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Bounded Parametric Control of Space Tether Systems Motion

Sergey P. Bezglasnyi

Abstract— **The control problem of planar motions of a space tether system (STS) is considered in this note. The STS is modeled by weight rod with two point masses. They are fixed on the rod. A third mass point can move along the rod. The control is the distance from the centre of mass of the STS to the movable mass. The control is a bounded continuous law of motion for a moving mass along the tether on the swing principle. New limited control laws processes of excitation and damping, diametrically reorientation and gravitational stabilization to the local vertical of the STS are constructed. The problem is solved by the method of Lyapunov's functions of the classical theory of stability. The theoretical results are illustrated by graphical representation of the numerical results.**

Index Terms— **orbit, space tether system, gravitational torque, asymptotic stability, swing principle**

I. Introduction

CURrently, a number of projects of interplanetary missions are being developed, which, in addition to the use of modern jet engines, require more energy efficient methods of motion in space. One promising area is the study of possibilities of the use of space tether systems (STS), under which two spacecraft connected by a tether with a length of tens or hundreds of kilometers are understood. The idea of using of STS for gravity simulation on the spacecraft was proposed by K.E. Tsiolkovsky [1]. Space tethers have received more attention in recent decades, with many notes and books available [1-6]. The fundamental note by Beletsky and Levin [2] presents the important results in providing the basis for the study of STS dynamics. Problems of simultaneous transport of loads by tether systems without changing the amount of system movement, which give the theoretical possibility of delivering a payload to a higher orbit and simultaneously lower the orbit of other cargo, such as space debris, without energy input, were discussed in [3].

In this paper we study the problem of gravitational stabilization of the relative equilibrium position of the STS in a circular orbit. We solve the problem of its reorientation by a movable mass swings principle. Swing can be simulated single-mass [7] or the two-mass [8, 9] pendulum of variable length. Many mechanical systems include flat pendulum motion, so swing models can be applied in the study of the dynamics and methods of control of such systems. Problems of swing and damping dual-mass pendulum resolved in [9], using the original continuous control law moving mass. The problem of the diametrical reorientation and stabilization of the gravitational plane motion of the satellite in a circular orbit was solved by the authors [10, 12], using the same law [9]. In [6] is presented the solution of the problem of orbital maneuvering the satellite using space tether system with a movable mass. The task of using the swing principle for gravitational stabilization problem and reorientation of a dumbbell shaped artificial satellite with movable mass in a circular orbit was solved in note [11]. There is flaw in the [9-11]. The movable mass can move without limitations. Limited control law on the principle of double pendulum swing was constructed in [13].

In this paper we consider the model of the STS. We build the limited laws of the movable mass motion for the control plane motion of the tether system in a circular orbit. The STS consists of two point masses connected by a weighty tether, along which a fourth point mass can be moved. Tether is modeled rigid rod. The center of mass of the STS moves in an orbit under the action of forces of central Newtonian gravity. Control is the distance from the common center of mass of the two ends of the rod to the cargo and moving cargo. Control imposed limitation. On the movement of the movable mass limitation imposed on both sides.

The new law for the model of the tether system solves the problem of gravitational stabilization of the radial equilibrium position of the STS relative to flat perturbations.

Control is built, which solves the problem of "swing" of the system and its reorientation in a diametrically opposite position with respect to an asymptotically stable equilibrium (a revolution of the STS at an angle π) in orbit.

II. Plane Motion Equation of the Tether System

Consider the STS motion in a central Newtonian gravitational field with center *O*. Tether system is modeled rigid rod with a mass m_4. Point mass m_1 and m_2 fixed to the ends of the rod. The movable mass m_3 is moved along the rod (Fig. 1). The common center of mass of payload and the rod is at the point O_1. We denote the distance from the point O_1 to the load m_3 as l and distance from the point O_1 to the center of mass O_2 as d. For them, the following relation holds true:

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S. P. Bezglasnyi is with the Further Mathematics Department, Samara University, Samara, 443086 Russia (corresponding author to provide phone: 8-917-104-1316; e-mail: bezglasnsp@rambler.ru).

$$\left(m_1+m_2+m_4\right)d=m_3(l-d) \tag{1}$$

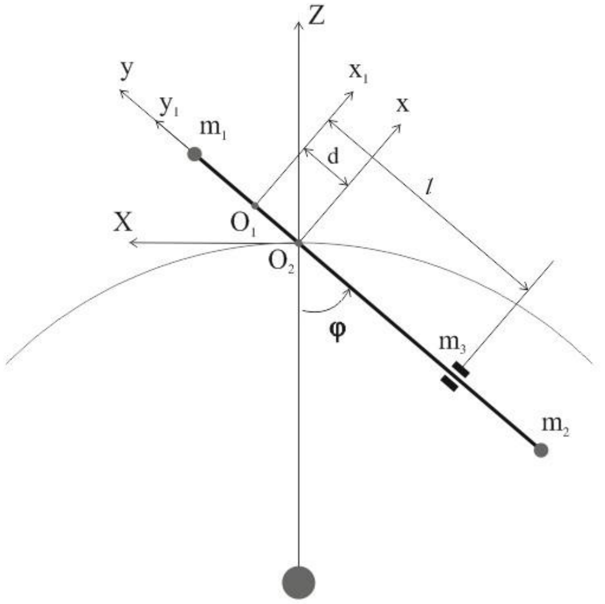

Fig. 1. Space tether system.

The orbital coordinate system O_2XYZ was selected. The axis O_2X is tangential to the orbit. The axis O_2Y is perpendicular to the plane of the orbit. The axis O_2Z completes the system of coordinates to the right hand third axis. O_1xyz is coordinate system associated with the STS.

The movement of the coordinate system O_1xyz relative to the orbital coordinates will be described by Euler angles ψ, θ, φ. We assume that the principal central moments of inertia of the system without movable mass: $B_1=0$,

$$A_1=C_1=\frac{L^2m_4}{12}+L^2\frac{4m_1m_2+m_1m_4+m_2m_4}{4\left(m_1+m_2+m_4\right)},$$

here L – length of the tether.

We obtain from relation (1):

$$d=\frac{m_3l}{m_1+m_2+m_3+m_4}, \tag{2}$$

here

$$m=\frac{\left(m_1+m_2+m_4\right)m_3}{m_1+m_2+m_3+m_4}. \tag{3}$$

Equation of plane motion about the center of mass of the STS in a circular orbit by the gravitational moment of the article [6]:

$$\varphi''=-2\frac{mll'}{A_1+ml^2}\left(\varphi'+1\right)-3\sin\varphi\cos\varphi \tag{4}$$

here the prime denotes the derivative with respect to new variable ν - true anomaly.

The distance from O_1 to the moving mass m_3, is considered the control:

$$l=l(\varphi,\varphi'). \tag{5}$$

III. Gravitational Stabilization of the STS

We will solve the problem of stabilization of the planar oscillations of a tether system relative radial position of equilibrium using a movable mass by swings principle. Control (5) are constructed according to the equations:

$$l=\begin{cases} l_0+a\varphi'\sin\varphi, \\ \text{when } -b\le a\varphi'\sin\varphi\le b,\ \text{a} \ge \text{b}>0; \\ l_0+bsign(\sin\varphi)\cdot sign(\varphi), \\ \text{when } a\varphi'\sin\varphi\le -b\bigcup a\varphi'\sin\varphi\ge b. \end{cases} \tag{6}$$

here $l_0=const>0,\ a=const>0$. Taking into account the equality:

$$l'=\begin{cases} a\varphi''\sin\varphi+a\varphi'^2\cos\varphi, \\ \text{when } -b\le a\varphi'\sin\varphi\le b,\ \text{a} \ge \text{b}>0; \\ 0,\ \text{when } a\varphi'\sin\varphi\le -b\bigcup a\varphi'\sin\varphi\ge b. \end{cases}$$

rewrite (4):

$$\begin{cases} \varphi''(A_1+ml(l_0+3a\varphi'\sin\varphi+2a\sin\varphi))= \\ =-2mla\cos\varphi(\varphi'+1)\varphi'^2- \\ -3(A_1+ml^2)\sin\varphi\cos\varphi, \\ \text{when } -b\le a\varphi'\sin\varphi\le b,\ \text{a} \ge \text{b}>0; \end{cases} \tag{7}$$

$$\begin{cases} \varphi''=-3\sin\varphi\cos\varphi, \\ \text{when } a\varphi'\sin\varphi\le -b\bigcup a\varphi'\sin\varphi\ge b. \end{cases} \tag{8}$$

Equation (7) has a zero solution $\varphi=\varphi'=0$ corresponding to the investigated relative system equilibrium. It is the equation of perturbed motion in the neighborhood of this equilibrium state. We will find a solution to the problem by using the second method of the classical theory of stability. We chose the Lyapunov function:

$$V=\frac{A_1+ml_0(l_0+3a\varphi'\sin\varphi+4a\sin\varphi)}{2}\varphi'^2+ \\ +\frac{3}{4}\left(A_1+ml_0(l_0+\frac{4}{3}a\varphi+\frac{a}{2}\varphi'\sin\varphi)\right)\times \\ \times(1-\cos2\varphi)\approx\frac{A_1+ml_0{}^2}{2}\left(\varphi'^2+3\varphi^2\right). \tag{9}$$

The function $V(\varphi,\varphi')$ can be represented by a power series in the neighborhood of the relative equilibrium state $\varphi=\varphi'=0$. The power series begins a positive definite quadratic form, so the function is positive definite according to the basis of definite function [14].

The derivative function (9) looks up to terms of the fourth degree in the variables φ, φ', by virtue of (7):

$$\frac{1}{Fn}\dot{V}=-\frac{9}{4}\varphi^4+\frac{12F}{G}\varphi^3\varphi'-\frac{21}{4}\varphi^2\varphi'^2-\frac{4F}{G}\varphi\varphi'^3-\frac{1}{2}\varphi'^4,$$

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here $F = mal_0 > 0$, $G = A_1 + ml_0^2 > 0$, n – the mean motion [2]. Derivative of Lyapunov function will be determined negatively function of its variables, if the equality $G > 2.1F$ holds, according to the Sylvester's criterion. The relative equilibrium state $\varphi = \varphi' = 0$ tether in a circular orbit is asymptotically stable based on Lyapunov's theorem on asymptotic stability [14]. Trivial solution $\varphi = \varphi' = 0$ is not asymptotic stability in general, but numerical calculations showed, that the for any initial deviations and velocity motion in the vicinity of the lower equilibrium position the STS.

A numerical integration equations of motion performed in the interval $\nu \in [0;300]$ *rad* for the following numerical values of system parameters $m_1 = 400$ *kg*, $m_2 = 300$ *kg*, $m_3 = 100$ *kg*, $m_4 = 200$ *kg*, $L = 3200$ *m*, $l_0 = 900$ *m*, $a = 500$ *m·s*, $b = 150$ and initial data: $\varphi(t_0) = 1{,}5$ *rad*, $\dot{\varphi}(t_0) = 0.1$ *rad/s*. These values were taken as an illustrative example. The phase portrait of system (7) with control (6) is

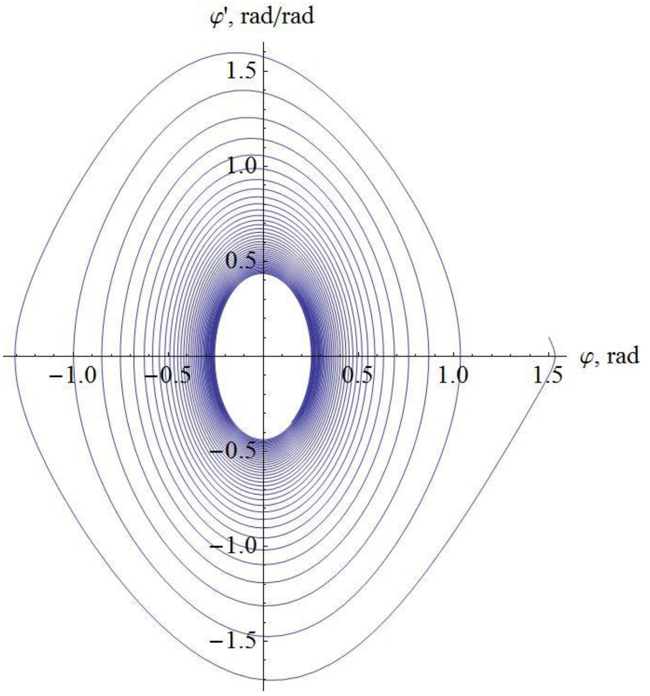

Fig. 2. Phase portrait.

shown in figure 2, which illustrating asymptotic damping of the amplitude and velocity vibrations of the of the STS around a zero equilibrium state. Amplitude and speed begins with sufficiently large initial deviations.

IV. Swinging and Reorientation of the Tether System

It is known [2] that the tether has two radial equilibrium states. The first, there is the relative equilibrium state in the orbit at which tether is directed along the radius of the local vertical. The second, there is the diametrically opposite equilibrium state. We solve the problem of the swinging of the STS from an arbitrary neighborhood of the relative equilibrium position and its diametrical reorientation. We will assume that control law parameter (2.1) is

$$a = const < 0. \tag{10}$$

The equation of controlled motion maintains the form (4). The function (9) is positive-definite in the vicinity of equilibrium $\varphi = \varphi' = 0$. We calculate the derivative of this function with respect to time by virtue of (1.4) up to terms of the fourth order we have the relation:

$$\begin{aligned} \frac{G}{n|F|}\dot{V}_{(2.3)} &= G\varphi'^2\left(\frac{1}{2}\varphi'^2 + \frac{15}{4}\varphi^2 + 4\frac{F}{G}\varphi\varphi'\right) + \\ &+3H\varphi^2\left(\frac{5}{2}\varphi'^2 + \frac{3}{4}\varphi^2 + 4\frac{F}{G}\varphi\varphi'\right) \end{aligned} \tag{11}$$

here $F = mal_0 < 0$.

Derivative (11) will be negative-definite, when the inequality $G > 4\sqrt{2/15}|F|$ satisfying, by Sylvester's criterion [14]. According to the Chetayev theorem of instability [14] the relative equilibrium state $\varphi = \varphi' = 0$ of the tether system in a circular orbit is unstable. Thus, the process of swinging of the STS with respect to radial position is implemented. Numerical calculations show that as a result of this swing we become diametric reversal of the system relative to its center of mass.

Let us show that after the STS diametric reversal control (6) under the conditions (10) stabilizes it in the neighborhood of the opposite equilibrium state $\varphi = \pi$, $\varphi' = 0$. We write the equation of perturbation motion by introducing the deviation $\varphi = \pi + x$:

$$\begin{aligned} &x''(A_1 + ml\ (l_0 - 3ax'\sin x - 2a\sin x)) = \\ &= 2mla\cos x(x'+1)x'^2 - \frac{3}{2}(A_1 + ml^2)\sin 2x \end{aligned} \tag{12}$$

If now $a = const < 0$, then equation (12) with control (6) will coincide with the first equation of the system (7) where $a = const > 0$. The solution $x = x' = 0$ will be

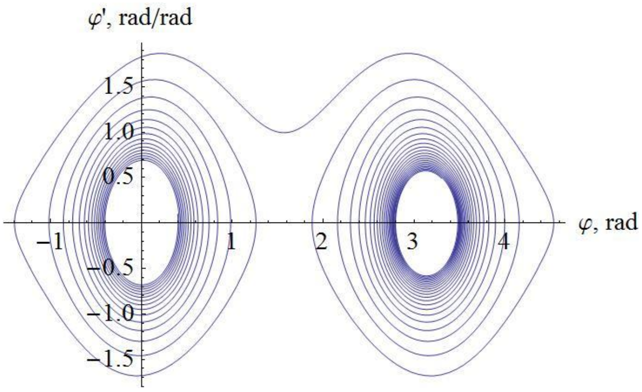

Fig. 3. Phase portrait.

asymptotically stable according to the results obtained in Section 2.

Thus, the control (6) under condition (10) implements asymptotically stable reorientation diametrically of the STS.

Figure 3 illustrates this process by graphs of numerical calculations. The integration was performed in the range $\nu \in [0;200]$ *rad*, parameter $a = -500$ *m·s* and the initial

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values: $\varphi(\nu_0) = 0.4$ *rad*, $\varphi'(\nu_0) = 0.1$ *rad/s*. Other parameters of the system were the same as they were before in Section 2. Phase portrait (Fig.3) shows the behavior of the angle φ. It shows swinging around the zero equilibrium state $\varphi = \varphi' = 0$ followed by an asymptotic approach to a new equilibrium state $\varphi = \pi$, $\varphi' = 0$.

Another result of the numerical research is a numerical integration of controlled motions for different values of the parameter a. It showed the choice of the values of this parameter can be used for control the direction of rotation of the tether system at the same initial conditions.

V. Conclusion

The equation of controlled planar motions relative to the center of mass of the dumbbell shaped satellite with a moving mass in a circular orbit under the action of the gravitational torque was obtained in this paper. New limit control laws of a moving mass motion was constructed. These laws are solve problems of gravitational stabilization with respect to planar perturbations of relative equilibrium of the dumbbell shaped satellite in a circular orbit and its diametrical reorientation by controlling motion of a movable mass. The Lyapunov functions necessary for a rigorous proof of the asymptotic stability and instability of studied movements were constructed for the proposed control. A numerical integration confirmed the findings.

The results of this paper further develop results from notes [13, 17] and can be used for projecting control systems for space tether systems.

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