Proceedings of the World Congress on Engineering 2017 Vol II
WCE 2017, July 5-7, 2017, London, U.K.

The Interaction between a Coated Foundation and a Rigid Punch with Rough Surfaces

Alexander V. Manzhirov, *Member, IAENG,* Kirill E. Kazakov

***Abstract*—Contact problem for a double layer foundation and a rigid punch is considered. It is assumed that the shape of the thin upper layer (roughness) and the shape of the punch base can be described by different rapidly changing functions. The basic integral equation of the problem is obtained. A projection method is developed which allows one to obtain the solution of the equation with high accuracy that cannot be done by known methods. An algorithm of numerical-analytical calculation is described. Quantitative mechanical effects are discussed. A model example is presented.**

***Index Terms*—coating, foundation, projection method, rapidly changing functions, roughness**

INTRODUCTION

THE problem of contact interaction between an elastic foundation with a coating of constant thickness and a rigid punch whose surface is described by a certain function was studied in the works [1]–[3]. The solution of this problem was obtained as a series in normed Legendre polynomials. Later, a similar problem was posed and solved for inhomogeneous aging viscoelastic foundations (see, e.g., [4]). In this problem, it was assumed that the lower layer is made of a single material and ages homogeneously, while the upper layer (the coating) has an inhomogeneity in both the vertical and horizontal coordinates and its thickness experiences weak fluctuations. Its solution was also sought as a series in Legendre polynomials but already with time-dependent coefficients. Soon it became clear that, in the cases where either the punch base shape or the coating thickness or the inhomogeneity is described by oscillating or even discontinuous functions (for example, if the coating is made of two materials with different properties), the solution representation as a series in classical orthogonal polynomials is inefficient [5]. Therefore, a demand arose for new approaches and methods for solving the problems with parameters described by such functions. One of such methods is the generalized projection method, allowed one to solve: (a) the problem of conformal contact in the case of actual profiles of surfaces [5]–[7], i.e., in the case, where the punch base shape and the shape of the coating surface in unstrained state repeat each other; (b) the problem of contact interaction between a punch and a foundation with surface-inhomogeneous coating in the case, where the coating inhomogeneity is given by an oscillating or even discontinuous function [8]. In the present paper, we study the contact interaction between a viscoelastic foundation with a homogeneous thin rough coating and rigid punch in the case, where the punch base surface and coating width are described by different rapidly changing functions.

Manuscript received March 20, 2017; revised April 3, 2017. This work was financially supported by the Russian Foundation for Basic Research under Project No. 17-51-80010 BRICS_a.

A. V. Manzhirov is with the Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, Vernadsky Ave 101 Bldg 1, Moscow, 119526 Russia; the Bauman Moscow State Technical University, 2nd Baumanskaya Str 51, Moscow, 105005, Russia; the National Research Nuclear University MEPhI, Kashirskoye shosse 31, Moscow, 115409, Russia; the Moscow Technological University, Vernadsky Ave 78, Moscow, 119454, Russia; e-mail: manzh@inbox.ru.

K. E. Kazakov is is with the Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, Vernadsky Ave 101 Bldg 1, Moscow, 119526 Russia; the Bauman Moscow State Technical University, 2nd Baumanskaya Str 5/1, Moscow, 105005, Russia; e-mail: kazakov-ke@yandex.ru

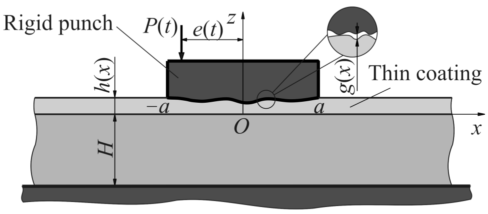

Fig. 1. Basic scheme of contact interaction

I. STATEMENT OF THE PROBLEM

We assume that a layer of an arbitrary thickness H with a thin coating of an thickness $h(x)$ lies on a rigid basis. This layers are made of different viscoelastic aging materials. We denote the moments of coating and lower layer production by τ_1 and τ_2, respectively. We also assume that the coating rigidity is less than the rigidity of the lower layer or they are of the same order of magnitude. There is smooth contact or perfect contact between layers and between the lower layer and the rigid base.

At time τ_0, the force $P(t)$ with eccentricity $e(t)$ starts to indent a rigid punch of width $2a$ (Fig. 1) into the surface of such a foundation. The function $g(x)$ describes distance between contact surfaces in nondeformable state and called backlash function. If the function $f(x)$ describes the form of the punch then $g(x) = f(x) - h(x) + h_0$, where $h_0 = -\min_{x\in[-a,a]}[f(x) - h(x)]$. In the case of conformal contact (see, for example, [6], [7]) $g(x) \equiv 0$. Contact area is constant and equal to $2a$. The coating is assumed to be thin compared with the contact area, i.e., its thickness satisfies the condition $h(x) \ll 2a$.

The integral equation of this problem can be written in the form

$$\begin{aligned}(\mathbf{I}-\mathbf{V}_1)&\frac{\theta q(x,t)h(x)}{E_1(t-\tau_1)} \\ &+(\mathbf{I}-\mathbf{V}_2)\frac{2(1-\nu_2^2)}{\pi E_2(t-\tau_2)}\int_{-a}^{a}k_{\rm pl}\Big(\frac{x-\xi}{H}\Big)q(\xi,t)\,d\xi \\ &=\delta(t)+\alpha(t)x-g(x)\quad(-a\leqslant x\leqslant a),\end{aligned}\tag{1}$$

$$\mathbf{V}_k f(x,t)=\int_{\tau_0}^{t}K^{(k)}(t-\tau_k,\tau-\tau_k)f(x,\tau)\,d\tau,$$

$$K^{(k)}(t,\tau)=E_k(\tau)\tfrac{\partial}{\partial\tau}\big[E_k^{-1}(\tau)+C^{(k)}(t,\tau)\big],\quad k=1,\,2,$$

ISBN: 978-988-14048-3-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

where $\delta(t)$ is the punch settlement and $\alpha(t)$ is its tilt angle; $E_k(t)$ are the Young moduli of the coating ($k=1$) and the lower layer ($k=2$) and ν_2 is Poisson's ratio of the lower layer; $\mathbf{I}$ is the identity operator; $\mathbf{V}_k$ are the Volterra integral operators with tensile creep kernels $K^{(k)}(t,\tau)$ ($k=1,\,2$); $C^{(k)}(t,\tau)$ ($k=1,\,2$) are the tensile creep functions; θ is a dimensionless coefficient depending on the contact conditions between coating and lower layer; in the case of a smooth coating-layer contact, we have $\theta=1-\nu_1^2$, and in the case of an perfect contact, $\theta=(1-\nu_1-2\nu_1^2)/(1-\nu_1)$, where ν_1 is Poisson's ratio of the coating; $k_{\rm pl}[(x-\xi)/H]$ is known kernel of the plane contact problem, which has the form [1]

$$k_{\rm pl}(s)=\int_0^\infty\frac{L(u)}{u}\cos(su)\,du,$$

and, in the case of a smooth contact between the lower layer and the rigid base,

$$L(u)=\frac{\cosh 2u-1}{\sinh 2u+2u},$$

and in the case of a perfect contact,

$$L(u)=\frac{2\varkappa\sinh 2u-4u}{2\varkappa\cosh 2u+4u^2+1+\varkappa^2},\quad \varkappa=3-4\nu_2.$$

We supplement Eq. (1) with the condition of the punch equilibrium on the foundation

$$\int_{-a}^{a}q(\xi,t)\,d\xi=P(t),\quad \int_{-a}^{a}\xi q(\xi,t)\,d\xi=M(t). \tag{2}$$

Here $M(t)=e(t)P(t)$ denotes the moment of application of the force $P(t)$.

Let us make the change of variables in (1) and (2) by the formulas

$$\begin{gathered}
x^*=x/a,\quad \xi^*=\xi/a,\quad t^*=t/\tau_0,\quad \tau^*=\tau/\tau_0,\\
\tau_1^*=\tau_1/\tau_0,\quad \tau_2^*=\tau_2/\tau_0,\quad \lambda=H/a,\quad g^*(x^*)=g(x)/a,\\
\delta^*(t^*)=\frac{\delta(t)}{a},\quad \alpha^*(t^*)=\alpha(t),\quad c^*(t^*)=\frac{E_2(t-\tau_2)}{E_1(t-\tau_1)},\\
m^*(x^*)=\frac{\theta}{1-\nu_2^2}\frac{h(x)}{2a},\quad q^*(x^*,t^*)=\frac{2(1-\nu_2^2)q(x,t)}{E_2(t-\tau_2)},\\
P^*(t^*)=\frac{2P(t)(1-\nu_2^2)}{E_2(t-\tau_2)a},\quad M^*(t^*)=\frac{2M(t)(1-\nu_2^2)}{E_2(t-\tau_2)a^2},\\
\mathbf{V}_k^*f(x^*,t^*)=\int_1^{t^*}K_k(t^*,\tau^*)f(x^*,\tau^*)\,d\tau^*,\quad k=1,\,2,\\
K_1(t^*,\tau^*)=\frac{E_1(t-\tau_1)}{E_1(\tau-\tau_1)}\frac{E_2(\tau-\tau_2)}{E_2(t-\tau_2)}K^{(1)}(t-\tau_1,\tau-\tau_1)\tau_0,\\
K_2(t^*,\tau^*)=K^{(2)}(t-\tau_2,\tau-\tau_2)\tau_0,\\
\mathbf{F}^*f(x^*,t^*)=\int_{-1}^{1}k_{\rm pl}^*(x^*,\xi^*)f(\xi^*,t^*)\,d\xi^*,\\
k_{\rm pl}^*(x^*,\xi^*)=\frac{1}{\pi}k_{\rm pl}\Big(\frac{x-\xi}{H}\Big)=\frac{1}{\pi}k_{\rm pl}\Big(\frac{x^*-\xi^*}{\lambda}\Big).
\end{gathered} \tag{3}$$

Then, omitting the asterisks, we obtain a mixed integral equation and additional conditions in the dimensionless form

$$\begin{gathered}
c(t)m(x)(\mathbf{I}-\mathbf{V}_1)q(x,t)+(\mathbf{I}-\mathbf{V}_2)\mathbf{F}q(x,t)\\
=\delta(t)+\alpha(t)x-g(x)\quad(-1\leqslant x\leqslant 1),\\
\int_{-1}^{1}q(\xi,t)\,d\xi=P(t),\quad \int_{-1}^{1}\xi q(\xi,t)\,d\xi=M(t).
\end{gathered} \tag{4}$$

In what follows, we construct the solution of the two-dimensional equation with the auxiliary conditions (4), which contains integral operators with constant as well as variable limits of integration and two different rapidly changing functions.

There exist four different versions of the substitution: 1) the settlement and the tilt angle of the punch are known (i.e., the right-hand side of the integral equation is given); 2) the punch settlement and the force moment are known; 3) the tilt angle of the punch and the indenting force are known; 4) the indenting force and its moment application are known. Each of these statements is a separate problem with its specific integral operator, and it is necessary to construct four systems of eigenfunctions for these problems.

In what follows we will construct the solution of the fourth problem.

II. The solution for known force and moment

Now we introduce the notation

$$\begin{aligned}
&Q(x,t)=\sqrt{m(x)}\Big[q(x,t)+(\mathbf{I}-\mathbf{V}_1)^{-1}\frac{g(x)}{c(t)m(x)}\Big],\\
&k(x,\xi)=\frac{k_{\rm pl}(x,\xi)}{\sqrt{m(x)m(\xi)}},\\
&\mathbf{A}Q(x,t)=\int_{-1}^{1}k(x,\xi)Q(\xi,t)\,d\xi.
\end{aligned}$$

Then integral equation and auxiliary conditions (4) can be reduced to the following integral equation with the Hilbert–Schmidt kernel $k(x,\xi)$ (see, e.g. [9], [10]):

$$\begin{gathered}
c(t)(\mathbf{I}-\mathbf{V}_1)Q(x,t)+(\mathbf{I}-\mathbf{V}_2)\mathbf{A}Q(x,t)\\
=\frac{\delta(t)}{\sqrt{m(x)}}+\frac{\alpha(t)x}{\sqrt{m(x)}}+\frac{\tilde c(t)\tilde g(x)}{\sqrt{m(x)}}\quad(-1\leqslant x\leqslant 1),\\
\int_{-1}^{1}\frac{Q(\xi,t)}{\sqrt{m(\xi)}}\,d\xi=\tilde P(t),\quad \int_{-1}^{1}\frac{Q(\xi,t)}{\sqrt{m(\xi)}}\xi\,d\xi=\tilde M(t),
\end{gathered} \tag{5}$$

where

$$\begin{aligned}
&\tilde g(x)=\int_{-1}^{1}\frac{k_{\rm pl}(x,\xi)g(\xi)}{m(\xi)}d\xi,\\
&\tilde c(t)=(\mathbf{I}-\mathbf{V}_2)(\mathbf{I}-\mathbf{V}_1)^{-1}\frac{1}{c(t)},\\
&\tilde P(t)=P(t)+(\mathbf{I}-\mathbf{V}_1)^{-1}\frac{1}{c(t)}\int_{-1}^{1}\frac{g(\xi)}{m(\xi)}d\xi,\\
&\tilde M(t)=M(t)+(\mathbf{I}-\mathbf{V}_1)^{-1}\frac{1}{c(t)}\int_{-1}^{1}\frac{g(\xi)}{m(\xi)}\xi\,d\xi.
\end{aligned} \tag{6}$$

We seek the solution of Eq. (5) in the class of functions continuous in time t in the Hilbert space $L_2[-1,1]$ (e.g., see [4], [11]). To this end, we at first construct an orthonormal system of functions in $L_2[-1,1]$ which contains $1/\sqrt{m(x)}$ and remaining basis functions can be written as the products of functions depending on x and weight function $1/\sqrt{m(x)}$. The system of functions which satisfies the above conditions can be obtained by the following formulas [12]:

$$\int_{-1}^{1} p_i(\xi)p_j(\xi)\,d\xi = \delta_{ij}, \quad p_n(x) = \frac{p_n^*(x)}{\sqrt{m(x)}},$$
$$p_0^*(x) = \frac{1}{\sqrt{J_0}}, \quad J_n = \int_{-1}^{1} \frac{\xi^n}{m(\xi)}\,d\xi,$$
$$p_n^*(x) = \frac{1}{\sqrt{d_{n-1}d_n}} \begin{vmatrix} J_0 & J_1 & \cdots & J_n \\ J_1 & J_2 & \cdots & J_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x & \cdots & x^n \end{vmatrix}, \quad (7)$$
$$d_{-1} = 1, \quad d_n = \begin{vmatrix} J_0 & J_1 & \cdots & J_n \\ J_1 & J_2 & \cdots & J_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ J_n & J_{n+1} & \cdots & J_{2n} \end{vmatrix}.$$

Note that if $m(x) = \text{const}$ then the polynomials $p_n(x)$ are the orthonormal Legendre polynomials.

The Hilbert space $L_2[-1,1]$ can be presented as the direct sum of orthogonal subspaces $L_2[-1,1] = L_2^{(1)}[-1,1] \oplus L_2^{(2)}[-1,1]$, where $L_2^{(1)}[-1,1]$ is the Euclidean space with the basis $\{p_0(x), p_1(x)\}$ and $L_2^{(2)}[-1,1]$ is the Hilbert space with the basis $\{p_2(x), p_3(x), \ldots\}$. The integrand and the right-hand side of integral equation (5) can also be presented in the form of the algebraic sum of functions continuous in time t and ranging in $L_2^{(1)}[-1,1]$ and $L_2^{(2)}[-1,1]$, respectively, i.e.,

$$Q(x,t) = Q_1(x,t) + Q_2(x,t), \quad f(x,t) = f_1(x,t) + f_2(x,t),$$
$$Q_1(x,t) = z_0(t)p_0(x) + z_1(t)p_1(x),$$
$$\begin{aligned} f_1(x,t) &= \frac{\delta(t)}{\sqrt{m(x)}} + \frac{\alpha(t)x}{\sqrt{m(x)}} + \frac{\tilde{c}(t)\tilde{g}_1(x)}{\sqrt{m(x)}} \\ &= \left[\sqrt{J_0}\delta(t) + \frac{J_1}{\sqrt{J_0}}\alpha(t) + g_0\tilde{c}(t)\right]p_0(x) \quad (8) \\ &+ \left[\sqrt{\frac{J_0J_2 - J_1^2}{J_0}}\alpha(t) + g_1\tilde{c}(t)\right]p_1(t), \end{aligned}$$
$$f_2(x,t) = \frac{\tilde{c}(t)\tilde{g}_2(x)}{\sqrt{m(x)}}.$$

Here $Q_1(x,t), f_1(x,t) \in L_2^{(1)}[-1,1]$, $Q_2(x,t), f_2(x,t) \in L_2^{(2)}[-1,1]$, $\tilde{g}(x) = \tilde{g}_1(x) + \tilde{g}_2(x)$, $g_1(x)/\sqrt{m(x)} = g_0p_0(x) + g_1p_1(x) \in L_2^{(1)}[-1,1]$, $g_2(x)/\sqrt{m(x)} \in L_2^{(2)}[-1,1]$. Coefficients g_0 and g_1 can be determined by formulas

$$g_i = \sum_{l=0}^{\infty} R_{il} \int_{-1}^{1} \frac{p_l(\xi)g(\xi)}{\sqrt{m(\xi)}}d\xi, \quad i = 1,2,$$

where R_{mn} are expansion coefficients of the kernel $k(x,\xi)$:

$$k(x,\xi) = \sum_{m,l=0}^{\infty} R_{ml}p_m(x)p_l(\xi), \quad (9)$$
$$R_{ml} = \int_{-1}^{1}\int_{-1}^{1} k(x,\xi)p_m(x)p_l(\xi)\,dx\,d\xi, \quad m,l = 0,1,\ldots,$$

Note that the formula for $Q(x,t)$ contains known term $Q_1(x,t)$ which is determined by the first auxiliary condition (5)

$$z_0(t) = \frac{\tilde{P}(t)}{\sqrt{J_0}}, \quad z_1(t) = \frac{J_0\tilde{P}(t) + J_1\tilde{M}(t)}{\sqrt{J_0(J_0J_2 - J_1^2)}}, \quad (10)$$

and the term $Q_2(x,t)$ must be found. Conversely, for the right-hand side, one should find $f_1(x,t)$, while $f_2(x,t)$ is known and determined be the function $\tilde{g}_1(x)$. These peculiarities permit one to class the resulting problem as a specific case of the generalized projection problem stated in [13], [14].

We can introduce the orthogonal projection operator mapping the space $L_2[-1,1]$ onto subspace $L_2^{(1)}[-1,1]$

$$\mathbf{P}_1\phi(x,t) = \int_{-1}^{1} \phi(\xi,t)[p_0(x)p_0(\xi) + p_1(x)p_1(\xi)]\,d\xi.$$

Obviously, the orthoprojector $\mathbf{P}_2 = \mathbf{I} - \mathbf{P}_1$ maps the space $L_2[-1,1]$ onto $L_2^{(2)}[-1,1]$. It is clear, that $\mathbf{P}_i f(x,t) = f_i(x,t)$, $\mathbf{P}_i Q(x,t) = Q_i(x,t)$ $(i = 1,2)$.

Using [13], we apply the orthogonal projection operator $\mathbf{P}_2$ to integral equation (5). As a result, we obtain the equation for determining $Q_2(x,t)$ with a known right-hand side

$$\begin{aligned} c(t)(\mathbf{I} - \mathbf{V}_1)Q_2(x,t) + (\mathbf{I} - \mathbf{V}_2)\mathbf{P}_2\mathbf{A}Q_2(x,t) \\ = -(\mathbf{I} - \mathbf{V}_2)\mathbf{P}_2\mathbf{A}Q_1(x,t) + \frac{\tilde{c}(t)\tilde{g}_2(x)}{\sqrt{m(x)}}. \quad (11) \end{aligned}$$

It is necessary to construct its solution in the form of an expansion in the eigenfunctions of the operator $\mathbf{P}_2\mathbf{A}$ which is a compact, strong positive, and self-adjoint operator from $L_2^{(2)}[-1,1]$ into $L_2^{(2)}[-1,1]$. The system of eigenfunctions of such an operator is a basis in the space $L_2^{(2)}[-1,1]$. The spectral problem for the operator $\mathbf{P}_2\mathbf{A}$ can be written in the form

$$\mathbf{P}_2\mathbf{A}\varphi_k(x) = \gamma_k\varphi_k(x),$$
$$\varphi_k(x) = \sum_{i=2}^{\infty} \varphi_i^{(k)}p_i(x), \quad k = 2,3,\ldots, \quad (12)$$

and hence

$$\sum_{n=2}^{\infty} R_{mn}\varphi_n^{(k)} = \gamma_k\varphi_m^{(k)}, \quad k,m = 2,3,\ldots,$$

where coefficients $R_{m,n}$ determined by (9). Solving this system we can obtain γ_k and $\varphi_m^{(k)}$ $(k,m = 2,3,\ldots)$.

We expand the functions $Q_2(x,t)$ and $g_2/\sqrt{m(x)}$ with respect to the new basis functions $\varphi_k(x)$ $(k = 1, 2, \ldots)$ in $L_2^{(2)}[-1,1]$, i.e.,

$$Q_2(x,t) = \sum_{k=2}^{\infty} z_k(t)\varphi_k(x), \quad \frac{g_2(x)}{\sqrt{m(x)}} = \sum_{k=2}^{\infty} g_k\varphi_k(x),$$

where coefficients g_k defined by

$$g_k = \sum_{i=2}^{\infty} \varphi_i^{(k)} \sum_{l=0}^{\infty} R_{il} \int_{-1}^{1} \frac{p_l(\xi)g(\xi)}{\sqrt{m(\xi)}}d\xi, \quad k = 2,3,\ldots$$

Substituting this equation into (11) and taking into account that the unknown expansion functions $z_k(t)$ $(k = 1, 2, \ldots)$ can be determined by the formula

$$\begin{aligned} z_k(t) = (\mathbf{I} + \mathbf{W}_k)\big[(\mathbf{I} - \mathbf{V}_2)[g_k(\mathbf{I} - \mathbf{V}_1)^{-1}c^{-1}(t) \\ - z_0(t)K_k^{(0)} - z_1(t)K_k^{(1)}]\big]/[c(t) + \gamma_k], \end{aligned}$$
$$K_k^{(0)} = \sum_{i=2}^{\infty} R_{0i}\varphi_i^{(k)}, \quad K_k^{(1)} = \sum_{i=2}^{\infty} R_{1i}\varphi_i^{(k)}, \quad (13)$$
$$\mathbf{W}_k f(x,t) = \int_1^t R_k^*(t,\tau)f(x,\tau)\,d\tau,$$

ISBN: 978-988-14048-3-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

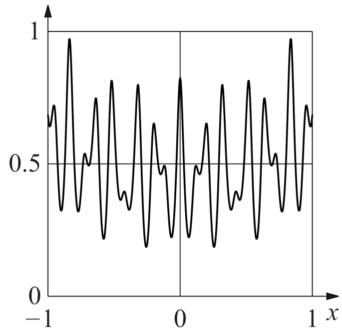

Fig. 2. Contact stress for a model with rough surfaces

where $R_k^*(t,\tau)$ $(k = 1, 2, \ldots)$ is the resolvent of the kernel

$$K_k^*(t,\tau) = \frac{c(t)K_1(t,\tau) + \gamma_k K_2(t,\tau)}{c(t) + \gamma_k}.$$

Note that the final solution has the following structure

$$q(x,t) = \frac{1}{m(x)}\left[z_0(t)P_0(x) + z_1(t)P_1(x) + \ldots - \frac{g(x)}{c(t)}\right],$$

i.e., one can explicitly write out the weight functions $m(x)$ and $g(x)$ in the solution. Note that the coating thickness is related to $m(x)$ and backlash function is related to $g(x)$ in the relations of change of variables (3). The formulas obtained permit obtaining efficient analytic solutions for the layers with rough coatings which can be described by complicated and rapidly oscillating functions. Such a result can hardly be done by other known methods.

In order to find the unknown punch settlement and tilt angle we act integral equation (5) by operator $\mathbf{P}_1$

$$\alpha(t) = \sqrt{\frac{J_0}{J_0 J_2 - J_1^2}}\Bigg\{c(t)z_1(t) + (\mathbf{I} - \mathbf{V}_2)\bigg[-(\mathbf{I} - \mathbf{V}_1)^{-1}\frac{g_1}{c(t)} + R_{10}z_0(t) + R_{11}z_1(t) + \sum_{k=2}^{\infty} K_k^{(1)} z_k(t)\bigg]\Bigg\},$$

$$\delta(t) = \frac{1}{\sqrt{J_0}}\Bigg\{c(t)(\mathbf{I} - \mathbf{V}_1)z_0(t) + (\mathbf{I} - \mathbf{V}_2)\bigg[-(\mathbf{I} - \mathbf{V}_1)^{-1}\frac{g_0}{c(t)} + R_{00}z_0(t) + R_{01}z_0(t) + \sum_{k=2}^{\infty} K_k^{(0)} z_k(t)\bigg]\Bigg\} - \alpha(t)\frac{J_1}{J_0}.$$

III. Conclusions

- We stated and solved the plane problem of contact interaction between a viscoelastic aging foundation with a coating and a rigid punch in the case when the punch base shape and the coating roughness are described by rapidly changing functions.
- The solution of the problem is obtained analytically. The expression for the contact stresses contains the coating width and backlash function explicitly. This allows one to perform computations for actual shapes of interacting surfaces using small number of expansion terms. (Fig. 2).
- Other known methods for the solution of this problem diverge with the increase of the time parameter or give an error for contact stresses up to 100%.

References

[1] V. M. Alexandrov and S. M. Mkhitaryan, *Contact Problems for Bodies with Thin Coatings and Interlayers [in Russian]*. Moscow: Nauka Publ., 1983.

[2] A. V. Manzhirov, "Axisymmetric contact problems for non-uniformly aging layered viscoelastic foundations." *J Appl Math Mech*, vol. 47, no. 4, pp. 558–566, 1983.

[3] A. V. Manzhirov, "On a method of solving two-dimensional integral equations of axisymmetric contact problems for bodies with complex rheology." *J Appl Math Mech*, vol. 49, no. 6, pp. 777–782, 1985.

[4] N. Kh. Arutyunyan and A. V. Manzhirov, *Contact Problems in the Theory of Creep [in Russian]*. Yerevan: Izd-vo NAN Armenii, 1999.

[5] K. E. Kazakov and A. V. Manzhirov, "Conformal contact between layered foundations and punches." *Mech Solids*, vol. 32, no. 3, pp. 512–524, 2008.

[6] A. V. Manzhirov and K. E. Kazakov, Conformal contact between foundations and punches, in N. K. Gupta and A. V. Manzhirov (Editors) *Topical Problems in Solid Mechanics*. New Delhi: Elite Publishing, 2008, pp. 92–104.

[7] A. V. Manzhirov and K. E. Kazakov, "Contact problem for a foundation with a rough coating." Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2016, WCE 2016, 29 June - 1 July, 2016, London, U.K., pp. 877-882.

[8] K. E. Kazakov, "Modeling of contact interaction for solids with inhomogeneous coatings." *J Phys Conf Ser*, vol. 181(012013), 2009.

[9] E. Goursat, *Cours d'Analyse Mathématique*, Tome III. *Intégrales Infiniment Voisines; Équations aux Dérivées Partielles du Second Ordre; Équations Intégrales; Calcul des Variations*. Paris: Gauthier–Villars, 1927.

[10] S. G. Mikhlin, *Integral Equations: And Their Applications to Certain Problems in Mechanics, Mathematical Physics and Technology*. London: Pergamon Press, 2014.

[11] V. S. Vladimirov, *Equations of Mathematical Physics*. New. York: Marcel Dekker, 1971.

[12] G. Szegö, *Orthogonal Polynomials*. Providence: Amer. Math. Soc, 1959.

[13] A. V. Manzhirov, "A mixed integral equation of mechanics and a generalized projection method of its solution." *Dokl Phys*, vol. **61**, no. 10, pp. 489–493, 2016.

[14] A. V. Manzhirov, "Integral equations with several different operators and their application to mechanics." Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2016, WCE 2016, 29 June - 1 July, 2016, London, U.K., pp. 10–15.

ISBN: 978-988-14048-3-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)