting chamber without considering action of the embedded gas on the newly generated cracks. Mean while, a novel search algorithm of fracture network is proposed by compiling a simple recursive function for the search of complex fracture network; and a very simple method is used to handle the complex problem. A new potential contact force calculation method is proposed using a unified calibration by Yan et al. [26]. The method redefines the potential function as the potential of a point in triangular proportional to the shortest distance from the point to the three sides of the triangular.

### C. Graphical User Interface

Numerical modeling of the discontinuous materials is very popular in recent decades. The FDEM is a state-of-the-art numerical modeling approach pioneered in the middle 1990s.

The two-dimensional FDEM research code named Y2D was presented by Munjiza in 2004 [9]. The main limitation of Y2D lies in two aspects: (a) the inability of dealing with heterogeneous media; (b) all pre-processing has to be finished directly in an ASCII input file without any graphical user interface. Mahabadi et al. presented the first GUI and pre- processor named Y-GUI, which is developed for Y2D. In addition, Y-GUI carries out a new algorithm allowing for the use of heterogeneous materials. Y-GUI V2.0 can carry out more features. The process in setting up input files for the standard format permitted in the FDEM has been greatly simplified [27].

On the basis of image processing method and FDEM, an innovative method is developed by Yan et al. [28]. This provides investigation on mechanical feature and failure mechanism of heterogeneous material that the interface of different materials can be identified the from a cross-section image.

### D. Applications

Applications of FDEM are another main aspect in the abundant literatures, which are as follows:

Single-hole blast-induced damage in a granitic outcrop has been assessed through both controlled experiments and numerical simulations with the FDEM [29]. Smoljanovic et al. presented the non-linear material and geometric analysis of dry stone masonry structural response under monotonic, cyclic and seismic loads using a combined finite-discrete element method (FDEM) [30]. Ma et al. performed numerical modeling of triaxial compression tests [31] using a combined FEM and discrete-element method (FDEM). In the framework of FDEM, the method proposed by Bangash et al. [32] was used to analyze the transient dynamics behavior of the pre-failure and post-failure in the einforced concrete beam structures. In the beam element, which is the basis of the method, the reinforcement was taken as simplified stiffness matrix. However, crack initiation and propagation through the structure cannot be monitored according this concept. The new numerical model of reinforcing bar was added in the FDEM in order to have the ability to analyze and predict the collapse of the reinforced concrete structures. Within the basic frame of the new approach constant strain triangular finite elements in the concrete structure were meshed while the linear one-dimensional elements implemented in the finite elements of concrete were employed to model the reinforcing bars. By means of the contact elements between the concrete and steel, which are embedded in a finite element mesh, the approach can analyze the material nonlinearity of fracture, fragmentation and cyclic behavior during dynamic loading. In order to realistically describe the interactions between the concrete and reinforcement, the combined FDEM codes were added several numerical algorithms, which are (1) the reinforcing bars embedded models; (2) the interaction between the reinforcement and concrete by means of steel strain–slip relation; (3) the slip of reinforcing bar influenced by the adjacent cracks; (4) local slip near the crack plane of the reinforcing bar while the high plastic deformation happening in bars under reversed cyclic loading; (5) yield stress reduction of the steel influenced by the curvature of the reinforcing bar; (6) the concrete and steel cyclic behavior added in the existing crack models, which is an important mechanism for energy loss in post-fracture response of the concrete structures. The crack initiation and propagation, inertial effects due to motion, contact impact and state of rest, energy dissipation by nonlinear effects, inertial effects due to motion can be analyzed in this model. In the dynamic conditions, the realistic description of cracking and collapse prediction in reinforced concrete structures were proposed in FDEM by Zivaljic et al. [33-34]. Zivaljic et al. presented and discussed some computational results in reinforced concrete structures by FDEM [35]. Lisjak et al. presents the effectiveness of using a hybrid finite-discrete element method (FDEM) code in a problem that related to the excavation of underground openings in brittle rock formations [36]. In the reference [37], a finite-discrete element combined method was applied to study crack initiation and propagation on a pre-existing plane interface in a Brazilian disc.

## III. OUTLOOKS OF FDEM

There has been considerable progress in the field of mechanical analysis of rock and concrete in the last decades. However, the existing numerical methods show their disadvantages in the analysis of mechanical behavior of the materials of rock and concrete because of the complexity of their geometry, component and constitutive relationship. A unified analysis modeling from continuum to discontinuum, including continuous modeling (FEM, FDM) and discontinuous modeling (DEM), will be the key research direction. This must promote the further development of FDEM, which will be as follows.

### A. Fundamental Theory, Method and Features

High-efficiency contact detection algorithm is the crux for improving the computational efficiency of FDEM. The effective solution of large-scale FDEM problems relies on a robust contact detection algorithm that dependences on the problems, such as dense packing and loose packing problems, quasi-static problems (where relative motion of individual bodies is restricted) and dynamic problems [38-42]. Improving the contact detection algorithm and increasing efficiency and veracity is the important subject in FDEM.

Fracture mechanical models have their own complexity and diversity because of the complexity of every material. In the existing method of FDEM, the unique fracture mechanical model is the double sections model. If the FDEM will be applied to more fields, such as Mining Engineering, Petroleum Exploration, Materials Processing Engineering, Earthquake Engineering Geology, Aerospace Composite Materials Engineering, Ice Engineering, Warehousing Project, Fluid-Solid Coupling, et al., more abundant fracture models is the key subject.

Research on the coupled multiphase FDEM is the important research direction of continuum-discontinuum models. The seepage in rock mass is the key problem in the deformation and stability analysis of engineering slopes, the basis of structures and underground structures. On the other hand, the interaction of thermal- hydraulic-mechanical has aroused the attention of many researchers in the field of underground nuclear waste-storage containers. Thus the research on the coupled multiphase FDEM will be one of the hot issues in the simulation of rock mechanics and engineering.

### B. Software and Graphical User Interface

FDEM is a robust and efficient two-dimensional research code that enables to modeling continuum-discontinuum behavior. However, the first challenge is the creation and verification of input files. Graphical user interface is not available and the entire input file has to be typed in an ASCII text editor. Although there are some developments in the graphical user interface, the practicability of the software for FDEM need more research.

### C. Three Dimensional Problems

FDEM have been employed in the area of rock mechanics and civil engineering to simulate complicated industrial problems, such as deep mining techniques (tunneling [43], pillar strength [44], etc.), rock blasting [45], block caving, seismic waves [46], packing problems, coastal protection [47-48], dam stability [49], rock slope stability [50], masonry wall stability [51], and rock mass strength characterization problems [44, 52-53]. For most of these were performed in a 2D circumstance. In the exiting FDEM, there are many problems to be studied, such as three dimensional fracture models, three dimensional contact detection algorithm, three dimensional graphical user interfaces, et al.

## IV. CONCLUSION

In the past decades, there are many developments in the field of FDEM because of its capacity of modeling continuum-discontinuum behavior. Even so, the further development of FDEM is necessary for its wider range of applications.

## ACKNOWLEDGMENT

Alexander M. Korsunsky acknowledges funding received for MBLEM laboratory in the University of Oxford through EU FP7 project iSTRESS (604646), and access to the facilities at the Research Complex at Harwell (RCaH), under the Centre for In situ Processing Studies (CIPS). The authors have declared no conflict of interest.

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