Stability of Perforated Plates with an In-plane Pre-load on Central Cutout

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Abstract — **The stability of perforated plates with an in-plane pre-load at central circular opening were studied. Parametric investigations were conducted by varying the diameter of opening and the magnitude of in-plane force. The effect of aspect ratio (edge-length to thickness) was examined. Three sets of numerical models were prepared. First, the plate structures without any openings (pristine) were simulated for critical buckling loads. Results were verified with theoretical closed form solutions. Second, circular openings were introduced on the same models and simulations were repeated. Finally, all models were simulated with in-plane pre-loads at cutouts. Plots representing Eigen values versus plate slenderness ratio were presented. Critical stresses for stability were calculated in relation to applied in-plane load at cutouts. Finally, a design equation for critical stress values was developed for a perforated plate.**

Keywords—**Buckling, Perforated plate, ABAQUS, Simply supported, 316L.**

Notation:

FE —Finite Element
LPS — Local Positioning System
SEL — Shell Edge Load
σ_{cr} — Critical stress for instability
σ_{cr}^{p}— Critical stress for instability with pre-load
σ_x — Normal stress parallel to x axis
p_x — Distributed load parallel to x axis
E — Elastic Modulus
ϑ — Poisson's ratio
t — Thickness of plate
K — Plate buckling coefficient
C — Plate buckling coefficient for a perforated plate
b — Width of plate
a — Length of plate
φ — Aspect ratio of plate (a/b)
S — Slenderness ratio of plate (a/t)
m — Number of Half-waves
p_{cr} — Critical unit compressive force
d — Diameter of circular opening
p_a — Total force applied
A_{sel} — Applied Shell Edge Load
f — Multiplication factor

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I. INTRODUCTION

The stability of structures is an important area in civil, mechanical, marine, aerospace and many other fields of engineering. Therefore structural stability problems can manifest in a myriad of ways. The field of solid mechanics examines many classic problems on stability criterion, and there is an extensive literature available on fundamental problems in applied mechanics [1], [2]. However, solutions to problems with varying boundary conditions and various possibilities of loads is a challenge, where there are many gaps in knowledge. There are many facets to engineering problems that require further study; one such area is in multi-loading scenerios involving perforated plates. In this paper, a plate without any cutout and with a cutout will be referred to as pristine and perforated.

Most published information is largely on pristine plates, whereas perforated plates are still an area requiring further research, especially under complex loading scenerios. Most literature on perforated plates pertain to loads subjected to pure uniaxial, biaxial or shear loading ([3]-[4]). However, problems involving how pre-loads on a centrally located opening affect stability have not been given thorough attention which the current investigations focus upon. The critical load under edge compression is computed for specific dimensions of a perforated plate. Investigations are limited to elastic buckling cases with intention for future research to extend to inelastic problems, with critical instability stresses examined for various slenderness ratios of plates.

II. PROBLEM DESCRIPTION AND JUSTIFICATION OF RESEARCH

A schematic of a perforated rectangular perforated plate lateral and longitudinal dimensions of $203.2 \times 254mm^2$ and a circular cutout at its centre is shown in Fig 1. Its critical buckling stress diminishes when this plate is subjected to an inplane distributed pre-load on the circular opening. The aim is to develop an equation for critical buckling stress with a known pre-load at its circular cutout. The plate shown in Fig 2 is of a blast resistant enclosure with its front face reinforced with steel and Al foam designed to withstand low yield frontal blast loading. Such reinforcements can be extended to one or more faces depending on the requirements.

The enclosures have the ability to with-stand high-pressure loads generated in a typical explosion. It is the intention of this research to build enclosures to protect sensor (viz: strain, thermal, LPS and acceleration) and wireless waspnote units. The back face shown in Fig 3 has a circular opening that allows passing wiring harness, and also allows the fixing of several sensors, antennas and media recording equipment, as shown in Fig 4. The back face is pre-loaded due to the mounting of aforementioned units causing the stress distribution to change around the cutouts. In this regard, the work described in this paper will go some way in developing numerical solutions for critical loads for pre-loaded plates.
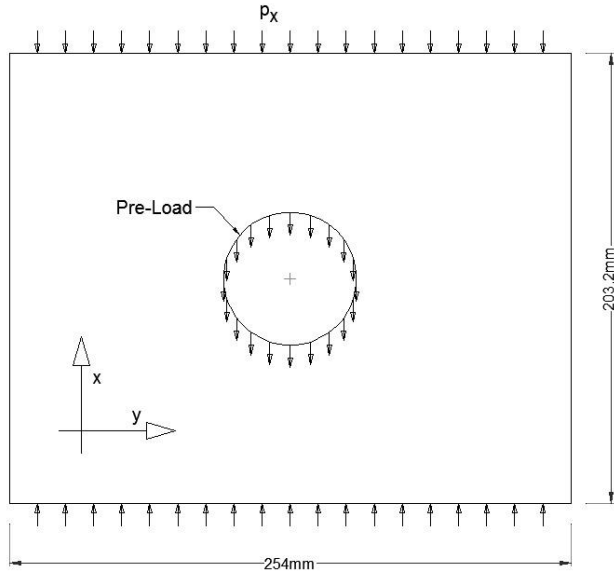

Fig 1: Problem description

Fig 2: Front and back faces of enclosure

Fig 3: Circular cutout on the back face of enclosure

Fig 4: Illustration of a pre-load to back face

III. ANALYTICAL THEORY

An analytical procedures on pristine plate with an edge-compression is presented here in Fig 5 the longitudinal (a) and lateral (b) edges are parallel to x and y axis respectively. This plate is subjected to in-plane edge-load, with a uniformly distributed load acting parallel to the x axis given by:

$$p_x = t\,\sigma_x \tag{1}$$

Theoretical critical buckling stress for this case involving uniformly distributed edge compression on two opposite edges, parallel to the y axis, is given by [2]:

$$\sigma_{cr} = K\frac{\pi^2 E}{12(1-\vartheta^2)}\left(\frac{t}{b}\right)^2 \tag{2}$$

Here K depends on ratio (φ) and number of half-waves (m) formed in the buckled configuration of plate, and is defined by:

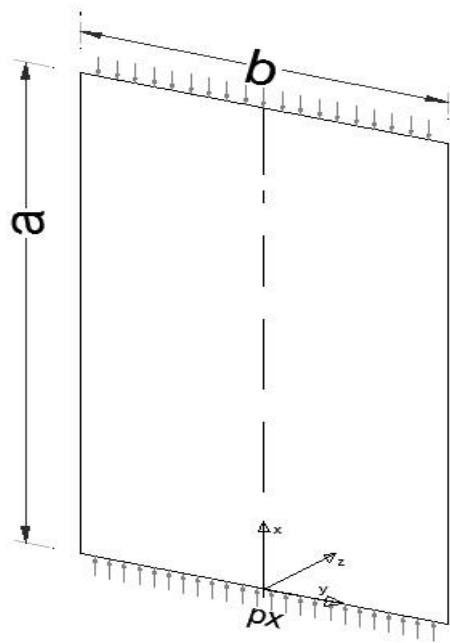$$K = \left(\frac{\varphi}{m} + \frac{m}{\varphi}\right)^2 \tag{3}$$

Fig 5: Simply supported pristine plate under pure edge compression

The product of critical stress σ_{cr} and area (tb), which gives the critical unit compressive force, expressed as:

$$p_{cr} = K \frac{\pi^2 E}{12(1-\vartheta^2)} \frac{t^3}{b}$$ (4)

Buckling will occur in one half-wave shape for short plates with small aspect ratios (φ). On or above $\varphi = \sqrt{2}$, two half-waves will be formed; for a short plate $(\varphi < \sqrt{2})$, the dependence of parameter K on the aspect ratio is shown in Fig 6. Note that the critical loads for higher values of K are larger than a half-wave mode of buckling. In the current analysis, the aspect ratio of plate investigated is 0.8; hence, only one-half-wave based displacement profile is considered in the analysis. Analytical procedures are tedious for plates with cutouts (perforated), especially, under complex set of boundary conditions and loads, ergo numerical methods are commonly used.
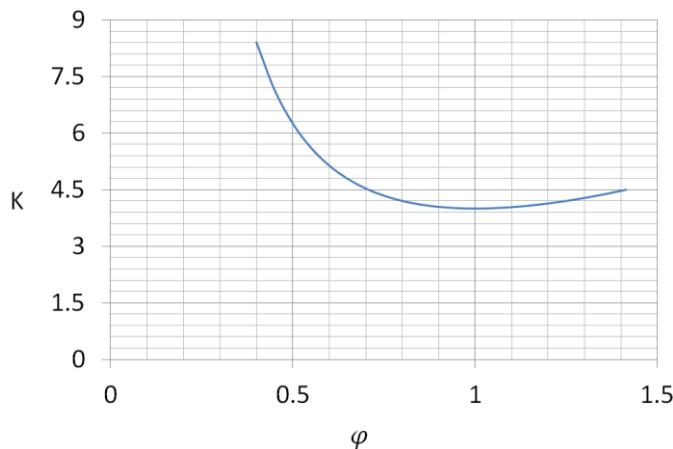

Fig 6: Relationship between K and φ

IV. PLATE WITH A PRE-LOAD AT CUTOUT

A perforated plate problem is solved after initial verification of results for a pristine plate. FE models of a pristine plate were compared with theoretical results. An in-plane load on cutout is defined that is equal to product of a multiplication factor (f) and theoretical critical load of a pristine plate. With this pre-load, the critical edge-load that will cause instability is estimated for a perforated plate.

The critical edge-load that causes buckling is dependent on multiple parameters. Note that the forgoing procedures are quite general involving consideration of just edge-compression. However, at a certain magnitude of pre-load at the cutout, there will be an appreciable depreciation in critical buckling load causing the plate to buckle out of its plane. An illustration of the buckled configuration of plate with one and two-half waves is shown in Fig 7. This phenomenon can be verified experimentally by the observation of lobes. This critical edge-load is smaller for problems with a higher pre-load.
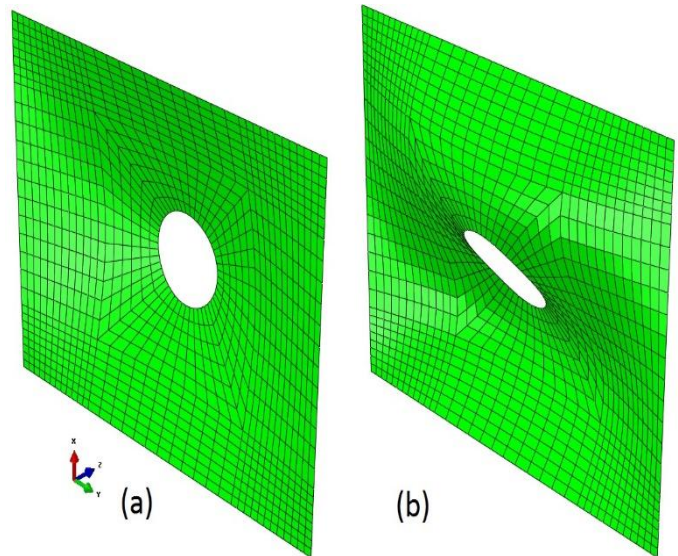

Fig 7: Buckled shapes with (a) one and (b) two-half waves

The FE solution scheme employed here involves two stages; initially a standard static load is applied in. A schematic of pre-load at central cutout is shown in Fig 8. In the second stage a buckling analysis is carried-out. Introducing a cutout in a plate leads to a reduction in p_{cr}, as discontinuities in a structure distrupt the smooth flow of stresses, causing the the direct normal stresses for plates in compression are affected. FE analyses give Eigen values which are scaling factors of initial applied load that will lead to buckling and are recorded for all configurations investigated.
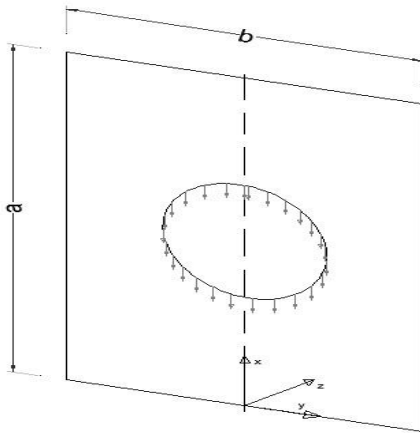
Fig 8: Illustration of a perforated plate with a pre-load at cutout

Numerical code ABAQUS was used to conduct the investigations with much of the parametric modelling accomplished using PYTHON programming language scripts. FE models with a range of slenderness ratios (40.6 to 225.7) were prepared, with thickness varying between 0.9-5 mm. The dimensions of the rectangular plates used in simulations are detailed in Table I. Yield stress versus plastic strain properties of 316L grade alloy used for FE model is in Table II, an elastic modulus of $200\,GPa$ and poisson's ratio 0.3 were assigned.

Table I: Plate thickness simulated

t (mm)	a (mm)	b (mm)	$\frac{a}{t}$
0.9	203.2	254	225.7
1.2	203.2	254	169.3
1.5	203.2	254	135.4
2.0	203.2	254	101.6
2.2	203.2	254	92.3
2.5	203.2	254	81.2
2.7	203.2	254	75.2
3.0	203.2	254	67.7
3.2	203.2	254	63.5
3.5	203.2	254	58.0
4.0	203.2	254	50.8
4.5	203.2	254	45.1
5.0	203.2	254	40.6

Table II: Plasticity data for 316L

Yield Stress (MPa)	Plastic Strain
238.4	0.0
302.8	0.0078
383.9	0.0341
447.9	0.0642
501.6	0.092
594.1	0.146
669.6	0.1998
729.8	0.2516
780.4	0.3035
819.2	0.3559

The material properties were obtained from tensile testing according to EN 10002-1. Note that only elastic properties were considered in this paper. In literature [5] materials are also assumed to deform plastically with no work hardening after yield stress. Such models also referred to as the elastic and perfectly plastic material model. FE models with two cutout diameters of 20.32 and 40.64 mm were considered. They were descritized into a grid of quadrilateral shell elements using adaptive meshing for modelling curvature around the circular cutouts. A fine mesh was used in zones around the cutout with the best mesh density gained from a grid independence study. High aspect ratios (length to width of elements) in the structured mesh were avoided to minimize inaccuracies illustrated in Fig 9. The general-purpose shell (4-NODE, S4R type ABAQUS [8]) elements that are capable of bending, axial deformation and transverse shear were used. Five integration points were used over the thickness for each element.
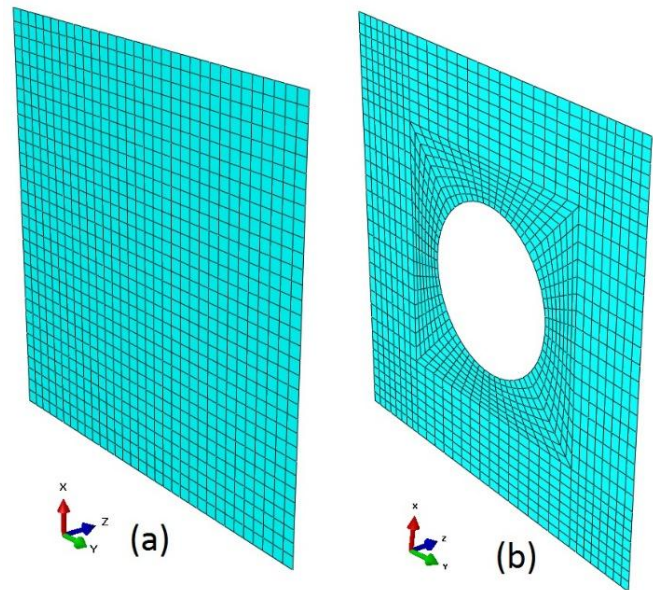

Fig 9: Illustration of adaptive mesh (a) pristine (b) with cutout

It is assumed that the free edges on the boundary are simply supported in an out-of-plane direction. The bottom end nodes of FE model were constrained against any movement along the global x direction. So the boundary conditions reflect a case of simply supported edge condition. An in-plane uniform SEL is applied on the top edge based on the following formulation:

$$A_{sel} = \frac{\pi^2 E}{12(1-\vartheta^2)}\frac{t^3}{b^2} \tag{5}$$

An illustration of the edge load applied is highlighted in Fig 10. The product of A_{sel} and Eigen values provided by the buckling analysis, gives the critical Shell Edge Load (SEL) of instability, for example a SEL of 2042.52 N/m is obtained from eq. (5) for a $0.9\,mm$ thick plate. The corresponding buckling analysis produced an Eigen value of 4.24; in a similar fashion an analysis is repeated for a perforated plate.

Eigen values of a plate of 20.32 mm diameter (d) cutout are shown along side a pristine plate in Fig 11. An Eigen value of 4.2433 for the pristine plate is within 1% of theoretical result. A lower value of 4.0652 was obtained for plate with a cutout.

A lower value in relation to a pristine plate is expected for perforated as an opening reduces the strength of plate. The critical shell edge-load that will cause instability therefore is $2042.52 \times 4.0652 = 8303.26 \; N/m$. FE results of perforated plates of cutout diameters (d) 20.32 and 40.64 mm were compared with a pristine plate. An in-plane uniform SEL is applied on the top edge based on equation (5). Corresponding critical Eigen values for various slenderness ratios are shown in Fig 12. The Eigen values are nearly constant independent of slenderness ratios for the cases considered. The K values for a perforated plate of thickness 0.9 mm and cutout diameter 20.32 mm is shown in Fig 13, a reduction was noted with an increase in cutout diameter.
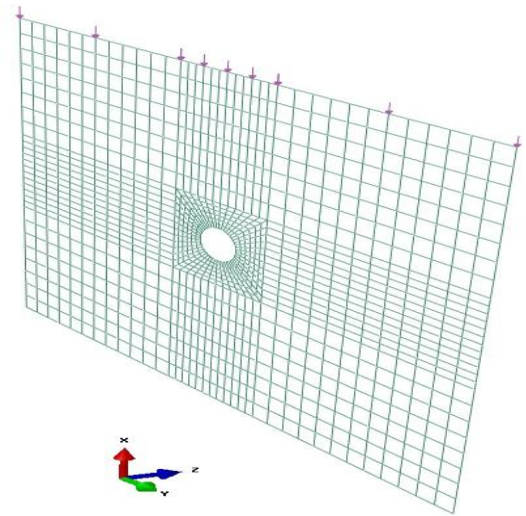

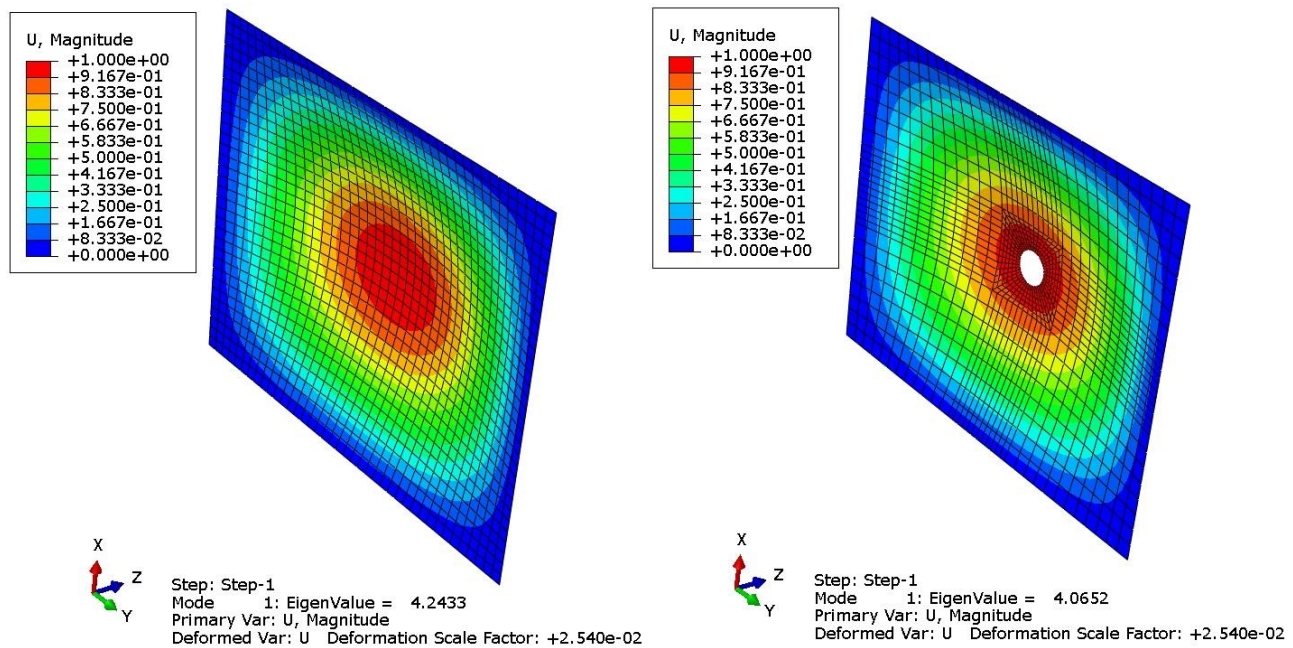Fig 10: Illustration of edge load

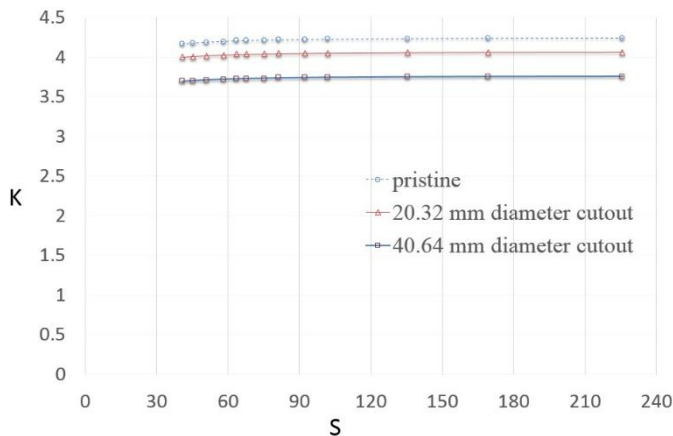Fig 11: Buckling analysis results of pristine (left) and perforated (right) plates
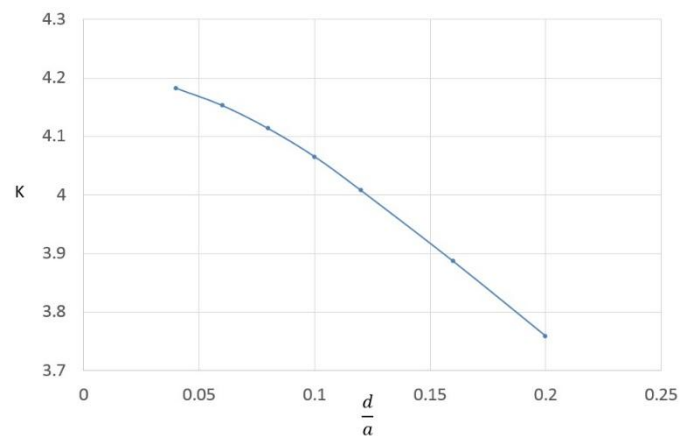

Fig 12: K values of models simulated

Fig 13: Effect of increasing the cutout dimeter

V. EIGEN VALUES UNDER PRE-LOADS AT CUTOUT

The FE models previously examined were subjected to additional in-plane pre-loads at cutouts; for example, a theoretical load (p_{cr}) of 2180.2 N is obtained from eq. (4) for a 0.9 mm thick pristine plate. The plate was pre-stressed by a load that is a multiplication factor (f) of p_{cr} on nodes around the cutout. The value of f was varied between 1 and 10% of p_{cr}. i.e. 21.802 and 218.02 N for the aforementioned configuration. An illustration of nodes around the cutout is highlighted in Fig 14. The SEL calculated from eq (5) was applied in the global x direction on top edge, and the bottom edge was restrained against moving in the applied direction. Buckling analysis was then carried-out and Eigen values were obtained. An illustration of results for a pre-load of 10% of f on 20.32 mm diameter cutout is shown in Fig 15.
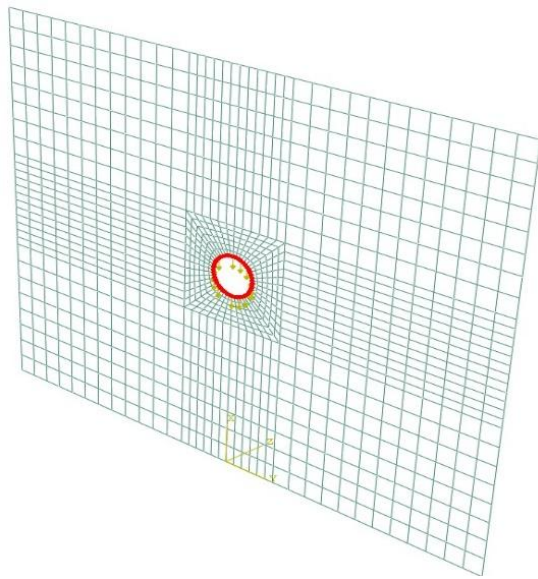

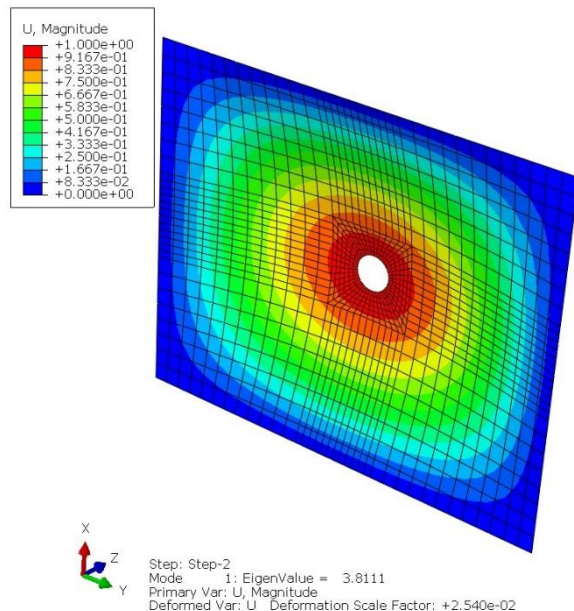Fig 14: Nodes around cutout with in-plane pre-load

Fig 15: FE result for a plate with a pre-load on cutout

VI. RESULTS AND DESIGN EQUATION DEVELOPMENT

The elastic stability of a perforated plate with pre-loads at the cutout was examined. A simple, easy to use formula was developed to plot the critical buckling stresses for all configurations of models examined. A buckling coefficient (C) was computed for the perforated plate. A curve fitting exercise was conducted to obtain coefficients through polynomial fits. The general method [5] in the development of design formulae involves a best-fit regression analysis that is a compromise between simplicity and accuracy. Design equations assume slenderness ratio and opening size as the two most important physical variables to affect stability behavior of plates. Generally, the lateral dimensions of the pristine plate are assumed as infinite compared to the diameter of cutout. For square plates ($a = b$), the buckling or ultimate loads can be considered as:

$$p_{cr} = g\left\{\frac{d}{a}, \frac{a}{t}\right\} \qquad (6)$$

The same procedure was employed in this paper for cases involving small cutout diameters that are much smaller compared to the dimensions of the plate. A simplified design equation is formulated that is specific to the plate examined. i.e. a $203.2 \times 254 \ mm^2$ plate. The objective is to keep the equation simple yet reliable thus less influencial parameters are avoided. This simplified equation considers the slenderness ratio as a function to estimate σ_{cr}^p. The critical unit compressive stress at simply supported edges is given by:

$$\sigma_{cr}^p = C \frac{\pi^2 E}{12(1-\vartheta^2)}\left(\frac{t}{b}\right)^2 \qquad (7)$$

Here C is a function of single independent variable, the slenderness ratio S. It is calculated with a cubic polynomial fit and is given by:

$$C = C_1 S^3 + C_2 S^2 + C_3 S + C_4 \qquad (8)$$

The coefficients C_1, C_2, C_3 & C_4 are listed in Table III. Note that in this investigation, the maximum in-plane loading at the cutout was limited to 10% of A_{sel}. The diameter of cutout was kept constant as 40.64 mm. There is a significant reduction in critical buckling load when the pre-load is increased beyond this limit. This is especially true if the diameter of the cutout is increased beyond 20% of edge dimension for a thinner plate. The accuracy of a quadratic fit was not significantly greater than a third order polynomial estimate. This equation will facilitate in the design of enclosures for blast resistant applications.

Note that it is required to find the limitations of this equation. The analysis conducted in this paper is a linear buckling analysis, however, a nonlinear analysis with fully defined material data is recommended for applications in real world scenario. In blast loading scenerios, the imperfections and material plasticity under large deformations influence structural collapse. Also note that out-of-plane loading and moment effects were not considered.

Table III: List of coefficients for design equation

In-plane load applied at cutout	C_1	C_2	C_3	C_4
1% of f	3.6001×10^{-8}	-1.7491×10^{-5}	2.7712×10^{-3}	3.8921
3% of f	3.6046×10^{-8}	-1.7504×10^{-5}	2.7721×10^{-3}	3.8415
5% of f	3.5973×10^{-8}	-1.7478×10^{-5}	2.7692×10^{-3}	3.7908
7% of f	3.608×10^{-8}	-1.7516×10^{-5}	2.7735×10^{-3}	3.7398
10% of f	3.5971×10^{-8}	-1.748×10^{-5}	2.7705×10^{-3}	3.6633

VII. CONCLUSIONS

Stability of plates containing circular cutouts subjected to in-plane pre-loads at central opening were investigated. A simplified design formula to estimate critical buckling loads was successfully presented. This design equation can be applied to a plate with a cutout diameter $20.32\,mm$. It is demonstrated here there is a possibility of developing generic formulae for any diameter cutout. To summarise:

- o Both cutout diameter and in-plane pre-load affect the critical buckling load. However, small in-plane pre-loads on a diameter cutout (less than 10% longer edge length) do not significantly reduce strength.
- o The critical stress of instability of a plate always decreases with increase in slenderness ratio and this decrease is greater for larger diameter (more than 20% of edge a) cutout.
- o Increasing the cutout diameter size to 20% of longitudinal edge a, reduces buckling load by 11.4% for a plate of aspect ratio 0.8.

ACKNOWLEDGMENT

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