Infant Mortality and Economic Growth: Modeling by Increasing Returns and Least Squares

I. C. Demetriou and P. Tzitziris

***Abstract*—Annual cross-country data from the World Bank database during the years 1998 and 2014 demonstrate wide variation in infant mortality rates (IMR) and gross domestic product per capita (GDPpc). All the datasets show that there is a range of high infant mortality at low GDPpc levels and there is a range of high GDPpc levels with low infant mortality. Unfortunately, the data are so noisy that there cannot be stated any simple relationship between IMR and GDPpc. However, using least squares estimates of infant mortality rates and assuming that GDPpc is subject to non-decreasing returns offers a meaningful method that is quite appropriate for modeling these datasets. Indeed, we understand at once the increasing slope of the IMR curve and we provide a quantitative explanation to what economists have observed: that at highest GDPpc levels, IMR increases after bottoming out. The advantage of our method is that it gives a clear quantitative model for confirmation of assumptions which so far had only qualitative support.**

***Index Terms*—convexity, gross domestic product, increasing returns, infant mortality rates, least squares, quadratic programming.**

I. Introduction

Infant mortality rate (IMR) is the number of deaths per 1,000 live births of children under one year of age [8]. High levels of poverty and low health and sanitation standards contribute to these deaths. The per capita Gross Domestic Product (GDPpc), among some other factors, 'as a proxy for income significantly affects infant mortality rate' [7]. GDP measures the monetary value of final goods and services produced in a country in a year [4], while the ratio of GDP to the total population of a country is the GDPpc. In this paper, GDPpc is measured on current US dollars.

Data for these indicators are maintained in the World Bank. Cross-country data demonstrate wide variation in infant mortality rates as GDPpc varies in its range. They show that infant mortality rates follow a convex descent trend, where there is a range of high infant mortality rates at low GDPpc levels and there is a range of high GDPpc levels with low infant mortality rates.

Motivated by an example of Georgiadou and Demetriou [5] concerning analogous data for 1995, we claim that a clear link between IMR and GDPpc is provided by assuming that GDPpc is subject to increasing returns. This is equivalent to assuming that IMR comes from an unknown underlying convex relationship on the IMR observations at the GDPpc observations [6], but convexity has been lost due to errors of measurement. The data are the coordinates $(x_i, \phi_i) \in \mathcal{R}^2$, for $i = 1, 2, \ldots, n$, where the abscissae x_i are the GDPpc observations in ascending order and ϕ_i is the measurement of the relationship of IMR at x_i. We regard the measurements as components of an n-vector $\underline{\phi}$.

We seek estimates y_i of the ϕ_i that are derived by minimizing the objective function

$$\Phi(\underline{y}) = \sum_{i=1}^{n} (y_i - \phi_i)^2 \tag{1}$$

subject to the convexity constraints

$$y[x_{i-1}, x_i, x_{i+1}] \geq 0, \ i = 2, 3, \ldots, n-1, \tag{2}$$

where

$$y\left[x_{i-1}, x_i, x_{i+1}\right] = \frac{y_{i-1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + \frac{y_i}{(x_i - x_{i-1})(x_i - x_{i+1})} + \frac{y_{i+1}}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)} \tag{3}$$

is the ith second divided difference on the components of $\underline{y}$.

In this paper we provide empirical evidence for our claim, by analyzing yearly cross-country data for 132 countries for a period of about 20 years.

In Section II we present the data for our computation. In Section III we give an outline of the method of Demetriou and Powell [3] for calculating the solution of the optimization problem that has been stated, we apply this method to the datasets of Section II and demonstrate the suitability of our assumption on the increasing slopes of the IMR curve.

The results are typically intended for use as a guide to policy makers.

II. Data

The data have been obtained from the World Bank database, which is freely available on the website [8]. The selection of our sample was based on the availability of IMR and GDPpc indicators for the years 1998, 2002, 2006, 2010 and 2014, which resulted to 132 world countries. Therefore in this paper we are able to present results that provide sufficient details of our quantitative modeling. Some basic descriptive statistics of our sample are displayed on Table I. The calculated statistics include the average, quartiles, median, minimum, maximum, range and standard deviation of each data set of the sample. We see in Table I that Average, Median and Quartiles of IMR decreased during the period 1998 to 2014, while those corresponding to GDPpc increased over the same period. Moreover, the IMR Range was narrowed, in contrast to the GDPpc Range which was expanded. These figures lead to the general conclusion that IMR has been reduced significantly over the last two decades, but without achieving such progress as the growth rates of GDPpc.

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TABLE I
DESCRIPTIVE STATISTICS FOR THE INDICATORS INFANT MORTALITY AND GDP PER CAPITA BY YEAR (SAMPLE SIZE: 132)

Year	Indicator	Average	1st Quartile	Median	3rd Quartile	Minimum	Maximum	Range	Standard Deviation
1998	IMR	42.4	10.9	29.0	70.1	3.5	148.1	144.6	37.2
	GDPpc	6645.8	469.3	1845.4	6630.6	124.7	41487.7	41363.0	9850.8
2002	IMR	36.9	9.3	24.8	61.4	2.6	137.7	135.1	33.2
	GDPpc	7170.8	510.3	2086.6	7290.2	111.5	43061.2	42949.6	10639.7
2006	IMR	31.5	8.1	20.1	50.2	2.3	124.5	122.2	28.8
	GDPpc	11482.1	964.8	3588.3	14265.3	154.9	74114.7	73959.8	16436.1
2010	IMR	27.2	7.1	16.7	42.2	1.9	109.6	107.7	25.4
	GDPpc	13372.0	1447.5	4979.2	16776.4	214.2	87646.3	87432.0	17953.5
2014	IMR	23.5	6.1	14.5	35.8	1.6	98.8	97.2	22.1
	GDPpc	15206.4	1939.3	6058.6	19357.8	286.0	97429.7	97143.7	20343.8

The data are too many to be presented as raw numbers in these pages, but we may easily capture their main features by looking at Figs. 1, 2, 3, 4 and 5 for the years 1998, 2002, 2006, 2010 and 2014 respectively. We notice a descending convex pattern of IMR with very high concentration of observations in the left-hand range of GDPpc and spare outliers. Further, the variation of IMR is wider at the low levels than at the middle and higher levels of GDPpc. Most interesting, except of the 2002 dataset, there exist IMR data at the highest GDPpc that have higher values than those in their vicinity. Theoretician economists explained this phenomenon by relying upon urban theories [1].

III. FITTING IMR SUBJECT TO INCREASING RETURNS

We provide an outline of the method of Demetriou and Powell [3] for calculating the solution to the problem of Section I and then we apply the method to the datasets of Section II. The description of the method is very instructive to our analysis.

Since the constraints (2) on $\underline{y}$ are linear, we introduce the scalar product notation

$$y[x_{i-1}, x_i, x_{i+1}] = \underline{a}_i^T \underline{y},\ i = 2, 3, \ldots, n-1, \qquad (4)$$

where $\underline{a}_i$, for $i = 2, 3, \ldots, n-1$ denote the constraint normals with respect to $\underline{y}$ and the superscript "T" is used to distinguish a row from a column vector. By taking into account that each divided difference depends linearly on only 3 adjacent components of $\underline{y}$, one can immediately see that the constraint normals are linearly independent vectors. Since in addition, the objective function (1) is strictly convex, we have to solve a strictly convex quadratic programming problem that has a unique solution, $\underline{y}^*$ say. Throughout the paper we use occasionally the descriptive term *convex fit* for the estimated IMR values.

The Karush-Kuhn-Tucker conditions provide necessary and sufficient conditions for optimality. They state that $\underline{y}^*$ is optimal if and only if the constraints (2) are satisfied and there exist nonnegative Lagrange multipliers $\{\lambda_i^* : i \in \mathcal{A}^*\}$ such that the first order conditions

$$\underline{y}^* - \underline{\phi} = \frac{1}{2} \sum_{i \in \mathcal{A}^*} \lambda_i^* \underline{a}_i, \qquad (5)$$

hold, where $\mathcal{A}^*$ is a subset of the constraint indices $\{2, 3, \ldots, n-1\}$ with the property

$$y^*[x_{i-1}, x_i, x_{i+1}] = 0,\ \ i \in \mathcal{A}^*. \qquad (6)$$

Equation (6) implies that the points (x_{i-1}, y_{i-1}^*), (x_i, y_i^*) and (x_{i+1}, y_{i+1}^*) are collinear. Hence and since it is usual in practice that there exist indices $j \notin \mathcal{A}^*$ such that $y^*[x_{j-1}, x_j, x_{j+1}] > 0$, the best convex fit is a piecewise linear curve, which interpolates the points $\{(x_i, y_i^*) : i = 1, 2, \ldots, n\}$. We say that the separate linear pieces are joined at the *knots* x_j, where the knots are all in the set $\{x_j : \ j \in \{1, 2, \ldots, n\} \setminus \mathcal{A}^*\}$. It is important to note that the knots are determined automatically by the quadratic programming method.

The quadratic programming algorithm generates a finite sequence of subsets $\{\mathcal{A}^{(k)} :\ k = 1, 2, \ldots\}$ of the constraint indices $\{2, 3, \ldots, n-1\}$ with the property

$$\underline{a}_i^T \underline{y} = 0,\ i \in \mathcal{A}^{(k)}. \qquad (7)$$

For each k, we denote by $\underline{y}^{(k)}$ the vector that minimizes (1) subject to the equations (7) and we call each constraint in (7) an active constraint. All the active constraints constitute the active set. Since the constraint normals are linearly independent, unique Lagrange multipliers $\{\lambda_i^{(k)} : i \in \mathcal{A}^{(k)}\}$ are defined by the first order optimality condition

$$\underline{y}^{(k)} - \underline{\phi} = \frac{1}{2} \sum_{i \in \mathcal{A}^{(k)}} \lambda_i{}^{(k)} \underline{a}_i, \qquad (8)$$

while, by strict complementarity, $\lambda_j^{(k)} = 0,\ j \notin \mathcal{A}^{(k)}$. The method chooses $\mathcal{A}^{(k)}$ so that each $\lambda_i^{(k)}$ satisfies the conditions

$$\lambda_i^{(k)} \geq 0,\ i \in \mathcal{A}^{(k)}. \qquad (9)$$

The method begins by calculating an initial approximation to the convex fit that requires only $O(n)$ computer operations. This is an advantage to the included quadratic programming calculation, because in all our runs the initial approximation came quite close to $\mathcal{A}^*$. Quadratic programming starts by deleting constraints if necessary from the active set of the initial approximation until all the remaining active constraints have nonnegative Lagrange multipliers. This gives $\mathcal{A}^{(1)}$. If $\mathcal{A}^{(k)}$, for $k \geq 1$ is not $\mathcal{A}^*$, then the quadratic programming algorithm adds to $\mathcal{A}^{(k)}$ the most violated constraint and deletes constraints with negative multipliers alternately, until the Karush-Kuhn-Tucker conditions are satisfied. This method is far faster than a general quadratic programming algorithm because it takes into account the band structure of the constraints when solving (7) and (8). For proofs on its efficiency one may consult the reference.

TABLE II
INFANT MORTALITY AND GDP PER CAPITA DATA FOR 2014, AND LEAST SQUARES CONVEX FIT [2]

No	GDPpc	IMR	Fit	Lagrange	Second difference	No	GDPpc	IMR	Fit	Lagrange	Second difference
1	286.00	55.80	58.50	-	-	67	6147.34	26.20	20.17	70547.64	-
2	362.25	45.10	57.46	411.44	0	68	6345.84	19.00	19.90	67233.55	0
3	431.38	58.40	56.52	2493.60	0	69	6472.10	34.40	19.72	65352.01	0
4	467.13	37.00	56.04	3436.13	0	70	6549.39	13.60	19.62	61931.43	0
5	539.62	62.80	55.05	8107.21	0	71	7153.44	35.60	18.78	42466.48	0
6	573.57	42.90	54.59	9768.75	0	72	7587.29	9.80	18.19	13894.57	0
7	615.94	62.40	54.01	12833.01	0	73	7640.65	7.80	18.11	11275.39	0
8	622.64	58.50	53.92	13205.19	0	74	7851.27	9.70	17.82	5280.97	0
9	630.00	53.60	53.82	13546.68	0	75	7886.46	28.90	17.77	4851.06	0
10	697.63	32.70	52.90	16714.74	0	76	7918.08	14.10	17.73	3761.11	0
11	701.68	30.50	52.85	17068.11	0	77	8025.30	3.50	17.58	843.60	0
12	713.46	62.20	52.69	18622.50	0	78	9680.12	19.50	15.30	2421.54	0
13	714.57	39.10	52.67	18747.85	0	79	10002.87	12.20	14.86	19.01	0
14	792.58	90.20	51.61	29675.39	0	80	10011.79	10.10	14.84	0	2.05E–06
15	830.15	53.50	51.10	32038.81	0	81	10303.90	12.30	14.62	2149.10	0
16	842.11	75.90	50.94	32733.85	0	82	10350.81	11.90	14.59	2712.04	0
17	903.46	65.70	50.11	33236.71	0	83	10415.46	8.60	14.54	3835.15	0
18	954.62	36.20	49.41	32060.62	0	84	10772.06	37.00	14.27	14264.09	0
19	1024.67	86.70	48.46	32301.30	0	85	11307.06	6.20	13.86	5584.00	0
20	1067.13	42.30	47.88	29199.88	0	86	11728.80	14.40	13.54	5200.44	0
21	1086.80	32.10	47.62	27982.77	0	87	12324.94	11.50	13.08	3628.45	0
22	1094.58	26.30	47.51	27742.80	0	88	12712.43	15.10	12.79	3832.81	0
23	1113.37	39.70	47.26	27960.32	0	89	13154.84	13.50	12.45	2019.55	0
24	1279.77	20.10	44.99	32400.93	0	90	13480.65	3.80	12.20	0	5.26E–07
25	1315.27	67.40	44.51	35115.76	0	91	13902.14	8.50	12.05	4469.96	0
26	1368.49	36.60	43.79	36749.46	0	92	14021.90	5.30	12.00	6589.45	0
27	1407.40	58.60	43.26	38503.27	0	93	14566.15	7.20	11.80	23516.72	0
28	1441.64	44.20	42.79	38996.08	0	94	15366.29	12.30	11.51	55766.05	0
29	1545.94	68.50	41.38	40204.00	0	95	16489.73	3.60	11.09	99261.92	0
30	1576.82	39.30	40.96	38886.50	0	96	16737.97	9.10	11.00	112592.33	0
31	1725.97	44.70	38.93	33017.42	0	97	18501.43	6.10	10.35	213990.24	0
32	1751.40	52.30	38.59	31723.31	0	98	18918.28	70.30	10.20	241501.39	0
33	1875.84	48.80	36.89	21977.28	0	99	19309.61	10.00	10.05	220286.76	0
34	1960.49	19.40	35.74	13331.89	0	100	19502.42	2.90	9.98	209853.86	0
35	2052.32	17.80	34.50	6954.88	0	101	20147.78	2.50	9.74	184071.63	0
36	2244.76	13.90	31.88	17.07	0	102	21317.45	18.70	9.31	154283.79	0
37	2434.28	18.00	29.31	0	3.10E–05	103	21627.35	3.70	9.20	140571.71	0
38	2560.52	28.30	28.83	2843.05	0	104	22124.37	3.00	9.01	124043.08	0
39	2872.51	22.80	27.65	10198.47	0	105	22217.49	10.20	8.98	122066.06	0
40	3065.16	8.10	26.92	16607.46	0	106	24001.88	2.20	8.32	79820.49	0
41	3124.08	31.70	26.69	20784.90	0	107	24406.47	12.90	8.17	75193.54	0
42	3190.31	24.60	26.44	24817.40	0	108	24855.22	5.60	8.00	65816.55	0
43	3203.24	71.50	26.39	25652.30	0	109	27245.74	2.60	7.12	27361.09	0
44	3365.71	21.00	25.78	21486.17	0	110	29718.50	3.60	6.21	9948.61	0
45	3477.15	45.80	25.36	19693.49	0	111	35179.65	3.00	4.19	0	5.01E–08
46	3499.59	23.60	25.27	18414.99	0	112	36152.69	2.10	4.15	552.61	0
47	3641.11	21.30	24.74	10824.92	0	113	37206.18	3.30	4.10	5468.68	0
48	3666.59	25.10	24.64	9633.41	0	114	42546.84	3.60	3.85	38935.12	0
49	3852.88	8.60	23.93	750.10	0	115	43593.70	7.70	3.80	46018.39	0
50	3873.53	13.20	23.85	398.65	0	116	43962.71	6.10	3.78	45637.60	0
51	4028.16	32.60	23.27	1062.05	0	117	46278.52	3.70	3.68	32519.34	0
52	4102.06	14.90	22.99	0	1.39E–05	118	47767.00	3.20	3.61	24013.91	0
53	4201.74	19.90	22.85	180.15	0	119	49864.58	2.00	3.51	13729.33	0
54	4328.90	12.60	22.68	1160.69	0	120	50185.48	4.40	3.49	13123.36	0
55	4429.65	11.30	22.54	3968.04	0	121	51148.36	3.00	3.45	9557.16	0
56	4588.65	12.90	22.32	11972.11	0	122	52036.73	1.60	3.41	7061.59	0
57	4712.87	18.10	22.15	20565.33	0	123	52138.68	3.30	3.40	7143.37	0
58	4830.98	15.80	21.98	29691.97	0	124	54321.29	3.10	3.30	9334.68	0
59	4851.66	5.40	21.96	31545.76	0	125	54398.46	5.70	3.30	9442.83	0
60	4884.37	14.60	21.91	35561.04	0	126	56007.29	2.20	3.22	3959.38	0
61	5112.38	19.40	21.60	66884.18	0	127	58899.98	2.40	3.08	0	9.31E–09
62	5119.22	13.90	21.59	67853.89	0	128	61330.91	3.00	3.09	0	6.55E–09
63	5232.69	98.80	21.43	85685.09	0	129	61995.83	3.20	3.11	121.10	0
64	5342.94	33.40	21.28	85950.42	0	130	85610.84	3.50	3.65	0	1.54E–09
65	5484.07	22.00	21.08	82868.82	0	131	96732.53	7.00	4.50	3280.76	0
66	5969.94	10.90	20.41	71370.14	0	132	97429.71	2.20	4.55	-	-

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The software package L2CXFT of Demetriou [2] implements this method and includes several useful extensions. It is the main tool of our work. The actual calculations were carried out by supplying the data to L2CXFT, while only a few iterations were needed for termination. We present the best convex fit to the most recent data set, namely that of the year 2014, in Table II, in order to give the reader an idea of the obtained results relating to the fit of Fig. 5. We have tabulated GDPpc, IMR, convex fit, Lagrange multipliers and central second differences at $\underline{y}^*$ corresponding to x_i, ϕ_i, y_i^*, λ_i^* and $y^*[x_{i-1}, x_i, x_{i+1}]$. Let us note that the numbers in the first column correspond to the countries of our sample (see [9]). If $y^*[x_{i-1}, x_i, x_{i+1}] > 0$ (sixth column) then x_i is a knot of the resultant fit, which implies that $\lambda_i^* = 0$ (fifth column). Moreover, the larger the magnitude of a Lagrange multiplier, the stronger the underlying linearity of the IMR indicator. Despite the existence of outliers, we see that the convex fit follows quite satisfactorily the trend of the data. Analogous results for the optimal fits displayed in Figs. 1–4 are demonstrated by Tzitziris [9].

However, associated with Figs. 1–5, we provide Tables III–VII that summarize the results of these runs. They display the knot indices $j = 0, \ldots, n-1-card(\mathcal{A}^*)$, the data indices of the knots, the GDPpc values, the estimated IMR values (i.e. the best convex fit) and the first differences of the fit (namely, the slopes of the line segments that join two consecutive knots).

Table III presents results associated with the 1998 data. We see in the fourth column of this table that the estimated IMR values decline over the GDPpc range [124.68, 30901.05], reaching a minimum value that is approximately equal to 4.65, and then increase over the range [30901.05, 41487.69]. The rates of change are negative up to 30901.05 and subsequently become positive, increasing from –0.1437292 up to 0.0000381.

Table IV presents results associated with the 2002 data. The estimated IMR values decline over the full range of values of GDPpc, with negative and increasing rates from –0.0745481 up to –0.0000012, while GDPpc reaches 43061.15.

Table V presents results associated with the 2006 data. The estimated IMR values decline over the GDPpc range [154.92, 33410.75], reaching a minimum equal to 4.37 and then increase over the range [33410.75, 74114.70], with a rate of change equal to 0.0000178.

Table VI presents results associated with the 2010 data. The estimated IMR values decline over the GDPpc region [214.23, 42935.25], reaching a minimum equal to 3.59 and then increase over the range [42935.25, 87646.27], with a rate of change equal to 0.0000194.

Table VII presents results associated with the 2014 data. The estimated IMR values decline over the GDPpc range [286.00, 58899.98], reaching a minimum equal to 3.08 and then increase over the range [58899.98, 97429.71], with increasing rates of change successively equal to 0.0000028, 0.0000230 and 0.0000764.

TABLE III
GDPPC AND KNOTS OF THE ESTIMATED IMR VALUES FOR 1998

j	Knot	GDP per capita	Estimated IMR	First difference
0	1	124.68	115.03	–
1	24	360.60	81.12	–0.1437292
2	49	869.11	42.94	–0.0750854
3	50	951.87	39.53	–0.0411843
4	66	1834.85	32.84	–0.0075749
5	97	5650.33	16.73	–0.0042237
6	106	12202.69	9.24	–0.0011421
7	117	22252.36	6.13	–0.0003098
8	119	25101.37	5.63	–0.0001766
9	127	30901.05	4.65	–0.0001692
10	132	41487.69	5.05	0.0000381

TABLE IV
AS IN TABLE III FOR 2002

j	Knot	GDP per capita	Estimated IMR	First difference
0	1	111.53	93.95	–
1	31	477.11	66.70	–0.0745481
2	48	1000.78	38.51	–0.0538358
3	57	1453.64	30.17	–0.0184195
4	95	6053.72	14.82	–0.0033368
5	107	14110.31	6.93	–0.0009787
6	108	15988.28	6.39	–0.0002901
7	119	26351.38	5.12	–0.0001227
8	132	43061.15	5.10	–0.0000012

TABLE V
AS IN TABLE III FOR 2006

j	Knot	GDP per capita	Estimated IMR	First difference
0	1	154.92	78.10	–
1	43	1448.76	36.39	–0.0322383
2	65	3394.43	23.52	–0.0066172
3	73	4428.52	21.78	–0.0016773
4	110	26455.13	5.89	–0.0007216
5	111	28482.61	4.60	–0.0006361
6	112	33410.75	4.37	–0.0000470
7	132	74114.70	5.09	0.0000178

TABLE VI
AS IN TABLE III FOR 2010

j	Knot	GDP per capita	Estimated IMR	First difference
0	1	214.23	68.54	–
1	36	1631.54	39.65	–0.0203841
2	45	2819.65	28.18	–0.0096595
3	64	4514.94	22.08	–0.0035941
4	91	11938.28	14.29	–0.0010495
5	110	30736.36	5.78	–0.0004526
6	119	42935.25	3.59	–0.0001802
7	132	87646.27	4.45	0.0000194

TABLE VII
AS IN TABLE III FOR 2014

j	Knot	GDP per capita	Estimated IMR	First difference
0	1	286.00	58.50	–
1	37	2434.28	29.31	–0.0135888
2	52	4102.06	22.99	–0.0037872
3	80	10011.79	14.84	–0.0013787
4	90	13480.65	12.20	–0.0007617
5	111	35179.65	4.19	–0.0003690
6	127	58899.98	3.08	–0.0000468
7	128	61330.91	3.09	0.0000028
8	130	85610.84	3.65	0.0000230
9	132	97429.71	4.55	0.0000764

IV. CONCLUSION

Our modeling approach states a relationship between IMR and GDPpc that is derived by the missing underlying convexity property, determines the IMR rates of change at GDPpc instants and provides a quantitative explanation to what economists have observed: that at high GDPpc, IMR increases after a bottoming out. Indeed, we assume that our data come from a convex process, but convexity has been lost due to errors. We impose the missing property as a constraint

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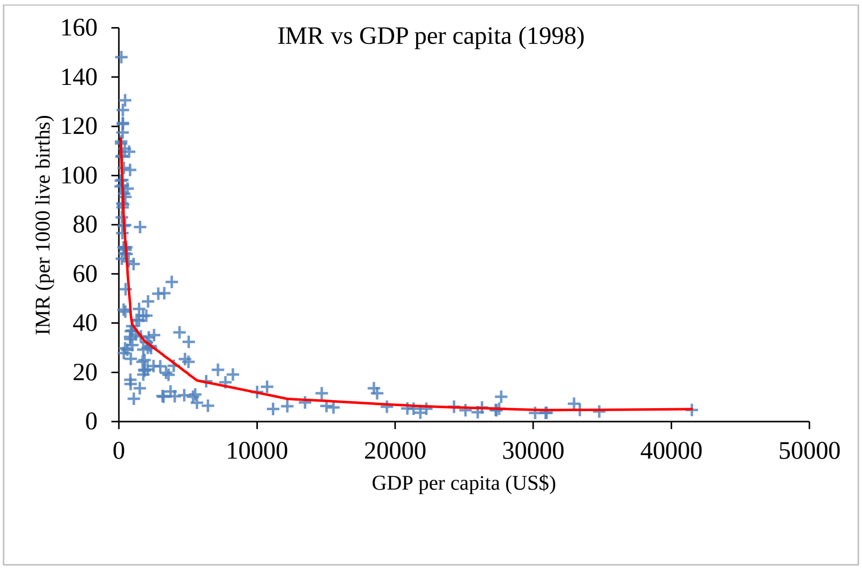

Fig. 1. Least squares convex fit (line) to infant mortality rate data (cross) against gross domestic product per capita for 1998

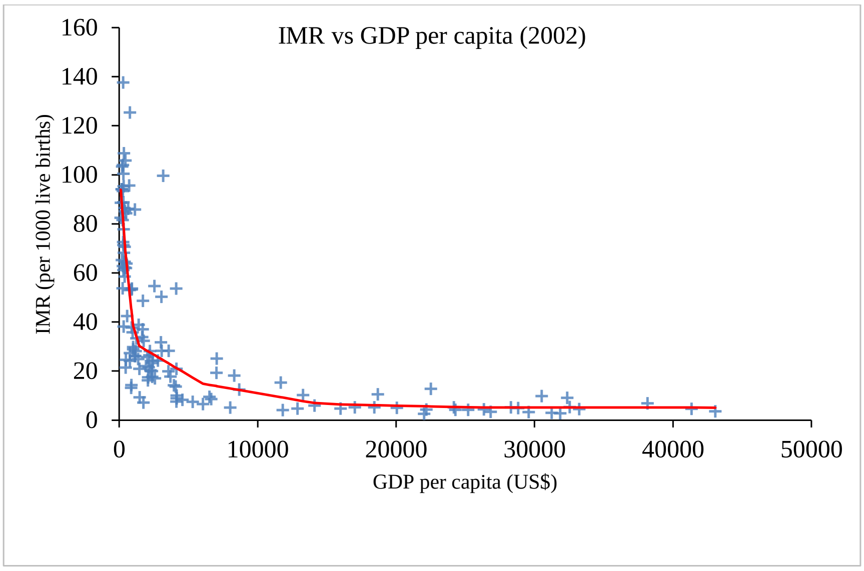

Fig. 2. As in Fig. 1 for 2002

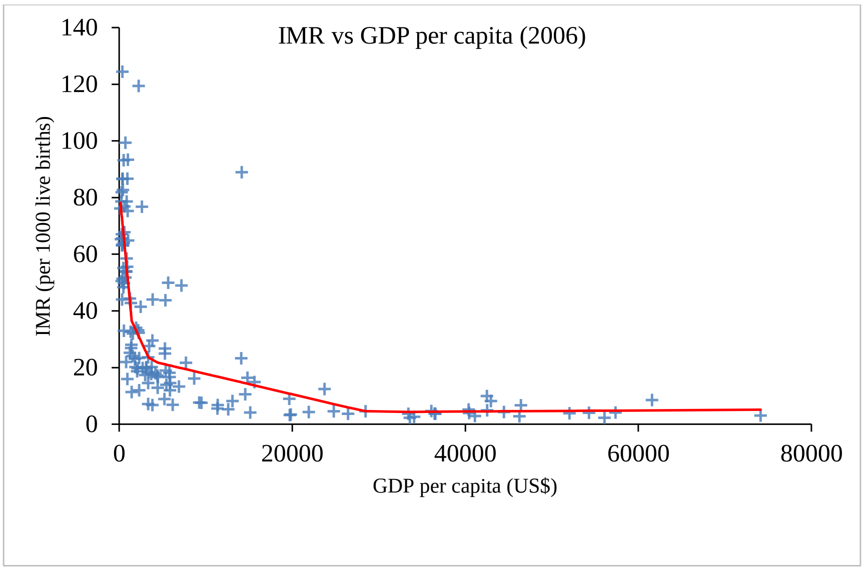

Fig. 3. As in Fig. 1 for 2006

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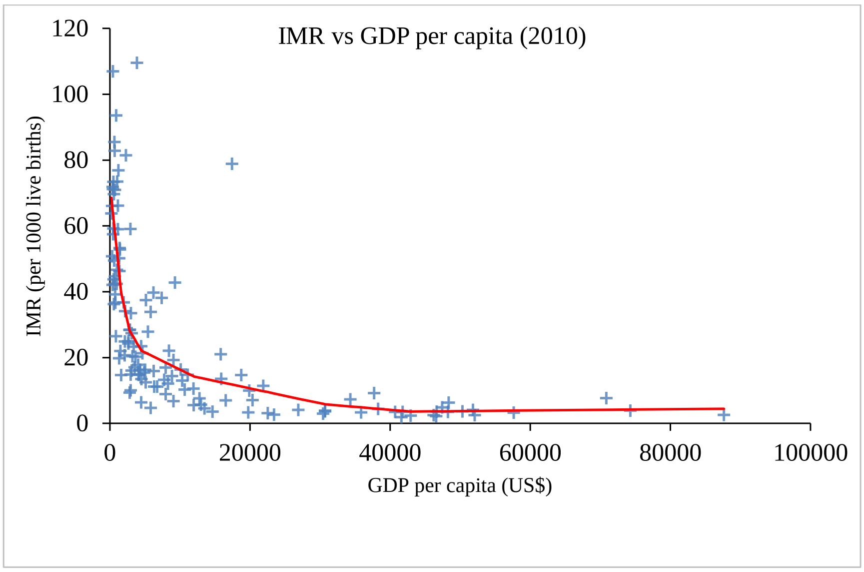

Fig. 4. As in Fig. 1 for 2010

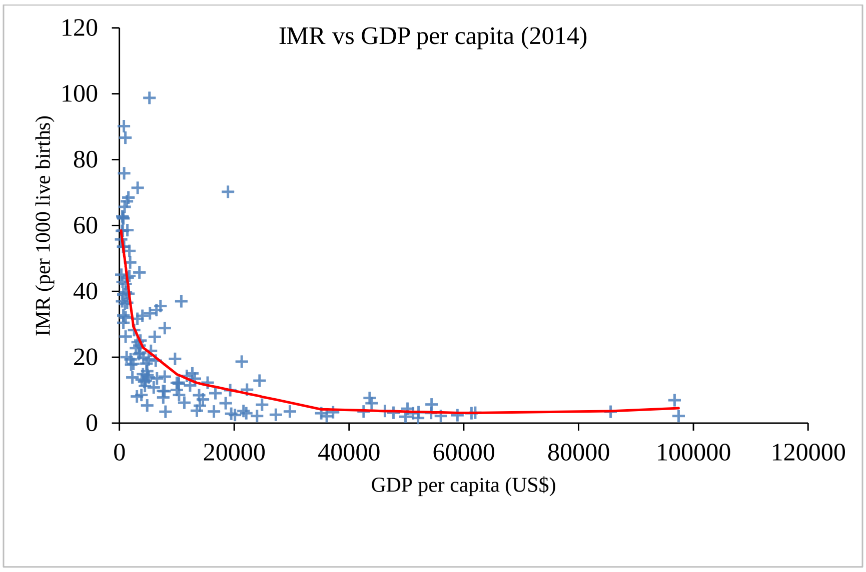

Fig. 5. As in Fig. 1 for 2014

and derive the estimated IMR values by an optimization calculation. Numerical evidence of Section III shows that IMR reaches a point at which it has its lowest value and then increases again while GDPpc is at the highest levels, indicating the ominously increasing slope of the IMR curve.

The results so far following from our analysis of real data suggest that the assumption that Infant Mortality Rates and Gross Domestic Product per capita follow increasing returns not only adequately describes reality, but also it is able to capture imperceptible features of the underlying process. Future research will be directed to design quantitative monitoring of the convexity method so as to provide an analytic tool for policy actions. This may be used by policy makers as part of the information on which decisions may be made.

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