

Solar Radiation Polyhedron Sensor with Self-calibration Facility

P. Salgado, P. Mestre, *Member, IAENG*, C. Serodio, *Member, IAENG*, and P. Afonso

Abstract—The aim of this study was to develop an intelligent sensor for acquiring solar radiation components data, and estimate cloudiness indexes, through a solid polyhedron surfaces coated by LDR sensors that measure the surface incident radiation. Based on geometrical aspect of sensors, a mathematical model is used to compute the sun pairwise azimuth and elevation angles as well as the direct and diffuse radiation. However the numerical results are frequently contaminated by systematic and aleatory errors. Even more, error on parameters of sensor model, namely due to the polyhedron surfaces erroneous orientation or due to badly sticking LDR, they can significantly impair the accuracy of the measurement process and compromise the sensor performance. To solve this problem in this paper the actual angles of surface parameters of the linearised model are estimated by the EM algorithm, which is iteratively applied. The *E* (expectation) step of the algorithm determines the expected value of the likelihood function given the radiation of surfaces (observations) and the current estimate of the angles of surfaces (unknown parameters), while the *M* (maximization) step computes new estimates by maximizing the expectation of direct solar irradiance, magnitude and direction, and diffuse irradiance (the likelihood function). The method was validated using a simulation in the presence of gross errors for the angles of the sensor surfaces.

Index Terms—radiation sensor, sun orientation, expectation-maximization algorithm, calibration of sensor

I. INTRODUCTION

SOLAR irradiation is the power per unit area received from the Sun. Their data are fundamental inputs for planning and management of solar energy applications, particularly on photovoltaic systems [1], solar thermal systems [2], greenhouses for agriculture [3], lighting and thermal regulation in buildings [4]. It also affects plant metabolism and animal behaviour.

Irradiance may be measured on the Earth's surface. This process depends on the tilt of the measuring surface, the height of the sun above the horizon, and atmospheric conditions. One part of Total Solar Irradiance (*TSI*) that reaches

on the Earth's upper atmosphere is reflect back to space and the remain is propagate to surface in two ways. The light scattered by the atmosphere reaches the ground from all points of sky, excluding circumsolar radiation. This is the Diffuse Irradiance, *D*. Direct irradiance, *I*, is the remaining part of the extraterrestrial irradiance, discounting the atmospheric losses due to absorption and scattering, which has a straight direction. Losses depend of cloud cover, gas atmosphere moisture content, and it is affected by light paths in atmosphere related with the apparent daily sun movement, namely with solar elevation angle and azimuth. All irradiance components are dependent on the time of the day. Less significant is the *TSI* variation with time of year due to the variation of distance between the Earth and the sun by about 6.6%, or sun activity variation with less of 1%. A third component is the reflected radiation, *R*, normally by mountains, neighbouring buildings and/or other objects, whose value may not be negligible.

Global Solar Radiation, *G*, includes the three above components linked up as follows:

$$G_{\varphi} = I_{\varphi} + D_{\varphi} + R_{\varphi} \quad (1)$$

All of these components is affected by orientation of the surface, i.e. with the surface angles $\varphi = [\alpha, \beta]$. α is the angle that surface do with the South direction of its geographical lies location, and β the inclination angle of surface in relation to horizontal plan of local.

From a *TSI*, with an average annual value around 1367 Wm^{-2} (outer atmosphere), only a maximum value around 1000 Wm^{-2} reaches a normal surface located at sea level, with cloudless sky and with sun in zero zenith angle. Under these conditions and during a typical summer day, the 1367 Wm^{-2} distribute its value to 1050 Wm^{-2} of direct beam radiation, that reaches an horizontal surface at ground level, and a diffuse radiation of low value near to 100 Wm^{-2} . Global radiation, *G* is a value of 1120 Wm^{-2} [5].

For all systems that are influenced by solar irradiation, the known about daily data of direct *I* and diffuse *D* radiation on horizontal or sloping surfaces is very useful. Such an information allows us to obtain an accurate model[6] for the impact of solar energy gains [7][8] as well as to monitor the performance of installed solar panel systems [9][10], solar gains in vertical glass or glass walls of buildings [11], daylight levels, evaluation of the climatic conditions of agricultural processes or to control the manage flux of energy in others systems sun radiation dependent [12]. Solar data are also essential for engineers to design various solar energy conversion devices, to make an intelligent use of the energy of the Sun. So, the quality and accuracy of information about the global solar radiation hitting the earth surface is important for many natural and technological systems.

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Weather stations can provide reliable records of solar radiation. However they usually record the radiation on the horizontal surface and its information is often not accessible on-time or in an inappropriate format. For example, solar radiation on undulating terrain is affected by multiple light reflections and its measure requires more complex processing that it would by taking only into account the variability of the tilted guidance relative to the Sun [11]. Often this measurement process is incomplete, failing to provide the light direction and radiation components, important information in many real-world applications. A review article about sun position sensors used in solar applications can be found in [13].

In a recent work [14] it was proposed a low-cost sensor based on a set LDRs (Light-Dependent Resistors) attached to an equal number of surfaces of a regular polyhedron. These surfaces are oriented in various direction in order to cover the entire Sun annual motion, from a daily journal from the sunrise to sunset. These LDRs sensor signals are acquire by a microcontroller-based acquisition board, which processes them to remove noise and erroneous readings. Finally, it sends the data to a computer. An application receives these values, form the microcontroller and processes them to compute direct and diffuse radiation and estimate the sun position (elevation and azimuth angles). The number and typologies of polyhedron surfaces (equal of number of LDR sensors) are in order to provide with hight level of information redundancy about the solar radiation, from that it is possible to computing the above described physical variables and unknown sensor model parameters. Moreover, in this work authors extend the capability of the sensor to automatically tune its model parameters, namely the angles of its surfaces.

Exact position of the Sun can be computed by a set of mathematical equations, resulted of cinematic relation about relative moving between the sum and the Earth. For that, it is only need to specify the date and hour of the day and the geographic localization the observer on Earth [15]. With these useful information in hand, it is theoretically possible to align the photovoltaic panel with the Sun's rays, thus increasing the energy production [16]. However, often weather conditions and erroneous panel's reference orientation compromise the results. For instance, incorrect clock setting and bad orientation of the sensor reference relatively to the geographic North are frequent, particularly in mobile platforms such as robots, caravans, or boats.

This paper is organized as follows. The next section introduces the mains concepts about solar radiation components to be measured by the sensor. Geometric structure of the sensor and its electric configuration are presented in section III. Fundamental equations that relate the measured surface irradiation values with the intensity of the radiation vector and the diffuse radiation components are explained. The problem about the sensibility of sensor model parameters relating with surfaces orientation errors is here formulated. In section IV it is proposed the self-tuning algorithm for automatic auto-calibration of sensor, namely describing its functionalities and characteristics. In section V, computational results of the testing and model validation and algorithm structure are presented. The sensor's performance is analysed and validated by computational tests. Finally, we present the necessary

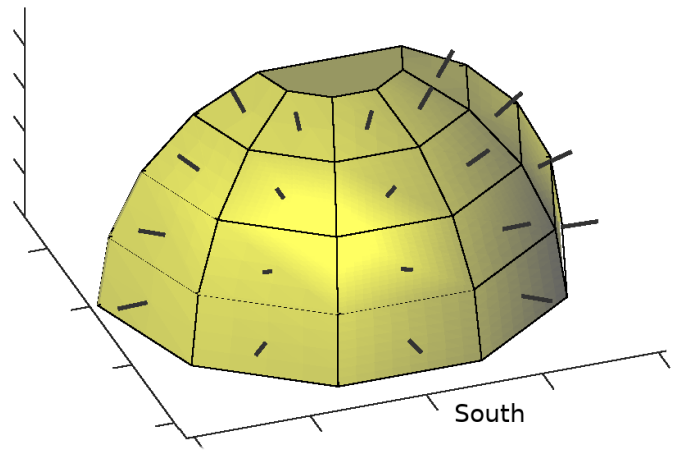


Fig. 1. Polyhedron sensor with $5 \times 4 + 1$ surfaces

conclusions about the proposed sensor's performance and discuss its real practical merit.

II. DIRECT AND DIFFUSE RADIATION

Direct radiation is characterized by being parallel light rays with a given direction and amplitude, which is adequately represented by the vector \vec{I} with dimensions 3×1 . The direction of vector \vec{I} can be defined by a pair of angles, azimuth (Az) and elevation (El) according to the following matrix equation:

$$\vec{I}_1 = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} -\cos Az \times \cos El \\ \sin Az \times \cos El \\ \sin El \end{bmatrix} \quad (2)$$

As explaining in next section, the \vec{I} vector and the diffuse radiation D can be computed from the measured values of radiation of sensor surfaces, R . So, if the estimation of \vec{I} is satisfactory, with good accuracy level, then it is possible to calculate the Elevation (El) and Azimuth (Az) angles in the following way:

$$El = tg^{-1} \left(\frac{I_z}{\|\vec{I}\|} \right) \quad (3)$$

and

$$Az = \pi - tg^{-1} \left(\frac{I_y}{I_x} \right) \quad (4)$$

III. THE SENSOR SYSTEM

Geometrical shape of proposed radiation sensor has polyhedron as illustrated in Fig. 1, an example with 5×4 oblique surfaces and a top horizontal surface. The number of polyhedron surfaces, ns , must be high enough in order to guarantee redundancy. This property is important, not only for establish a convenient relation between the measured surfaces radiation with intensity of diffuse and direct radiation as well as to estimate its direction, but also to allow us to implement an automatic strategy to auto-tuning or auto-calibration of the sensor mechanism, which will be described in section IV.

Polyhedron surfaces are represented by vectors perpendicular to the surface plane, $\vec{S}_i = (S_{i,x}, S_{i,y}, S_{i,z})$, for

$i = 1, \dots, ns$, with i^{th} surface associated to the i^{th} LDR sensor. Their magnitudes are here considered as unitary. The sensor has an orientation such that it is aligned with the South direction. From this frame, orientation of the surfaces are characterized by the angle α , which measure its direction relative to Earth South, and by an angle β what measure the surface inclination in relation to the horizontal ground plane. Taking into account the reference x_0, y_0, z_0 , with x_0 aligned with the South direction, the unitary surface vector is given by:

$$\vec{S}_i = \begin{bmatrix} S_{i,x} \\ S_{i,y} \\ S_{i,z} \end{bmatrix} = \begin{bmatrix} \cos \alpha_i \cdot \cos \beta_i \\ \sin \alpha_i \cdot \cos \beta_i \\ \sin \beta_i \end{bmatrix} \quad (5)$$

The radiation on the i^{th} surface R_i is the sum the scalar value of diffuse radiation D with the projection of the direct incoming light intensity \vec{I} on received surface:

$$R_i = \vec{S}_i \cdot \vec{I} + D = \vec{S}_i^T \vec{I} \quad (6)$$

with $\vec{S}_i = [\vec{S}_i^T \ 1]^T$ and $\vec{I} = [\vec{I}^T \ I_0]^T$. \vec{I} is a vector with dimension 4×1 and represent the components of global radiation, $\vec{I} = [I_x \ I_y \ I_z \ I_0]^T$. Surface vector $\vec{S}_i = [\vec{S}_i^T \ 1]^T$ is now in homogeneous coordinates. In this way, the diffuse radiation, $I_0 = D$, which comes from all direction, reaches with same value all LDR sensors.

The measured radiation of i^{th} sensor is given by $R_i = (\cos \alpha_i \cos \beta_i I_x + \sin \alpha_i \cos \beta_i I_y + \sin \beta_i I_z) + I_0$ with the surface plane of the sensor defined by α_i and β_i angles.

However, in a real situation surface angles are not precise. Their real value are $\alpha_i = \bar{\alpha}_i + \Delta\alpha_i$ and $\beta_i = \bar{\beta}_i + \Delta\beta_i$, where $\bar{\alpha}_i$ and $\bar{\beta}_i$ are the surface angles that are assumed to be true values, while $\Delta\beta_i$ and $\Delta\alpha_i$ are the unknown quantity errors. So, the vector of the measured radiation sensor is:

$$\vec{R} = \bar{R} + \Delta\vec{R} \quad (7)$$

where $\vec{R} = [R_1 \ \dots \ R_i \ \dots \ R_{ns}]^T$ is the vector of the measured radiation sensors and $\Delta\vec{R}$ is the vector that measure the discrepancy form the estimated values sensors, \bar{R} , from measured values, \vec{R} . So, $\Delta\vec{R}$ reflect an erroneous of matrix of vectors of sensor surfaces.

Light intensity \vec{I} is computed by polyhedron sensor taking the measures values of the ns LDR surface sensors. If there are no errors in the vectors surfaces this problem can be solved by:

$$\vec{I} \approx \bar{\Omega}^\dagger \cdot \vec{R} \quad (8)$$

where \vec{I} is the estimation of \vec{I}^T . $\bar{\Omega}^\dagger$ is the Moore–Penrose inverse of $\Omega = [S_1 \ \dots \ S_i \ \dots \ S_n]^T$, the matrix of the surface vectors, which provides a least squares solution to a linear equations system (Eq. 8).

From components of \vec{I} the sun elevation and azimuth angles of Sun position is estimated based on Eq. (3) and Eq. (4).

However, here we assume that matrix Ω is not perfectly known due to badly surface orientation. So the real Ω matrix differs from the theoretical expected matrix $\bar{\Omega}$ by a perturbation matrix $\Delta\Omega$, i.e. $\Omega = \bar{\Omega} + \Delta\Omega$. The solution of this problem is not trivial. In next section we proposed

the use of EM algorithm to estimate $\Delta\Omega$. With this resulted others variables can be estimated with more accuracy.

IV. EM ALGORITHM

The output of polyhedron sensor must be in concordance with equation 6. If it is an exact model, for each real-intensity irradiance \vec{I} , the estimate \bar{R} is equal to the measured values R (here designated as y). However, because the sensor model has an unknown set of parameters, the matrix $\theta = \Delta\Omega$, with $\theta \in \Theta \subset R^r$ the output data of the sensor are affected by an error value. Moreover, \vec{I} is a non-observed variable (here designed as z) with $z \in Z \subset R^{n-m}$. EM algorithm is useful for parameter estimation when:

- Direct access to all the data $x = \{y, z\}$, needed for the estimation, is not possible.
- Observations $y \in Y \subset R^m$, with $m < n$, provide data for estimation.
- Statistical model of the process is available, here considered as normal distribution $f(x|\theta)$ and $g(y|\theta)$.

In this work the EM algorithm is used to estimate the unknown parameters θ , based on the fact the distribution f and g are known. It is used to expected values of the unknown data given to obtain the maximum likelihood estimate of the parameters given the complete set of data (known data plus estimate of the unknown data). An initial estimate of the parameters and the known data is done. This process is used iteratively until the estimate converges, in two steps:

- *E(expectation)*: obtain expected values for the missing data using an initial parameter estimate. At time with parameter estimate $\theta^{(k)}$, compute the auxiliary function $Q(\theta|\theta^{(k)}) = E\{\ln[f(x|\theta)]|y, \theta^{(k)}\}$. Q is the expected value of the log-likelihood of the complete data $x = \{y, z\}$, given the current parameter estimate and the measurements y . For discrete data $Q(\theta|\theta^{(k)}) = \sum_{z \in Z} \ln[f(y, z|\theta)] f(z|y, \theta^{(k)})$.
- *M(aximization)*: obtain the maximum likelihood estimate of the parameters using the estimated data. Obtain the updated maximum likelihood parameter estimate using the function. $\theta^{(k+1)} = \arg \left\{ \max_{\theta} Q(\theta|\theta^{(k)}) \right\}$

Repeat until the estimate converges.

V. RESULTS

To test the algorithm and sensor model, a polyhedron sensor with 5×4 tilted surface and an upper top horizontal surface is used. All of them have their surfaces orientation angles (α 's and β 's) deviated from their expected value of random values in the range $\pm 10^\circ$.

A set of radiation data with variable magnitude values for azimuths angles, Az , between 80 and 280° (with $Az = 0$ for the north direction) and elevation, El , between 0 and 90 degrees, for angular increments of $\pm 2^\circ$ is used in learning process, where EM algorithm tuning the sensor model parameters. The accuracy results of the forecasting models will be measured by the sum of errors expressed in absolute $L1$ norm and Root Mean Square (RMS or $L2$):

$$L_1(x) = \frac{1}{n} \sum_{i=1}^n |\hat{x}_i - x_i| \quad (9)$$

TABLE I
AZIMUTH AND ELEVATION ANGLES ERRORS

	<i>Al error</i>		<i>Az error</i>	
	L_1	L_2	L_1	L_2
Untuned	1.49	1.46	4.77	10.98
Tuned	0.79	0.79	2.52	5.54

$$L_2(x) = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{x}_i - x_i)^2} \quad (10)$$

For measure the directional error between the incoming direct radiation \vec{I} and its model estimation \hat{I} , we are use the collinearity factor:

$$Ed(\vec{I}) = 1 - \frac{\vec{I} \cdot \hat{I}}{|\vec{I}| |\hat{I}|} \quad (11)$$

or $Ed(\vec{I}) = 1 - \cos(\delta)$, where δ is the angle made between the vectors \vec{I} and \hat{I} . When these vectors are the same direction $\delta = 0$ and $Ed(\vec{I}) = 0$, the error of directional estimation is null.

Initially, the untuned sensor and for above presented test condition, have an *RMS* error of the estimation about $\pm 11^\circ$ and around $\pm 1.5^\circ$, respectively, for *RMS* error of the angle *El*. The mean of absolute error about directional incoming direct radiation, δ , is about 5° . With these error values the use of sensor is not advisable for the vast majority of practical application. Next the EM algorithm is used to estimated the angles the surface vectors, by iteratively tuning its values. This process is performed in 100 iterations. Figures 3 and 4 shown the evolution of α and β errors for 21 surfaces. In general, the errors angles surface converge to a null value, with strong reduction of *RMS* error values.

With the progressive correction of the model, the estimation of the position of the sun from its azimuth and elevation values is significantly improved, as shown in Fig. 6 and 5. If at the beginning of this process the estimation error was high, at the end of learning process we have an error of about 0.8 degrees for the elevation and 2.5 degrees for the azimuth. These values are relatively good values in most practical application. Table I presents other error measure for the initial and for tuned Sensor. *Al* and *Az* are directly calculated by equations (3) and (4) from the estimated vector of direct radiation direction. Thus the errors of these variables are related to the error *ED*, equation (11), which is greatly reduced by the presented learning method, as can be observed in Fig. 7, with a value stabilized near of zero at end of iterative process. Also, during this process the discrepancy of values between the LDR radiation measure, *R*, and its estimation by sensor model \hat{R} is also strong reduced. In Fig. 2 is shown the error $\Delta \hat{R}$ during the iterative tuning process. In the right side of figure are legend the curves with the number of sensor surface.

VI. CONCLUSION

In this paper authors presented a low-cost and feasible sensor for measuring Sun radiation with automatic tuning

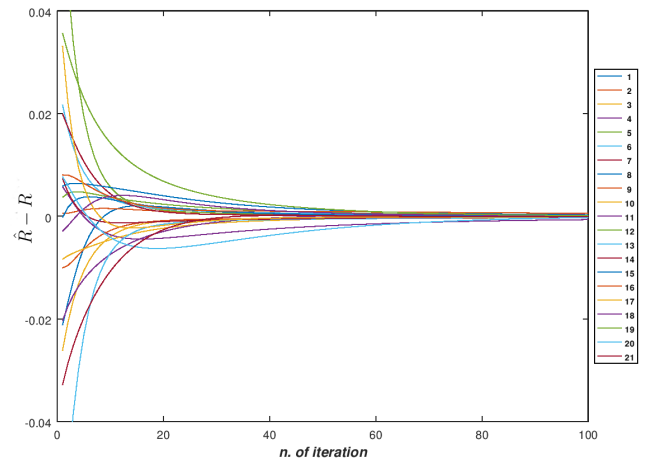


Fig. 2. Error of surface radiation of polyhedron

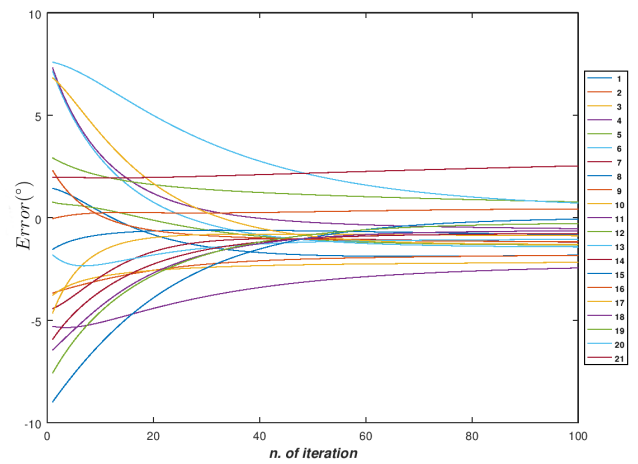


Fig. 3. Error estimation of α angle

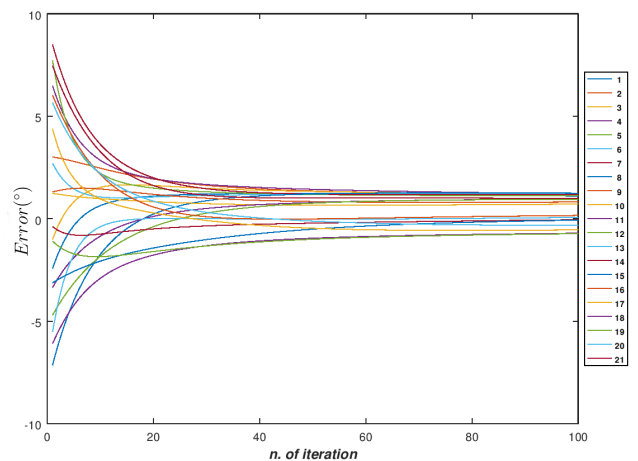


Fig. 4. Error estimation of β angle

capability. The sensor has the shape of a solid polyhedron, faceted with LDR electronic sensors, which output values are sent to a computer. Then, these values are used to estimate the Sun elevation and azimuth angles, as well to determine the values of direct and diffuse components of Sun radiation.

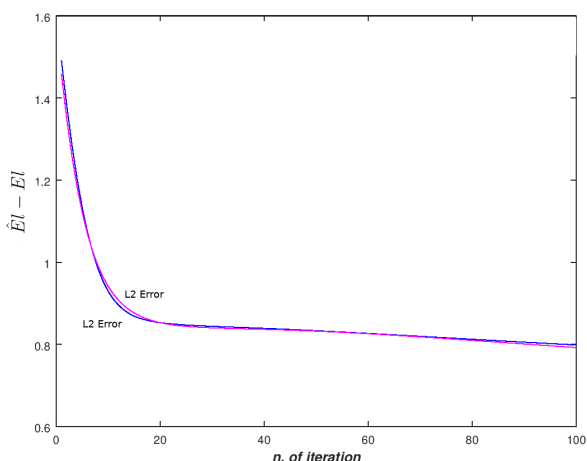


Fig. 5. Error estimation of Elevation angle

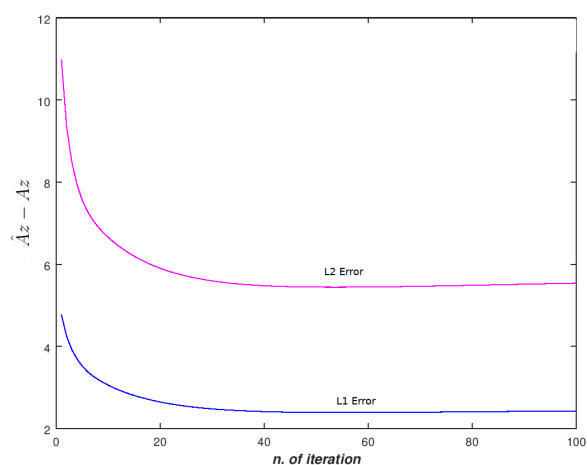


Fig. 6. Error estimation of Azimuth angle

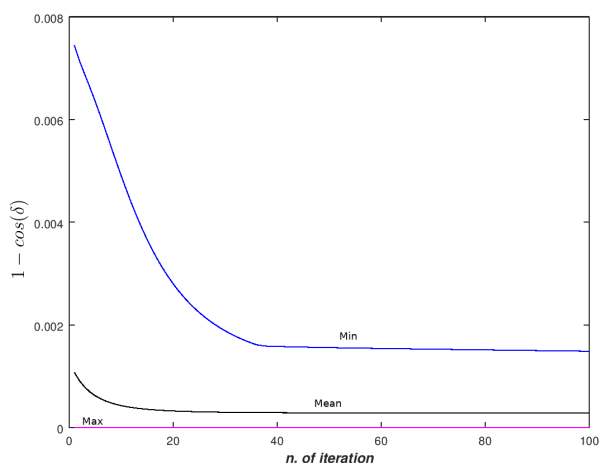


Fig. 7. Direction error of estimated incoming radiation, $E_d(\hat{I})$

Errors associated with surface orientation strongly deteriorate the performance of the sensor. Although there is a large amount and redundancy of information supplied by a significant number of LDR sensors, if the polyhedron sensor is not conveniently tuned, it loses its usefulness. In this

work, the redundant information is used by EM algorithm to calibrate the sensor. The results from the model simulation show that this method can solve this problem, as well as it has the potential to detect any faults in LDR sensors.

The experimental results show the convergence of the method and also a good agreement between the estimated and real measure variables, namely radiations measure of surfaces, R , diffuse radiation and incoming direct radiation \vec{I} , and its El e Az angles. However, the tests should continue, subjecting the sensor to a more and longer period of work test.

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