

Free Vibration Analysis of Elastic Orthotropic Rectangular Inclined Damped Highway Supported by Pasternak Foundation under Moving Aerodynamic Automobile

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Abstract— Various plates and plate like structures are often subjected to moving loads, such as aerodynamic automobiles. In this paper automobile highway was modelled as an elastic orthotropic rectangular plate. The effects of damping and drag force were put into consideration. The fourth order differential equation governing such plates resting on Pasternak foundation was expressed as first order differential equation. The equation was changed to its algebraic form using finite difference algorithm, then solved with the aid of MATLAB in conjunction with a computer program. Simple supported conditions were used. The effects of damping drag force, foundation and other physical phenomena were investigated and the results obtained are consistent with the ones existing in literature.

Index Terms— aerodynamics, automobile, inclined damped highway, Pasternak foundation, free vibration, orthotropic.

I INTRODUCTION

THERE has been great concern recently about the safety of the structures on which loads move. This is, partly, because of advances in all branches of transportation characterized by increasing weight and high speed of these moving loads.[2,8,9] Modern structure, like automobile highways and bridges, are therefore been subjected to vibration and dynamic stress more than ever before.[1,2,8] The importance of moving load problem is manifested in numerous applications in the field of Engineering, applied Mathematics, applied Physics and transportation.[4,9] Most recent developments and results can be found in some researchers works.[6,7] In this work we attempted to carry out a free vibration analysis of an aerodynamic highway, modeled as an orthotropic plate, on an elastic foundation under the influence of moving aerodynamic automobile. Aerodynamic drag is a force that restricts the forward velocity of an automobile. It also impacts the fuel economy of automobile.[10] In application some plates or plate like structures are not isotropic but orthotropic, which put into consideration the possibility of such plates not being uniform in all

directions[3] Viscous damping was also considered, which usually causes reduction or decay of motion.[3,5] Deflection profile of plates depends on the size of damping coefficient[3,5]. This work focused on free vibration of orthotropic damped plate with effect of drag force

II FORMULATION OF PROBLEM

A. Assumptions

In developing the governing partial differential equation the following assumptions were made:

- The small strain in the system is still governed by Hook's law.
- The plate is resting on elastic foundation.
- The load is taken to be a distributed time load.
- There is no deformation in the middle of the plate, i.e the plate remains the same before and after bending
- The damping and drag coefficient values respectively are taken to be very small
- $W(x,y,t) = W$ = deflection of the Mindlin plate
- Uniform gravitational field, g .
- m = constant mass moving on the plate

B. Governing Equation

The equation governing the vibration of damped simply supported orthotropic inclined plate resting on Pasternak foundation subject to a moving load can be written as follows[1,2,3]:

$$\alpha_1 \frac{\partial^4 w}{\partial x^4} + 2\alpha_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_3 \frac{\partial^4 w}{\partial y^4} + Kw + m \frac{\partial^2 w}{\partial t^2} + 2m\gamma C_d \frac{\partial w}{\partial t} + G_1 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = \frac{m}{r} \left[g(\cos \theta) - \left(\frac{\partial^2 w}{\partial t^2} + u \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial x} \right) + u^2 \frac{\partial^2 w}{\partial x \partial t} + u^2 \frac{\partial}{\partial x} \left(\frac{\partial w}{\partial x} \right) \right) \right] \left[H \left(x - vt + \frac{r}{2} \right) - H \left(x - vt - \frac{r}{2} \right) \right] \delta(y - y_1) \quad (1)$$

where,

$w = w(x,y,t)$ is the deflection of the plate.

t = time in seconds

E = Young's modulus

m = mass density per unit area

H = thickness of plate

r = length of the load

$g(\cos \theta)$, = acceleration due to gravity of the load on an inclined plane

v = velocity

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K, G_1 = foundation stiffness

γ = viscous damping coefficient

C_d = Drag coefficient value

α_1 = flexural rigidity in the x direction

α_3 = flexural rigidity in the y direction

α_2 = effective torsional rigidity

$H(x)$ = Heaviside step function

v = velocity

$\delta(x)$ = dirac delta function

θ = angle of inclination of the plate to the horizontal

For complete formulation of the problem, a simply supported rectangular plate is considered. The following initial and boundary conditions and have been considered as follows:

$$w(x, y, 0) = w_t(x, y, 0) = 0$$

$$w(0, y, t) = w(a, y, t) = w_{xx}(0, y, t)$$

$$= w_{xx}(a, y, t) = 0$$

$$w(x, 0, t) = w(x, b, t) = w_{yy}(x, 0, t)$$

$$= w_{yy}(x, b, t) = 0$$

III PROBLEM SOLUTION

The dynamic equilibrium equations which governs behaviour of damped orthotropic rectangular plate supported by Pasternak foundation is gotten by neglecting the terms representing the applied force in equation (1). The right hand side of equation (1) represents the applied load, $P(x,y,t)$. For free vibration, $P(x,y,t) = 0$. Therefore equation (1) becomes

$$\alpha_1 \frac{\partial^4 w}{\partial x^4} + 2\alpha_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_3 \frac{\partial^4 w}{\partial y^4} + Kw + m \frac{\partial^2 w}{\partial t^2} + 2m\gamma C_d \frac{\partial w}{\partial t} + G_1 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad (2)$$

$$\frac{\partial^3 w}{\partial x^3} = D_x, \quad \frac{\partial^3 w}{\partial y^3} = D_y, \quad \frac{\partial^3 w}{\partial t^3} = D_t, \quad \frac{\partial^3 w}{\partial x \partial y^2} = D_{xy}, \quad \frac{\partial w}{\partial x} = d_x, \quad \frac{\partial w}{\partial y} = d_y, \quad \frac{\partial w}{\partial t} = d_t \quad (3)$$

Equation (2) can now be written as follows

$$\alpha_1 \frac{\partial D_x}{\partial x} + 2\alpha_2 \frac{\partial D_{xy}}{\partial x} + \alpha_3 \frac{\partial D_y}{\partial y} + Kw + m \frac{\partial d_t}{\partial t} + 2m\gamma C_d \frac{\partial w}{\partial t} + G_1 \left(\frac{\partial d_x}{\partial x} + \frac{\partial d_y}{\partial y} \right) = 0 \quad (4)$$

A Finite Difference Algorithm

Equations (4) is solved using a numerical method based on the finite difference algorithm. This equation is to be transformed into its equivalent algebraic form. The finite difference definition of first order partial derivative of a function $E(x,y,t)$ with respect to x, y and t respectively are as follows:[1,3]

$$\frac{\partial E}{\partial t} = \frac{1}{4r} (E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} + E_{i,j+1}^{k+1} + E_{i,j}^{k+1} - E_{i+1,j+1}^k - E_{i+1,j}^k - E_{i,j+1}^k - E_{i,j}^k) \quad (5)$$

$$\frac{\partial E}{\partial x} = \frac{1}{4h} (E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} - E_{i,j+1}^{k+1} - E_{i,j}^{k+1} + E_{i+1,j+1}^k + E_{i+1,j}^k - E_{i,j+1}^k - E_{i,j}^k) \quad (6)$$

$$\frac{\partial E}{\partial y} = \frac{1}{4k} (E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} - E_{i,j+1}^{k+1} - E_{i,j}^{k+1} + E_{i+1,j+1}^k + E_{i+1,j}^k - E_{i,j+1}^k - E_{i,j}^k) \quad (7)$$

where E is the function value of the centre of a grid, which is well approximated by the average of its values at the grid nodes [5]

$$E\left(x + \frac{h}{2}, y + \frac{k}{2}, t + \frac{r}{2}\right) = \frac{1}{8} (E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} + E_{i,j+1}^{k+1} + E_{i,j}^{k+1} + E_{i+1,j+1}^k + E_{i+1,j}^k + E_{i,j+1}^k + E_{i,j}^k) \quad (8)$$

Hence,

$$\frac{\partial D_x}{\partial x} = \frac{1}{4h} (D_{x_{i+1,j+1}}^{k+1} + D_{x_{i+1,j}}^{k+1} + D_{x_{i,j+1}}^{k+1} + D_{x_{i,j}}^{k+1} - D_{x_{i+1,j+1}}^k - D_{x_{i+1,j}}^k - D_{x_{i,j+1}}^k - D_{x_{i,j}}^k) \quad (9)$$

$$\frac{\partial D_y}{\partial y} = \frac{1}{4k} (D_{y_{i+1,j+1}}^{k+1} + D_{y_{i+1,j}}^{k+1} + D_{y_{i,j+1}}^{k+1} + D_{y_{i,j}}^{k+1} - D_{y_{i+1,j+1}}^k - D_{y_{i+1,j}}^k - D_{y_{i,j+1}}^k - D_{y_{i,j}}^k) \quad (10)$$

$$\frac{\partial D_t}{\partial t} = \frac{1}{4r} (D_{t_{i+1,j+1}}^{k+1} + D_{t_{i+1,j}}^{k+1} + D_{t_{i,j+1}}^{k+1} + D_{t_{i,j}}^{k+1} - D_{t_{i+1,j+1}}^k - D_{t_{i+1,j}}^k - D_{t_{i,j+1}}^k - D_{t_{i,j}}^k) \quad (11)$$

$$\frac{\partial d_x}{\partial x} = \frac{1}{4r} (d_{x_{i+1,j+1}}^{k+1} + d_{x_{i+1,j}}^{k+1} + d_{x_{i,j+1}}^{k+1} + d_{x_{i,j}}^{k+1} - d_{x_{i+1,j+1}}^k - d_{x_{i+1,j}}^k - d_{x_{i,j+1}}^k - d_{x_{i,j}}^k) \quad (12)$$

$$\frac{\partial d_y}{\partial y} = \frac{1}{4r} (d_{y_{i+1,j+1}}^{k+1} + d_{y_{i+1,j}}^{k+1} + d_{y_{i,j+1}}^{k+1} + d_{y_{i,j}}^{k+1} - d_{y_{i+1,j+1}}^k - d_{y_{i+1,j}}^k - d_{y_{i,j+1}}^k - d_{y_{i,j}}^k) \quad (13)$$

$$\frac{\partial d_t}{\partial t} = \frac{1}{4r} (d_{t_{i+1,j+1}}^{k+1} + d_{t_{i+1,j}}^{k+1} + d_{t_{i,j+1}}^{k+1} + d_{t_{i,j}}^{k+1} - d_{t_{i+1,j+1}}^k - d_{t_{i+1,j}}^k - d_{t_{i,j+1}}^k - d_{t_{i,j}}^k) \quad (14)$$

$$\frac{\partial d_x}{\partial x} = \frac{1}{4h} (d_{x_{i+1,j+1}}^{k+1} + d_{x_{i+1,j}}^{k+1} + d_{x_{i,j+1}}^{k+1} + d_{x_{i,j}}^{k+1} - d_{x_{i+1,j+1}}^k - d_{x_{i+1,j}}^k - d_{x_{i,j+1}}^k - d_{x_{i,j}}^k) \quad (15)$$

$$w\left(x + \frac{h}{2}, y + \frac{k}{2}, t + \frac{r}{2}\right) = \frac{1}{8} (w_{i+1,j+1}^{k+1} + w_{i+1,j}^{k+1} + w_{i,j+1}^{k+1} + w_{i,j}^{k+1} + w_{i+1,j+1}^k + w_{i+1,j}^k + w_{i,j+1}^k + w_{i,j}^k) \quad (16)$$

The evaluation of the unknown variables in a particular time (t_i) for the above equation (16) can be calculated using the data at both immediate previous t and y steps. The unknown variables in equation (16) then doubles in number. Applying the finite difference approximation of the boundary conditions[1,9] yields the number of unknowns to be solved for. The resulting algebraic equation was therefore solved for the unknowns. With different values of the parameters the vibration behaviour of the plate was observed and reported in the next section.

IV RESULTS AND DISCUSSION

For this work the coupled differential equation (4) was solved using the central differential formula of finite Difference method. The following values of the various parameters were used:

$$\alpha_1 = 0.297, \alpha_2 = 0.21, \alpha_3 = 0.69, K = 0, 100, 200.$$

$$\gamma = 0, 0.554, 0.715$$

respectively. $C_d = 0.2, 0.65, 0.98$ respectively. Clearly, for free vibration, the effect of the angle of inclination of the highway is not relevant. The aerodynamic drag's Frictional force increases with automobile speed. The deflection profile depends largely on the value the damping coefficient. There is a positive correlation between the velocity of the aerodynamic automobile and the deflection of the plate under consideration. On the other hand, it was

observed that the foundation modulus K , is inversely proportional to the maximum amplitude. We recorded the maximum deflection when damping is lowest. The paper set out to investigate free vibration of orthotropic damped rectangular, considering the effect of aerodynamic drag. Aerodynamic drag and damping are very significant in the free vibration of automobile highway. Also the Pasternak foundation stabilised the deflection. It was observed that the maximum amplitude of the deflection of the plate resting on Pasternak foundation is lower than when it rests on non-Pasternak foundation.

V CONCLUSION

The paper set out to investigate free vibration of orthotropic damped rectangular, considering the effect of aerodynamic drag. Aerodynamic drag and damping are very significant in the free vibration of automobile highway. Also the Pasternak foundation stabilised the deflection. It was observed that the maximum amplitude of the deflection of the plat resting on Pasternak foundation is lower than when it rests on non-Pasternak foundation.

Appendix: Equations

$$\alpha_1 \frac{\partial^4 w}{\partial x^4} + 2\alpha_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_3 \frac{\partial^4 w}{\partial y^4} + Kw + m \frac{\partial^2 w}{\partial t^2} + 2m\gamma C_d \frac{\partial w}{\partial t} + G_1 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = \frac{m}{r} \left[g(\cos \theta) - \left(\frac{\partial^2 w}{\partial t^2} + u \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial x} \right) + u \frac{\partial^2 w}{\partial x \partial t} + u^2 \frac{\partial}{\partial x} \left(\frac{\partial w}{\partial x} \right) \right) \right] \left[H \left(x - vt + \frac{r}{2} \right) - H \left(x - vt - \frac{r}{2} \right) \right] \partial(y - y_1) \quad (1)$$

$$\alpha_1 \frac{\partial^4 w}{\partial x^4} + 2\alpha_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_3 \frac{\partial^4 w}{\partial y^4} + Kw + m \frac{\partial^2 w}{\partial t^2} + 2m\gamma C_d \frac{\partial w}{\partial t} + G_1 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad (2)$$

$$\frac{\partial^3 w}{\partial x^3} = D_x, \quad \frac{\partial^3 w}{\partial y^3} = D_y, \quad \frac{\partial^3 w}{\partial t^3} = D_t, \quad \frac{\partial^3 w}{\partial x \partial y^2} = D_{xy}, \quad \frac{\partial w}{\partial x} = d_x, \quad \frac{\partial w}{\partial y} = d_y, \quad \frac{\partial w}{\partial t} = d_t \quad (3)$$

$$\alpha_1 \frac{\partial D_x}{\partial x} + 2\alpha_2 \frac{\partial D_{xy}}{\partial x} + \alpha_3 \frac{\partial D_y}{\partial y} + Kw + m \frac{\partial d_t}{\partial t} + 2m\gamma C_d \frac{\partial w}{\partial t} + G_1 \left(\frac{\partial d_x}{\partial x} + \frac{\partial d_y}{\partial y} \right) = 0 \quad (4)$$

$$\frac{\partial E}{\partial t} = \frac{1}{4r} \left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} + E_{i,j+1}^{k+1} + E_{i,j}^{k+1} - E_{i+1,j+1}^k - E_{i+1,j}^k - E_{i,j+1}^k - E_{i,j}^k \right) \quad (5)$$

$$\frac{\partial E}{\partial x} = \frac{1}{4h} \left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} - E_{i,j+1}^{k+1} - E_{i,j}^{k+1} + E_{i+1,j+1}^k + E_{i+1,j}^k - E_{i,j+1}^k - E_{i,j}^k \right) \quad (6)$$

$$\frac{\partial E}{\partial y} = \frac{1}{4k} \left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} - E_{i,j+1}^{k+1} - E_{i,j}^{k+1} + E_{i+1,j+1}^k + E_{i+1,j}^k - E_{i,j+1}^k - E_{i,j}^k \right) \quad (7)$$

$$E \left(x + \frac{h}{2}, y + \frac{k}{2}, t + \frac{r}{2} \right) =$$

$$\frac{1}{8} \left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} + E_{i,j+1}^{k+1} + E_{i,j}^{k+1} + E_{i+1,j+1}^k + E_{i+1,j}^k + E_{i,j+1}^k + E_{i,j}^k \right) \quad (8)$$

$$\frac{\partial D_x}{\partial x} = \frac{1}{4h} \left(D_{x_{i+1,j+1}}^{k+1} + D_{x_{i+1,j}}^{k+1} + D_{x_{i,j+1}}^{k+1} + D_{x_{i,j}}^{k+1} - D_{x_{i+1,j+1}}^k - D_{x_{i+1,j}}^k - D_{x_{i,j+1}}^k - D_{x_{i,j}}^k \right) \quad (9)$$

$$\frac{\partial D_y}{\partial y} = \frac{1}{4k} \left(D_{y_{i+1,j+1}}^{k+1} + D_{y_{i+1,j}}^{k+1} + D_{y_{i,j+1}}^{k+1} + D_{y_{i,j}}^{k+1} - D_{y_{i+1,j+1}}^k - D_{y_{i+1,j}}^k - D_{y_{i,j+1}}^k - D_{y_{i,j}}^k \right) \quad (10)$$

$$\frac{\partial D_t}{\partial t} = \frac{1}{4r} \left(D_{t_{i+1,j+1}}^{k+1} + D_{t_{i+1,j}}^{k+1} + D_{t_{i,j+1}}^{k+1} + D_{t_{i,j}}^{k+1} - D_{t_{i+1,j+1}}^k - D_{t_{i+1,j}}^k - D_{t_{i,j+1}}^k - D_{t_{i,j}}^k \right) \quad (11)$$

$$\frac{\partial d_x}{\partial t} = \frac{1}{4r} \left(d_{t_{i+1,j+1}}^{k+1} + d_{t_{i+1,j}}^{k+1} + d_{t_{i,j+1}}^{k+1} + d_{t_{i,j}}^{k+1} - d_{t_{i+1,j+1}}^k - d_{t_{i+1,j}}^k - d_{t_{i,j+1}}^k - d_{t_{i,j}}^k \right) \quad (12)$$

$$\frac{\partial w_x}{\partial t} = \frac{1}{4r} \left(w_{t_{i+1,j+1}}^{k+1} + w_{t_{i+1,j}}^{k+1} + w_{t_{i,j+1}}^{k+1} + w_{t_{i,j}}^{k+1} - w_{t_{i+1,j+1}}^k - w_{t_{i+1,j}}^k - w_{t_{i,j+1}}^k - w_{t_{i,j}}^k \right) \quad (13)$$

$$\frac{\partial d_y}{\partial y} = \frac{1}{4k} \left(d_{y_{i+1,j+1}}^{k+1} + d_{y_{i+1,j}}^{k+1} + d_{y_{i,j+1}}^{k+1} + d_{y_{i,j}}^{k+1} - d_{y_{i+1,j+1}}^k - d_{y_{i+1,j}}^k - d_{y_{i,j+1}}^k - d_{y_{i,j}}^k \right) \quad (14)$$

$$\frac{\partial d_x}{\partial x} = \frac{1}{4h} \left(d_{x_{i+1,j+1}}^{k+1} + d_{x_{i+1,j}}^{k+1} + d_{x_{i,j+1}}^{k+1} + d_{x_{i,j}}^{k+1} - d_{x_{i+1,j+1}}^k - d_{x_{i+1,j}}^k - d_{x_{i,j+1}}^k - d_{x_{i,j}}^k \right) \quad (15)$$

$$w \left(x + \frac{h}{2}, y + \frac{k}{2}, t + \frac{r}{2} \right) = \frac{1}{8} \left(w_{i+1,j+1}^{k+1} + w_{i+1,j}^{k+1} + w_{i,j+1}^{k+1} + w_{i,j}^{k+1} + w_{i+1,j+1}^k + w_{i+1,j}^k + w_{i,j+1}^k + w_{i,j}^k \right) \quad (16)$$

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