Numerical Analysis of Latent Heat Storage with an Internal Fin using Flexpde

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I.

Abstract- Thermal energy storage plays an important role in a wide variety of industrial, commercial and residential applications. Phase change material (PCM) is used in these systems in order to store heat. During the phase change in a PCM storage system, the solid-liquid interface moves away from the heat transfer surface and the surface heat flux decreases due to the increasing thermal resistance of the molten or solidified medium. Heat-transfer enhancement techniques such as fins and honeycombs have to be used to increase the heat-transfer fraction in the storage. The purpose of this paper is to analyze a simplified numerical model which predicts the solid-liquid interface location and temperature distribution of the fin in the melting process with a constant end-wall temperature in the finned one-dimensional PCM storage. The numerical results were compared to the analytical results by [6]. The results show that the numerical model gives a satisfactory estimation for the fin temperature and the solid-liquid interface and the variation is significant when the length-to-half thickness ratio (L/D) of the fin is smaller than 200.

Index Terms—PCM, Latent heat storage, Flexpde, Numerical model

List of symbols

Cp	heat capacity, J kg ⁻¹ K ⁻¹		
Ď	half thickness of the fin, m		
E'	energy storage per unit length, W m	1	
E"	energy storage per unit area, W m ⁻²		
g	acceleration of gravity, m s ⁻²		
Н	latent heat of fusion, J kg ⁻¹		
h	heat transfer coefficient, W m ⁻² K ⁻¹		
k	heat conductivity, W m ⁻¹ K ⁻¹		
L	length, m		
q	heat flow, W		
q'	heat flow per unit length, W m ⁻¹		
q"	heat flux, W m ⁻²		
S	location of the phase change interface	e, m	
Т	temperature, ⁰ C		
t	time, s	Subscript	s and superscripts
Х	distance in the x-direction, m		
У	distance in the y-direction, m	с	convection
		f	fin
Greek symbols		i	initial
		1	liquid
α	thermal diffusivity, m ² s ⁻¹	m	melting
β	thermal expansion coefficient, ⁰ C ⁻¹	р	phase change
μ	dynamic viscosity, kg m ⁻¹ s ⁻¹		material
ρ	density, kg m ⁻³	s	solid
λ	root of the transcendental equation	w	wall

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conditions and constant thermal properties [2,9]. In recent years, phase change problems have

In recent years, phase change problems have motivated considerable research into numerical solution methods. A wide range of numerical methods applied to these problems has been reported [3]. During the last three decades phase change materials (PCM) for energy storage systems have developed rapidly. The most commonly used PCMs for thermal energy storage are paraffins, salt hydrates, and fatty acids [10]. Inspite of having significant latent heat of fusion during phase change, these materials are also having some disadvantages like low thermal conductivity, weak stability, lack of durability, and supercooling effects.

INTRODUCTION AND OBJECTIVES

A latent heat thermal energy storage system (LHTESS) is

preferable to sensible heat storage in applications with a

small temperature variation because of its nearly

isothermal energy storage and high energy storage density.

This change of phase involves exchange of heat and/or

mass, super-cooling, changes in thermo-physical properties

etc [1, 11-12]. This phenomenon of solidification and

melting is associated with many practical applications like

metal processing, solidification of castings, environmental

engineering, thermal comfort, electronic cooling, solar

water heating and thermal energy storage system in a solar

thermal plant. In these applications, matter is subject to a

phase change. During melting or solidification, a boundary

separating two different phases develops and moves in the

matter during the process. In early years, analytical

methods were the only means available to understand the behavior of physical processes involving the moving

boundary. Although analytical methods provide an exact solution, but due to their limitations, analytical solutions are mainly for the one-dimensional cases of an infinite or semi-infinite region with simple initial and boundary

The main components of a latent heat storage system are (i) PCMs (ii) Heat exchanger and (iii) Container material. A common problem in latent heat thermal storages is the poor conductivity of the PCMs [8].In a large PCM storage system the solid–liquid phase transition suffers from the small thermal conductivity of the PCMs, especially in the liquid state (for paraffins k=0.1–0.2 W/m K and for salt hydrates k=0.4–0.6 W/m K) [4]. PCM solidifies on the heat exchanging surface and acts as a self-insulator because of low thermal conductivity. So for practical usage some kind

of heat transfer enhancement technique has to be used in LHTESS to increase the heat transfer rate.

Heat transfer in LTHESS can be enhanced by the following techniques [5, 7]:

(a) Active methods like agitators/vibrators, scrapers and slurries.

(b) Using microencapsulated PCM.

(c) Using PCM containing dispersed high conductivity particles.

(d) Using PCM graphite composite material.

(e) Using extended surfaces such as fins and honeycombs.

Lamberg et al. [6] introduced a simplified analytical model based on a quasi-linear, transient, thin fin equation which predicts the temperature distribution in the fin, and liquidsolid interface as a function of time in storage. It is evident that fins and different kind of matrix structures enhance the internal heat transfer of a phase change material. Charging and discharging time should be optimized to achieve the best economical and technical benefit of the storage. The geometry of the latent heat thermal energy storage system plays a very important role.

This paper continues work already carried out on the solidification by [6] but using numerical method. The objective of this paper is to examine the melting process in a semi-infinite PCM storage integrated with a fin. The numerical model based on a quasi-linear, transient, thin-fin equation is presented which predicts the solid–liquid interface location and temperature distribution of the fin.

II. THERMAL ENERGY STORAGE SYSTEM MODEL

Thermal energy storage system model of melting process in a semi-infinite PCM storage with a thin fin is studied (see Figure 1) [6]. The thermal energy storage is 2dimensional and it is semi-infinite both in the x-direction $(0 < x < \infty)$ and y-direction $(0 < y < \infty)$ and the length of the fin approaches infinite. The end-wall with a constant temperature and the fin act as heat sources in the melting process.

In a storage the melting occurs in two different regions. The regions are shown in Figure 1. In region 1, the only heat source is the constant temperature end-wall. Here the fin is not influencing the melting process. Heat transferred from the wall is first melting the phase change material by conduction and later by natural convection. The liquid PCM starts to flow up along the vertical hot wall surface and fall down along the cold solid–liquid interface causing natural convection [6]. In region 2, both the wall and the fin are transferring heat to the phase change material. After a short period the fin plays the most important role in the heat transfer in region 2.

In this paper a simplified one-dimensional numerical model based on a quasi-linear, transient, thin-fin equation is presented. The model predicts the solid–liquid interface location and the temperature distribution of the fins during the solidification process in the storage.



Figure 1: Semi-infinite phase change material storage with a fin

In addition to the one-dimensional numerical model, the heat transfer in the PCM storage is calculated with a simplified one-dimensional analytical model given by [6]. The one-dimensional numerical calculation is carried out using a programme known as FlexPde, which is a simulation package solving systems and coupled equations through the finite element method in one-, two- and three-dimensions.



Figure 2: Methodology to find out the effect of numerical approach

A. ASSUMPTIONS

Due to the nonlinear, unsteady nature of the problem several assumptions have to be made to simplify the problem [6].

- 1) Initially the solid PCM and the fin are in the melting temperature of the phase change material $T_m=T_s=T_f$. Therefore, the heat conduction in a solid PCM is considered to be negligible.
- 2) The end-wall temperature T_w is kept constant and it is higher than the melting temperature of the phase change material Tm.
- 3) The temperature distribution of the thin fin is considered to be 1-dimensional in the x-direction.
- 4) The sensible heat of liquid PCM is assumed to be negligible. The latent heat of fusion is assumed to be the principal mode of energy storage.
- 5) In region 1 the heat is transferred from the wall to the solid–liquid interface 1-dimensionally in the x-direction. The main heat transfer mode is assumed to be conduction in liquid PCM. Natural

convection is assumed to be negligible. With these assumptions, it is possible to find an analytical solution for the solid–liquid interface location in the x-direction.

- 6) In region 2 it is assumed that heat transfer is 1dimensional from the fin to the solid–liquid interface in the y-direction because the fin plays the most important role in melting PCM in region 2. The main heat transfer mode is assumed to be natural convection in liquid PCM. Conduction is assumed to be negligible.
- 7) The physical properties for the phase change material and for the fin are assumed to be constant, because the temperature differences in phase change material storage are usually relatively small.

B. MATHEMATICAL FORMULATION

The mathematical formulation will also be handled in two parts. In region 1 the melting can be handled as 1dimensional one-phase Stefan problem [1], which is the simplest explicitly solvable moving boundary problem with constant imposed end-wall temperature and constant thermo-physical properties of the materials. The exact solution for the problem was found by Neumann in 1860 [5]. In a 1-phase Stefan problem the heat equation of a liquid phase change material T_1 and heat equation for solid–liquid interface with initial and boundary conditions are defined as [5]:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha_1} \frac{\partial T_l}{\partial t} , \quad t > 0$$
(1)

$$(\rho H)_l \frac{\partial S_x(t)}{\partial t} = -k_l \frac{\partial T_l(S_x,t)}{\partial x} , t > 0$$
⁽²⁾

$$S_{x}(0) = 0$$
 (3)
T (S, t) = T (4)

$$T_1(0,x,t) - T_m$$
 (4)
 $T_1(0,t) = T_w$ (5)

Where $S_x(t)$ is the location of the solid–liquid interface in the x-direction as a function of time, α_1 is thermal diffusivity of liquid PCM, ρ is density, H latent heat of fusion and k_1 conductivity of the liquid. In region 2, all heat transfer is assumed to occur only in the y-direction. An arbitrary differential element dx is separated from the PCM storage to outline energy balances. The element is shown in Figure 2. The energy balance of a differential element dx yields two equations, one for the fin and one for the PCM. The energy balance for the fin is

$$E_{f}^{''} = q_{x}^{''} - q_{x+dx}^{''} - q_{c}^{''}$$
(6)

Where E_f'' denotes the rate of heat storage to the fin, q_x'' is the heat flux by conduction at position x, while q_{x+dx}'' denotes heat flux by conduction at position x+dx and q_c'' refers to the convective heat flux from the fin to the liquidsolid interface. The rate equations are substituted into the energy balance Eq. (6) which can be rewritten with initial and boundary conditions as



Figure 3: Energy flows in the arbitrary differential element of the finned PCM

$$(\rho c_p)_f D \frac{\partial T_f}{\partial t} = k_f D \frac{\partial^2 T}{\partial x^2} - h(T_f - T_m), t > 0$$
(7)

$$T_{f}(x,0) = T_{m}$$
 (8)
 $T_{d}(0|t) = T_{m}$ (9)

$$T_{f}(\infty,t) = T_{m}$$
(10)

where T_f is the temperature of the fin, h the heat transfer coefficient from the fin to the solid–liquid interface, T_m the melting temperature of the PCM and D the half thickness of the fin.

The heat transfer coefficient is given by [6]

$$h = 0.072 \left[\frac{\left[g \left(\frac{T_W - T_m}{2} \right) \rho_l^2 k_l^2 \beta \right]}{\mu} \right]^{1/3}$$
(10.1)

The heat flows from the constant temperature end-wall and from the fin to the solid–liquid interface of the phase change material. The energy balance for the solid–liquid interface in the y-direction is the following:

$$E'_p = q'_w + q'_c \tag{11}$$

where E'_p is the rate of heat storage to the phase change material, q'_w is the rate of heat flow per unit length from the wall to the solid–liquid interface and q'_c the rate of heat flow per unit length from the fin to the solid–liquid interface. The stored heat due to melting in a dx wide element is

$$E'_{p} = (\rho H) \frac{\partial s_{y}}{\partial t} dx \tag{12}$$

Where S_y is the distance from the fin to the solid–liquid interface in the y-direction. The heat transfer from the fin to the solid–liquid interface is assumed to take place by convection and it is defined as

$$q_c' = h(T_f - T_m)dx \tag{13}$$

The rate of heat flow per unit length from the constant temperature end-wall to the solid–liquid interface q_w is determined by pure conduction through the liquid in the x-direction:

$$q'_{w} = \frac{k_{l}}{x} (T_{w} - T_{m}) \frac{\partial S_{y}}{\partial x} dx$$
(14)

Equation (14) is a first order approximation and it leads to inaccuracies in the model because natural convection is not

taken into account. However, according to Lamberg et al. [6] heat source q'_w is dominant only during the very early stages of melting before sufficient heat energy has diffused through the fin to provide the amount of heat which later becomes the dominant source of the melting phase change material. Therefore, the error made is assumed to be small. Now the Equations (12)–(14) are substituted into the energy balance Eq. (11) and it can be rewritten with an initial condition as

$$(\rho H)_l \frac{\partial S_y}{\partial t} = \frac{k_l}{x} (T_w - T_m) \frac{\partial S_y}{\partial x} dx + h (T_f - T_m) dx$$

t > 0 (15)
$$S_y(x,0) = 0$$
 (16)

Equations (1)–(16) are solved mathematically to achieve numerical solution for the problem.

III. SOLUTION

A. ANALYTICAL SOLUTION OF SIMPLIFIED ONE-DIMENSIONAL MODEL

Region 1

In region 1 the interface melts only in the x-direction. The Stefan problem Eqs. (1)–(5) has a well-known analytical solution solved by Neumann [1]. The location of the solid–liquid interface in each time step can be solved from Eq. (17):

$$S_x(t) = 2\lambda \sqrt{\alpha_l t} \tag{17}$$

where λ is a root of the transcendental equation

$$\lambda e^{\lambda^2} \operatorname{erf}(\lambda) = \frac{st_l}{\sqrt{\pi}} = \frac{c_{pl}(T_w - T_m)}{H\sqrt{\pi}}$$
(18)
Region 2

In region 2, the temperature distribution for the fin is solved analytically [6].

$$b = \rho_1 H \tag{26}$$

The solution of the interface location Sy in the y-direction is given by [6]

$$S_{y}(x,t) = h(T_{f} - T_{m})x\left(\frac{-bx + \sqrt{(bx)^{2} + 2abt}}{ab}\right)$$
(27)

B. NUMERICAL SOLUTION

The equations of the fin temperature distribution with initial and boundary conditions Eqs. (7)–(10.1) are solved numerically. The numerical calculation is carried out with a program called FlexPDE which is a simulation package that solves systems of coupled non-linear partial differential equations and linear partial differential equations through the finite element method in one, two and three dimensions. The analytical solution can be calculated from Eq. (19) & (27). The numerical solution for the temperature distribution of the fin and distance from fin surface is compared to the analytical solution by [6] to verify the accuracy of the numerical solution.

IV. VERIFICATION

The performance of the presented method is first verified with a one-dimensional phase change testing problem explained in [6]. The PCM used for analysis is laboratory grade pure n-octadecane paraffin with thermo-physical properties as listed in Table-1. The fin is assumed to be of aluminium. The temperature difference between the temperature of the wall and melting temperature of the PCM is set to be 20 $^{\circ}$ C. Thus, the wall temperature is 48 $^{\circ}$ C in the calculations. Initially, the fin and the PCM are in the melting temperature of the paraffin, 28 $^{\circ}$ C. The half thickness of the fin is assumed to be D=1 mm.

$$=\frac{(T_{w}-T_{m})\left\{e^{Bt-\sqrt{\frac{Bx^{2}}{A}}}-\frac{1}{2}e^{Bt-\sqrt{\frac{Bx^{2}}{A}}}\left(1-e^{2\sqrt{\frac{Bx^{2}}{A}}}+erf\left[\frac{x}{2\sqrt{At}}-\frac{\sqrt{Bx^{2}t}}{x}\right]+e^{2\sqrt{\frac{Bx^{2}}{A}}}+erf\left[\frac{x}{2\sqrt{At}}+\frac{\sqrt{Bx^{2}t}}{x}\right]\right)\right\}}{e^{Bt}}$$

$$+\frac{C}{B}$$

Where

 $T_f(x,t)$

$$A = \frac{k_f}{2} \tag{19}$$

$$(\rho c_p)_f$$

$$P = h$$
(21)

$$C = BT_m$$
(22)

The energy balance for the PCM interface location in the y-direction (Eq. 15) is a first order partial differential equation. It can be rewritten as

$$-\frac{a\partial S_y}{x\partial x} + b\frac{\partial S_y}{\partial t} = h(T_f - T_m), \qquad t > 0 \qquad (23)$$

S_y(x, 0) = 0 (24)

$$a = k_{I} (T_{w} T_{m})$$
(25)

The analytical and numerical results for the temperature distribution of the fin and distance from fin surface are shown in Figure 4 and Figure 6 respectively, when t=3600 s. The RMS error is 1.5×10^{-5} so, it can be concluded that there is good agreement between analytical and numerical results. Figure 5 shows the temperature distribution of the fin with five different time steps. It can be concluded that at these initial values the temperature of the fin stops changing at t=720 s. It happens due to constant value of heat transfer coefficient calculated from equation (10.1).

The thickness of the fin D has a big influence on the temperature distribution of the fin. Figure 7 shows the temperature distribution of the fin at different thicknesses of the fin when t=3600 s.

Table 1

Physical properties of the aluminium fin and n-octadecane paraffin

Property	Fin	PCM (l)
Density (p) kg m ⁻³	2713	777
Heat conductivity (k) Wm ⁻¹	180	0.149
K-1		
Heat capacity (c _p) J kg ⁻¹ K ⁻¹	960	2660
Latent heat of fusion (H) J		241360
kg ⁻¹		
Melting temperature (T _m)		28
⁰ C		
Viscosity (µ) kg m ⁻¹ s ⁻¹		0.00385
Thermal expansion		0.001
coefficient (β) K ⁻¹		



Figure 4: The comparison of analytical and numerical results for the temperature distribution of the fin when t=3600 s in a semi-infinite n-octadecane storage.



Figure 5: The x-directional temperature distribution of the fin at different time steps in a semi-infinite n-octadecane storage.



Figure 6: The comparison of analytical and numerical results for a solidliquid interface location when t=3600 s in a semi-infinite n-octadecane storage.



Figure 7: The x-directional temperature distribution of the fin when t=3600 s with different fin thickness in a semi-infinite n-octadecane storage.

V. RESULTS AND DISCUSSION

The effect of thickness (D) on temperature distribution with time is shown in Figure 8 and Figure 9 with the help of surface plots. Using these figures, we can conclude that the temperature of the fin increases when the thickness of the fin increases. But if we increase the parameter (D), it will also increase the weight and volume of the system. So we have to select an optimum value of (D).

We considered the values of D from 1 mm to 3 mm. Keeping this in mind, we included a dimensionless number i.e. aspect ratio (L/D) of fin to find out the optimum value for significant temperature distribution. From Figure 12 it can be concluded that for the given conditions if L/D ratio is greater than 200 the temperature distribution in fin is not significant.

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Figure 8: Surface plot of the temperature distribution of the fin with length and time for D=1 mm in a semi-infinite n-octadecane storage.



Figure 9: Surface plot of the temperature distribution of the fin with length and time for D=3 mm in a semi-infinite n-octadecane storage.



Figure 10: Solid-liquid interface location with length of fin and time for D=1 mm in a semi-infinite n-octadecane storage.



Figure 11: Solid-liquid interface location with length of fin and time for D=3 mm in a semi-infinite n-octadecane storage.



Figure 12: Surface plot of the temperature distribution of the fin with aspect ratio (L/D) and time in a semi-infinite n-octadecane storage.



Figure 13: Solid-liquid interface location with aspect ratio (L/D) of fin and time in a semi-infinite n-octadecane storage.

The effect of thickness (D) on solid-liquid interface with time is shown in Figure 10 and Figure 11 with the help of surface plots. Using these figures, we can conclude that the solid liquid interface moves away from fin as we move away from end wall and increases the value of D from 1 mm to 3mm. For a value of greater than 200 for aspect ratio there is insignificant change in solid-liquid interface. So we can finally conclude that for given conditions

$$\left(\frac{L}{D}\right)_{optimum} \le 200 \tag{28}$$

VI. CONCLUSIONS

The purpose of this research was to providing a numerical model for PCMs used for thermal energy storage where PCMs exchange heat with fin surface. The model was evaluated for the given assumptions, boundary and initial conditions. The results were compared with analytical solution given by [6] and found to be in good agreement. During the analysis we consider the effect of time, half thickness and L/D i.e aspect ratio. From the analysis it can be concluded that when time > 720 s the change in fin temperature is insignificant for a particular length of the fin and if we increase D the temperature of fin increases but it affects the weight and volume of storage system. Finally we found 200 as optimum value of aspect ratio for significant changes in the temperature of fin and solid-liquid interface.

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