Vibration Model of the Mobile Platform with Serial Industrial Manipulator for the Purpose of Suspension Design

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Abstract— **The content of the article develops the vibration model of mobile platform with serial manipulator mounted. Two main sources of vibration in mobile platform manipulation systems are the ground uncertainties and manipulator motors stiffness. Vibration can affect the behavior of the manipulator control system and decrease its dynamic properties. One of effective solutions to decrease vibration is to design vibration isolation system. To design isolators it is strictly important to perform frequency analysis of the system "mobile platform-manipulator" and to estimate its natural frequencies. Developed vibration model is based on the Denavit-Hartenberg convention. Platform and manipulator are assumed as one kinematic chain. This allows to decrease the number of system's degrees of freedom. Motors in manipulator joints are modeled by their stiffness and damping. Simulation and frequency analysis is performed. Based on simulation results, the model of the platform with vibration isolators (modeled by their stiffness and damping) is developed.**

***Index Terms*— Mobile manipulator platform, suspension, vibration isolation,**

I. Introduction

CURRENTLY, due to the increasing number of tasks for which the manipulators are used, wheeled mobile platforms with robots mounted are gaining more propagation. Due to the extremely high restrictions on the mobility and maneuverability of these systems, spherical wheels with high stiffness are often used in the construction. However, this solution has a major drawback – high level of vibration from the ground is transferred on manipulator and its end-effector.

The main elements of manipulator - actuating motors and gears connecting the motors with the manipulator links. When designing gears, as well as manipulator links, engineers usually strive to meet the requirements of high stiffness for these elements. However, it is not always possible to reduce the elastic compliance of the elements to the point where its effect becomes negligible, primarily because of the severe restrictions on the weight and sizes of the elements of the manipulator [1].

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Elements flexibility and vibration transmitted from the wheels leads to the appearance of elastic vibrations in the dynamics of manipulator and its working body. Elastic vibrations adversely affect the operation of the robot, causing an increase of dynamic loads on components, reducing speed and accuracy while implementing the motion control law, the emergence of non-damping vibrations, dangerous resonance phenomena, etc. Therefore, at the phase of design of the mobile platform, and the manipulator control system, elastic properties of the elements must be taken into account. It is important to perform full frequency analysis of the system "platform-manipulator" , which should be based on the entire system of differential equations, taking into account the structural features of the manipulator. This is due not only to the need to harmonize the range of operating frequencies of the designed control system and the frequency properties of the system, but also with the definition of achievable accuracy of the manipulator. The fact that the elastic vibrations are one of the causes of dynamic errors in motion leads us to the importance to estimate the contribution of the elastic elements to the total dynamic error [2].

One of the possible solutions for preventing the vibrations arising in the robot's motions on uncertain surfaces is the use of vibration isolators system. To design such a system is necessary to assess system's main frequencies. On the basis of this it is possible to choose the parameters of vibration isolators. The developed mathematical model allows us to do so without carrying out experimental research on real hardware.

II. Denavit-Hartenberg convention and homogeneous transformations

Denavit-Hartenberg (D-H) convention [3] is a commonly used approach to describe homogeneous transformations of the spatial kinematic chain, which is the kinematic model for the typical manipulation robot. In order to define position of a rigid body, six parameters are generally needed: three rotational and three translational. D-H convention allows to reduce the number of parameters from six to four. In this convention, coordinate frames are attached to the links and joins of the kinematic chains such that one transformation is for the joint, and the second one is for the link. Consider we have a serial manipulator consisting of n joints. Each homogeneous transformation A_i can be written as

$$A_i = Rot_{z_i\theta_i} Trans_{z_i d_i} Trans_{x_i a_i} Rot_{x_i \alpha_i} =$$

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$$= \begin{bmatrix} cos\theta_i & -sin\theta_i cos\alpha_i & sin\theta_i sin\alpha_i & a_i cos\theta_i \\ sin\theta_i & cos\theta_i cos\alpha_i & -cos\theta_i sin\alpha_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$i = \overline{1,n},$$

where $\theta_i, a_i, d_i, \alpha_i$ – are the four D-H parameters, joint angle, link length, link offset and link twist. There are four constraints on the relationship between the axes:

1. The axis x_i is the axis of revolution of joint j_{i+1}.
2. The axis x_i is perpendicular to the axis z_{i-1} and z_i.
3. The axis x_i intersects the axis z_{i-1}.
4. Coordinate frames are right-handed.

When the coordinate frames are assigned, D-H parameters values can be obtained as:
α_i is the distance along x_i from the intersection of the x_i and z_{i-1}axes to o_i.
d_i is the distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1}axes.
α_i is the angle from $z_{i-1}$to z_i measured about x_i.
θ_i is the angle from $x_{i-1}$to x_i measured about z_{i-1}.
Position and orientation of the link i-th in the inertial frame can be obtained by multiplication of the homogeneous transformation:

$$T_i^0 = A_0 \dots A_i = \begin{bmatrix} R_i^0 & o_i^0 \\ 0 & 1 \end{bmatrix},$$

where R_n^0 – rotation matrix, o_n^0 – translation vector.

III. Manipulator Jacobian and equations of motion

The manipulator Jacobian J∈$\mathscr{R}^{6\times n}$ is used in the task of velocity kinematics of the robot, and in derivation of the equations of motion. Jacobian represents the relation between the joint velocities $\dot{q}$ (q – vector of generalized coordinates of the system. If the robot has n joints, then q is a $n \times 1$ vector of joint angles) and the angular and linear velocities of the robot's end-effector:

$$\xi = J\dot{q},$$

where $\xi = \begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix}$ – vector of the linear and angular velocities.

Then, it possible to define two parts of the Jacobian – linear and angular:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}.$$

Jacobian for the center of mass of the link j can be written as [3]

$$J_{C,i} =$$
$$= \begin{bmatrix} z_0^0 \times (r_{C,j}^0 - o_0^0) & \dots & z_{j-1}^0 \times (r_{C,j}^0 - o_{j-1}^0) & 0 & \dots & 0 \\ z_0^0 & \dots & z_{j-1}^0 & 0 & \dots & 0 \end{bmatrix},$$

if the link j is revolute, and

$$J_{C,i} = \begin{bmatrix} z_0^0 \times (r_{C,j}^0 - o_0^0) & \dots & z_{j-1}^0 & 0 & \dots & 0 \\ z_0^0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix},$$

if the link j is translational.

Here, $r_{C,j}^0$ – is the vector pointing to the center of gravity of the link j, expressed in the inertial coordinate frame

$$o_0\ x_0\ y_0\ z_0\ ,\ z_0^0\text{=k},\ z_{j-1}^0 = R_{j-1}^0 k,\ \text{k=}[0,0,1]^T.$$

Commonly, vectors $r_{C,j}^j$ are known, which are pointing to the center of gravity of the link j, expressed in the body attached frame. The transformation to $r_{C,j}^0$ is simply denoted by:

$$r_{C,j}^0 = o_j^0 + R_j^0 r_{C,j}^j.$$

Kinetic energy of the rigid body is the sum of the translational and rotational kinetic energies:

$$K = \tfrac{1}{2} m v^T v + \tfrac{1}{2} \omega^T \mathcal{I} \omega,$$

where m – total mass of the object, v and ω are the linear and angular velocity vectors, respectively, $\mathcal{I}$ - inertia tensor, expressed in the inertial frame. It can be defined as:

$$\mathcal{I} = RIR^T.$$

Then, the total kinetic energy of the system equals

$$K = \frac{1}{2}\dot{q}^T \left(\sum_{i=1}^{n} \begin{pmatrix} m_i J_{vmi}(q)^T J_{vmi}(q) + \\ J_{\omega mi}(q)^T R_i(q) I_{mi} R_i(q)^T J_{\omega mi}(q) \end{pmatrix} \right) \dot{q}.$$

Inertia matrix of the system is obtained as

$$M = \sum_{i=1}^{n} \begin{pmatrix} m_i J_{vmi}(q)^T J_{vmi}(q) + \\ J_{\omega mi}(q)^T R_i(q) I_{mi} R_i(q)^T J_{\omega mi}(q) \end{pmatrix}.$$

In the case of developed model, there are two sources of potential energy in the system: gravity and torsional stiffness in the manipulator joints. We consider the joint configuration is fixed, so that gravity will have no effect on the vibration as it is a constant force. Potential energy of the joint i due to the motor stiffness can be expressed:

$$P_i = \frac{1}{2} k_i (\theta_i - \theta_{i0})^2,$$

where k_i – joint stiffness value, θ_i – joint angle, θ_{i0} – initial joint angle. For the generalized forces of the system, it is proposed to include the damping of the motors in the joints, which represents to each generalized coordinate i:

$$F_i = -c_i(\dot{\theta}_i - \dot{\theta}_{i0}),$$

where c_i – joint stiffness value, θ_i – joint angle, θ_{i0} – initial joint angle.

Equations of motion are obtained in the form of ordinary Lagrange equations:

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} = F_i, i = \overline{1,n}.$$

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As we consider a fixed robot configuration set, expression for the kinetic energy will contain no terms depending on generalized coordinates q_i. Thus,

$$\frac{\partial K}{\partial q_i} = 0.$$

IV. Vibration model of the platform with KUKA lightweight arm

Mobile manipulator consists of a mobile platform and a KUKA lightweight arm. The arm has 6 joints (and 6 DOF).

We use D-H parameters to describe the vibration behavior of the system “platform-arm”. It is proposed to model the platform as a single degree-of-freedom rigid body, with its parameters: m_s, I_s – mass and inertia tensor, correspondingly. As the main source of the vibration in the manipulator is from the irregularities in the wheels contact points, we introduce one degree-of-freedom for the platform – its vertical displacement z_s. Figure 1 shows a sketch of the platform with assigned coordinate frames. We introduce several coordinate transformations:

1. $o_I\ x_I\ y_I\ z_I\ - o_s\ x_s\ y_s\ z_s$ - transformation from the inertial coordinate frame to the frame assigned to the platform center of gravity.
2. $o_s\ x_s\ y_s\ z_s\ - o_1\ x_1\ y_1\ z_1$ - transformation from the platform frame to the frame attached to the first link. Next, there are 5 coordinate transformations for each following link.

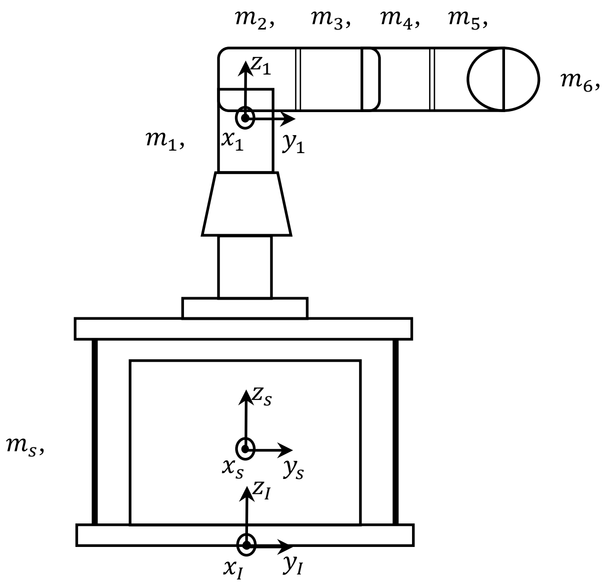

Figure 1 – Sketch of the mobile manipulator with the coordinate frames assigned.

Then, we obtain the D-H parameters, which are summarized in Table 1.

TABLE I
D-H PARAMETERS AND THEIR VALUES FOR THE MOBILE MANIPULATOR

Transformation	θ_i	d_i (m)	a_i (m)	α_i (rad)
I-S	0	$0.2 + z_s$	0	0
S-1	θ_1	0.08916	0	π/2
1-2	θ_2	0	-0.425	0
2-3	θ_3	0	-0.39225	0
3-4	θ_4	0.10915	0	π/2
4-5	θ_5	0.09465	0	- π/2
5-6	θ_6	0.0823	0	0

As can be seen from Table 1, the system has 7 DOF.

Then, we can define the vector of the (15) generalized coordinates as:

$$q = [z_s\ \theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6]^T.$$

Then, seven Jacobians are obtained as described in the previous sections, one for each coordinate transformation.

After the Jacobians are calculated, it is possible to define the kinetic energy of the system, and its inertia matrix. Kinetic energy is defined as the sum of 7 kinetic energies – of the platform and of the 6 joints:

$$K = \frac{1}{2}\dot{q}^T \left(\begin{pmatrix} m_s J_{vms}(q)^T J_{vms}(q) + \\ + J_{\omega ms}(q)^T R_s(q) I_{ms} R_s(q)^T J_{\omega ms}(q) \end{pmatrix} + \\ + \sum_{i=1}^{6} \begin{pmatrix} m_i J_{vmi}(q)^T J_{vmi}(q) + \\ + J_{\omega mi}(q)^T R_i(q) I_{mi} R_i(q)^T J_{\omega mi}(q) \end{pmatrix} \right) \dot{q}.$$

7×7 Inertia matrix is derived.

We use the following values of the joint stiffness, as identified in [4]:

$$k_1 = k_2 = \cdots = k_5 = 10000\frac{N}{m}, k_6 = 7500\frac{N}{m}.$$

For join i, consider the damping ratio ξ=0.1. Then, we obtain damping coefficients values as

$$c_i = \xi 2\sqrt{k_i I_i}.$$

After substitution equations (11)-(13) into (14) we obtain linearized system in the form[5]

$$M\ddot{q} + C\dot{q} + Kq = 0,$$

where M is a 7×7 inertia matrix of the system, C – diagonal dissipative matrix of the joints damping, K – diagonal stiffness matrix of the system.

For the purpose of numerical simulation the model can be rewritten in the form of its highest derivative:

$$\ddot{q} = -M^{-1}(C\dot{q} + Kq).$$

As the system has 7 DOF, M is 7×7 matrix. We define 1 DOF as an input of the system (vertical movement of the platform). We can derive two matrices from M: M_1 – 6×6 matrix, that is

$$M_1 = \begin{bmatrix} M_{2,2} & \dots & M_{2,7} \\ \dots & \dots & \dots \\ M_{7,2} & \dots & M_{7,7} \end{bmatrix},$$

and M_2 - 6×1 vector

$$M_2 = \begin{bmatrix} M_{2,1} \\ \dots \\ M_{7,1} \end{bmatrix}.$$

Then, the system can be rewritten as

$$\ddot{q}_\theta = -M_1^{-1}(C\dot{q} + Kq + M_2\ddot{x}),$$

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where q_θ=[$\theta_1, \ldots, \theta_6$].

V. Vibration Model of the Platform with KUKA Lightweight Arm with Vibration Isolation System

To model the vibration isolator system it is proposed to introduce one more degree of freedom in the system. Thus, the platform consists of bodies: sprung mass m_s with its inertia I_s, and unsprung mass m_u, with its inertia I_u. Figure 2 show the sketch of the manipulator.

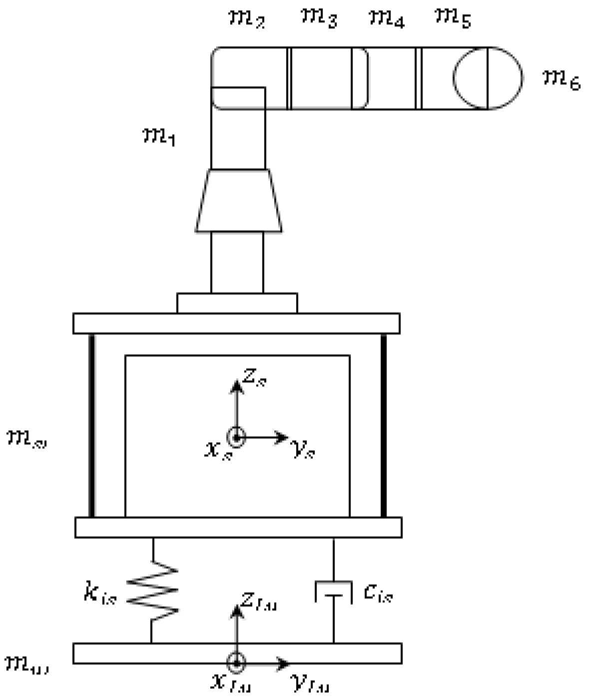

Figure 2 – Sketch of the mobile manipulator with vibration isolation system

We introduce one more coordinate transformation: $o_u\ x_u\ y_u\ z_u\ -o_s\ x_s\ y_s\ z_s$ - transformation from the coordinate frame assigned to the center of gravity of the unsprung mass of the platform to the frame assigned to the sprung mass center of gravity.

D-H parameters are summarized in Table 2.

TABLE II
D-H PARAMETERS AND THEIR VALUES FOR THE MOBILE MANIPULATOR

Transformation	θ_i	d_i (m)	a_i (m)	α_i (rad)
I-U	0	z_u	0	0
U-S	0	$0.2 + z_s$	0	0
S-1	θ_1	0.08916	0	π/2
1-2	θ_2	0	-0.425	0
2-3	θ_3	0	-0.39225	0
3-4	θ_4	0.10915	0	π/2
4-5	θ_5	0.09465	0	- π/2
5-6	θ_6	0.0823	0	0

As the system has now 8 DOF, the vector of generalized coordinates is:

$$q = [z_u\ z_s\ \theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6]^T.$$

The following procedures are the same, as for the previous model: we obtain 8 Jacobians for each coordinate transformation, write the expression for the kinetic and potential energies, and derive equations of motion in the form of Lagrange equations. Note, that stiffness and damping values of the isolation system are included in the expressions for potential energy and generalized forces, correspondingly.

As described above, the system can be written in the form

$$\ddot{q}_\theta = -M_1^{-1}(C\dot{q} + Kq + M_2\ddot{x}),$$

with M_1 is a 7×7 matrix

$$M_1 = \begin{bmatrix} M_{2,2} & \ldots & M_{2,8} \\ \ldots & \ldots & \ldots \\ M_{8,2} & \ldots & M_{8,8} \end{bmatrix},$$

and M_2 - 7×1 vector

$$M_2 = \begin{bmatrix} M_{2,1} \\ \ldots \\ M_{8,1} \end{bmatrix}.$$

VI. Simulation Results

Developed software can be used to perform frequency response analysis of the system. In order to do this, inputs and outputs of the system have to be assigned. We choose vertical displacement of the platform x as an input variable, and angle θ_6 as an output. An example is shown on Figure 3, where 100 input sinestream signals are generated with frequencies from 1 Hz to 100 Hz.

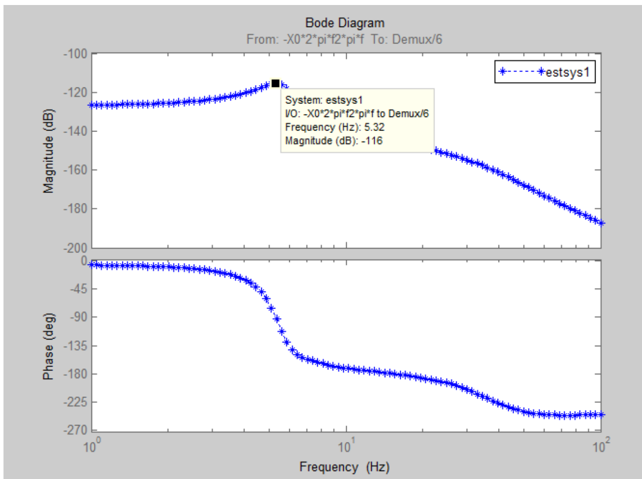

Figure 3 – Bode diagram of the system.

VII. Conclusion

The article describes the approach for developing vibration models of the mobile platforms with manipulators mounted. For this, mobile platform and manipulator are assumed as one kinematic chain with rigid connection. Using Denavit-Hartenberg convention allows to decrease the number of degrees-of-freedom of the system. Frequency analysis of the platform with KUKA lightweight arm mounted was performed, and natural frequency of the system was estimated. Vibration model of the platform with vibration isolators is developed.

(24)

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