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Three Axis Attitude Control of RASAT by Magnetic Torquers

Yusuf Acar, *Member, IAENG* and Cagatay Yavuzyilmaz

***Abstract*—Spacecraft magnetic attitude control has been largely investigated, but the focus has generally been the control of satellites meeting conditions of gravity gradient stability by either inertia parameters or deployed boom. In this study, we consider three axes magnetic control for Turkish earth observation satellite, RASAT. This satellite is significant case study for magnetic control problem since it does not meet gravity gradient stability conditions on pitch axis owing to physical properties such as greater inertia along z axis than x axis. However, near polar orbiting satellites' pitch axis is controllable under magnetic torques. On the other hand, the controller is supposed to be simple to consider computational efficiency of onboard computer. For this reason, state feedback control methodology is applied. However, the significances of the controller are that the controller gain matrix is constant and its calculation is based on linear quadratic law with averaging method of slowly time varying system. Furthermore, this approach can be extended to control off-nadir attitude.**

***Index Terms*— Dynamic Attitude Simulator, Magnetic Torque, Periodic Systems, Satellite Attitude Control, Stabilization, Time-varying systems.**

I. Introduction

ONE of the most typical attitude control modes in a satellite mission is the coarse acquisition mode used for the transition from a safe mode where angular velocities are controlled to the normal operation mode, used for applications requiring fine pointing, such as imaging. The normal operation mode is three axis stabilized and typically performed by Reaction / Momentum wheels (RWs / MWs), owing to the fact that they can satisfy the stringent requirements on manoeuvrability and attitude accuracy. However, it is impractical to activate the normal operation mode directly after a safe mode and a transition mode makes it possible to bring sensors to the right field of view before normal operations. In this case, magnetic torquers take supreme attention to generate the torque required to stabilize satellites according to mission requirements.

More generally, magnetic torquers are advantageous for small low earth orbiting (LEO) satellites since they have lighter weight, less power consumption and are relatively cheaper than wheel based control systems.

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Y. Acar is with The Scientific and Technological Council of Turkey (TUBITAK), Space Technologies Research Institute and Middle East Technical University, Ankara, 06800, METU Campus, Turkey. (corresponding author e-mail: yusuf.acar@tubitak.gov.tr).

C. Yavuzyilmaz is with The Scientific and Technological Council of Turkey (TUBITAK), Space Technologies Research Institute, Ankara, 06800, METU Campus, Turkey.

Magnetic torquers exploit the earth magnetic field to actively produce two axis control torques at any time. Fortunately, the direction of the Earth's magnetic field is changing with respect to orbit position, so three axis control torques can be generated over an orbital period. Apart from the typical transition mode, another potential application is the use of magnetic attitude control for normal operations. This feature would be useful for nanosatellites, with little space to accommodate momentum exchange devices. Therefore, magnetic attitude control of spacecraft has been getting attention. Due to time variant of magnetic field strength on spacecraft body, time varying control methods are studied. However, the small satellites especially nanosatellites are generally not capable to solve the dynamics online. In order to reduce the computational complexity, averaging methods to obtain time invariant system [1], ε- strategies [2] and PD based linear control laws [3][4] are considered. However most of researches are based on gravity gradient stable satellite [2]. This can be accomplished by either deployed boom or slightly different inertia parameters. Some of the researches study the satellite with constant spinning momentum wheel along pitch axes. This momentum makes satellite pitch axes aligning to orbit pitch as well as providing extra stiffness against disturbances.

In this paper, three axis magnetic control for RASAT which is naturally unstable earth observation satellite is considered. The controller design is based on linear quadratic law with averaging method of slowly time varying system. This control design is tested on Dynamic Attitude Simulator Environment (DASE) [5] consisting of an OBC flight computer (exactly the same as on the satellite), an OBC test equipment, power supply, a ground station PC and DASE PC. This environment is to make as if on-board computer operates an actual satellite in space whereas in fact it is connected to the test setup. It will be shown in this work that by carefully adjusting the weighting matrices, sufficient performance can be achieved with magnetic torquers in terms of attitude error.

II. Dynamics of the System

The satellite rigid body attitude dynamics can be defined as:

$$\hat{\mathrm{J}}_s\bar{\alpha}_s + \tilde{\omega}_s\hat{\mathrm{J}}_s\bar{\omega}_s = \bar{\tau}_m + \bar{\tau}_g + \bar{\tau}_{dist} \tag{1}$$

$\hat{\mathrm{J}}_s$ is inertia matrix of satellite, $\bar{\alpha}_s$ is angular acceleration, $\bar{\omega}_s$ is angular rates and $\bar{\tau}_{\mathrm{dist}}$, $\bar{\tau}_g$, $\bar{\tau}_m$ are disturbance torques, gravity gradient torque and magnetic torques, respectively. $\tilde{\omega}_s$ is the skew-symmetric matrix of the angular rates defined as follows:

$$\tilde{a} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \tag{2}$$

$$\bar{\tau}_g = 3\omega_o^2 \tilde{e}_3 \hat{J}_s \bar{e}_3 \tag{3}$$

$\bar{e}_3$ is column matrix of unit zenith vector expressed in the body axis coordinates and ω_o is orbital rate ($\omega_o^2 = \mu_g/R_e^3$). Here, μ_g is gravitational constant and R_e is the satellite's geocentric radial distance.

The magnetic torque generated by magnetic torques can be defined as cross product of the magnetic dipole moment vector and geomagnetic field strength vector in body frame as follows:

$$\bar{\tau}_m = -\tilde{B}\bar{m} \tag{4}$$

The simplified model of Earth magnetic field in orbital fixed frame is derived as a first dipole moment by the following manner [6], [7], [8]:

$$\bar{B}^{(o)} = \frac{\mu_m}{R_e^3} \begin{bmatrix} \cos(\eta_m)\sin(i_m) \\ -\cos(i_m) \\ 2\sin(\eta_m)\sin(i_m) \end{bmatrix} \tag{5}$$

Here, μ_m is magnetic dipole strength equal to 7.9x10^{15} Wb-m, i_m is the magnetic inclination and η_m is phase angle which can be rewritten as ω_ot.

III. Control of the System

Spacecraft magnetic control has been largely studied; however, the focus has generally been the control of satellites meeting conditions of gravity gradient stability by either deployed boom or inertia parameters. For our study, the gravity gradient torques do not provide fully stable system. To gain insight, linearized equations of motion in a circular orbit under gravity gradient torques (no other torques) given in [8] is considered, characteristic equation for the pitch axis is $s^2+[3\omega_o^2 (J_1-J_3)/J_2]=0$ and it is clear that one of the roots is positive real number, unstable, if $J_1<J_3$. The necessary and sufficient condition for pitch stability is $J_1>J_3$. Necessary and sufficient conditions for roll/yaw stability are given [8] as:

$$k_1 = (J_2 - J_3)/J_1, \quad k_3 = (J_2 - J_1)/J_3$$
$$k_1 k_3 > 0$$
$$1 + 3k_1 + k_1 k_3 > 0$$
$$(1 + 3k_1 + k_1 k_3)^2 - 16 k_1 k_3 > 0$$

In the literature, the papers are considering the case $J_3<J_1<J_2$ where gravity gradient stabilization exists. In our case, the earth observation satellite shows roll/yaw stability however the pitch axis is not stable due to inertia property as follows:

$$\hat{J} = \begin{bmatrix} 7.2295 & 0.2393 & 0.1475 \\ 0.2393 & 7.0031 & -0.0058 \\ 0..1475 & -0.0058 & 7.6666 \end{bmatrix}$$

Although the pitch axis is unstable under the gravity gradient torques, pitch control torque is fortunately possible with magnetic torquers due to highly inclined orbit. There is a potential stabilization controller if the orbit plane does not coincide with magnetic equator plane [9]. Due to cross product nature of the generated magnetic torque (instant deficiency), the control input cannot be produced along the geomagnetic field vector. This system can be called instantly underactuated.

The inverse of $\tilde{B}$ is required to determine the magnetic dipole moment, *m* (consequently, control input). However, it is a skew symmetric matrix. In this point, it is assumed that magnetic dipole moment determined by controller is supposed to be perpendicular to the magnetic field vector and magnetic torque. Thus, magnetic dipole moment can be defined as follows [6]:

$$m = \frac{1}{\|B\|^2}\tilde{B}u_d = -\frac{1}{\|B\|^2}\tilde{B}^T u_d \tag{6}$$

Substituting (6) into (4) yields the control torque as:

$$\tau_m = \frac{1}{\|B\|^2}\tilde{B}\tilde{B}^T u_d = G(t)u_d \tag{7}$$

where u_d is 3 by 1 desired control torque column matrix and *G(t)* is 3 by 3 time-varying control influence matrix.

$$B^{(b)} = \hat{C}^{(b,o)} B^{(o)}$$
$$\tilde{B}^{(b)} = \hat{C}^{(b,o)} \tilde{B}^{(o)} \hat{C}^{(o,b)}$$
$$\hat{C}^{(b,o)T} = \hat{C}^{(o,b)}$$
$$\tilde{B}^{(b)}\tilde{B}^{(b)T} = \hat{C}^{(b,o)}\tilde{B}^{(o)}\hat{C}^{(o,b)}\hat{C}^{(o,b)T}\tilde{B}^{(o)T}\hat{C}^{(b,o)T} = \hat{C}^{(b,o)}\tilde{B}^{(o)}\tilde{B}^{(o)T}\hat{C}^{(o,b)}$$

$$\tau_m = \frac{1}{\|B\|^2}\hat{C}^{(b,o)}\tilde{B}^{(o)}\tilde{B}^{(o)T}\hat{C}^{(o,b)} u_d \tag{8}$$

$\hat{C}^{(b,o)}$ is attitude matrix and identity for nadir pointing.

$$\tau_m = \frac{1}{\|B\|^2}\tilde{B}^{(o)}\tilde{B}^{(o)T} u_d \tag{9}$$

After rewriting (1),

$$\begin{aligned} \dot{x} &= f(x) + g(x,t)u_d \\ \dot{x} &= Ax + G(t)u_d \end{aligned} \tag{9}$$

To build computational efficient controller and have global convergence over orbital periods, averaging of time-varying periodic dynamics is a suitable approach for slowly time-varying periodic systems. Averaging method builds

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time-invariant system dynamics approximating the actual dynamics. [10]

$$G(t) = G(t+T)$$

$$G_a(t) = J_S^{-1} \frac{1}{T} \int_0^T G(t) dt$$

$$G_a(t) = J_S^{-1} \frac{1}{T} \int_0^T \frac{1}{\|B\|^2} \hat{C}^{(b,o)} \tilde{B}^{(o)} \tilde{B}^{(o)T} \hat{C}^{(o,b)} dt \tag{11}$$

For nadir-pointing satellite, orbit frame axes are aligned to body frame axes $\hat{C}^{(b,o)} = \hat{I}$, therefore earth magnetic field vector is in both frames equal to each other ($B^{(b)} = \hat{C}^{(b,o)} B^{(o)} = B^{(o)}$).

$$G_a(t) = J_S^{-1} \frac{1}{\|B\|^2} \left(\frac{\mu_m}{R_e^3} \right)^2 \begin{bmatrix} G_{a_{11}} & & \\ & G_{a_{22}} & \\ & & G_{a_{33}} \end{bmatrix} \tag{12}$$

$$G_{a_{11}} = 2 - \cos^2(i_m)$$
$$G_{a_{22}} = 2.5\left(1 - \cos^2(i_m)\right)$$
$$G_{a_{33}} = 0.5\left(1 + \cos^2(i_m)\right)$$

$$\dot{x} = Ax + G_a u_d \tag{13}$$

The eigenvalues of G_a are positive and this averaged matrix is non-singular satisfying the conditions of controllability[11] and stability[12]. Let $u_d = -Kx$ in (13) and periodic closed loop system is $\dot{x} = (A - G_a K)x = A_c x$ and stable if the poles of closed loop system are negative for averaged systems.

To calculate the control input $u_d = -Kx$ consider LQR controller minimizing the following general cost function:

$$J = \frac{1}{2} \int_0^T \left(x^T Q x + u^T R u \right) dt + \frac{1}{2} x^T(T) P_T x(T) \tag{14}$$

where Q, R, P are state weighting matrix, control input weighting matrix and solution of Riccati equation, respectively.

Reminding that increasing the state weighting matrix Q or reducing control input weighting matrix R will yield a faster response and use more control effort to produce large control magnetic dipole moment. In real application, this weighting matrices should be arranged through simulation due to saturation of actuators. However, it is better to have proper value for initial guess by using Bryson's Rule, also called inverse square law which is based on normalization of states and control inputs. For simplicity off-diagonal elements of weighting matrices can be selected as zero, meaning that the eigenvalues of weighting matrices are considered as:

TABLE I
RASAT PARAMETERS

Orbit	Eccentricity ≈ 0 Height ≈ 690 km Inclination ≈ 98.1^0
Magnetic Torquers	Max. Magnetic Dipole Moment = ±3 Am^2 (%50 duty)
Inertia (kgm^2)	$\hat{J} = \begin{bmatrix} 7.2295 & 0.2393 & 0.1475 \\ 0.2393 & 7.0031 & -0.0058 \\ 0..1475 & -0.0058 & 7.6666 \end{bmatrix}$

$$Q = diag\left(x_1^{-2}, x_2^{-2}, \ldots, x_n^{-2}\right)$$
$$R = diag\left(u_1^{-2}, u_2^{-2}, \ldots, u_r^{-2}\right)$$

In practice, attitude control problem is highly nonlinear after the separation from launch vehicle due to high angular rates. Fortunately, angular rates can be easily reduced to operating range and then linearization can be valid as well as sufficient. The stability needs to be check for time varying system. Through the simulation, proper gain can be selected. (see for further information ([12],[13])

IV. SIMULATIONS

The system is simulated by using parameters (Table 1) of RASAT, Turkish earth observation satellite which designed and produced by TUBITAK-Space Technologies Research Institute and was launched into a ~690 km circular, ~98.1 inclination orbit in August 2011.

The simulations are performed on Dynamic Satellite Attitude Simulator [5] which has the features that the actual flight software runs on an actual on-board computer and the systems simulates space environments as well as virtual models of sensors and actuators.

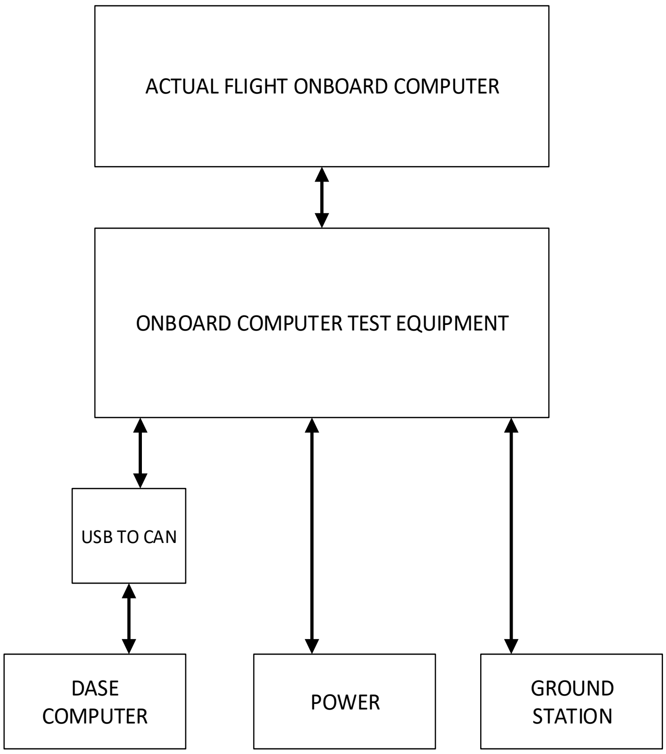

Fig. 1. Dynamic Attitude Satellite Simulator [5] (own idea, redrawn simply)

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DASE (Fig. 1) has following components: an OBC flight computer (exactly the same as on the satellite), an OBC test equipment, power supply, a ground station PC including actual ground station software and DASE PC. OBC test equipment establishes whole satellite communication between ground station and flight computer as well as simulating virtual models of actuators and sensors utilizing parameters like sensitivity, bias, noise, installation matrix, quantization. The environment has general perturbation satellite orbit model with atmospheric drag as well as Sun's position and Earth magnetic field to simulate satellite in a more realistic manner.

For simplicity off-diagonal elements of weighting matrices can be selected as zero as follows:

$$Q = diag(8.6,\ 8.6,\ 8.6,\ 0.01,\ 0.01,\ 0.01)$$

$$R = diag(0.1,\ 0.1,\ 0.1)$$

After separation of satellites, initial rates can be high. In order to see the performance of magnetic control and asymptotical behavior, initial rates for each axis are given high enough (1 deg/s) with respect to nominal operation points which is 0, -0.06, 0 deg/s respectively. Norm of angular velocity (Fig. 2 , Fig. 3) is asymptotically converges to 0.06 deg/s after 1 orbital period (1.5 hour) and coarse attitude is maintained after 2-3 orbital period.

As seen in Fig. 6 the magnetic controller is able to regulate the attitude angles. The real-time simulation duration is 6 hours under the magnetic controller action. The sampling period of the controller is chosen as 5 seconds. The signals in the figure are obtained from the flight software running on the OBC hardware. Comparing to the previous simulation result (Fig. 4, Fig. 5) attitude angle signals have some variations. The attitude information, the roll, pitch and yaw angles are the estimated attitude obtained from attitude

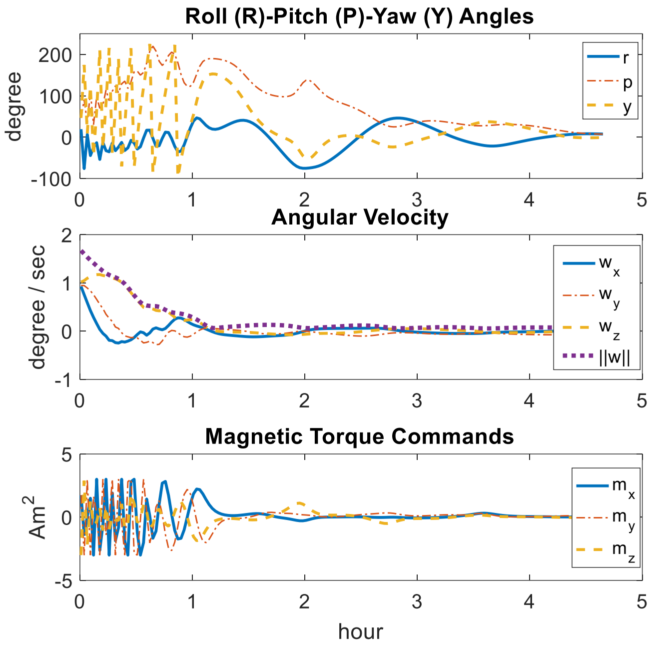

Fig. 2. Detumbling to 3 axis nadir pointing

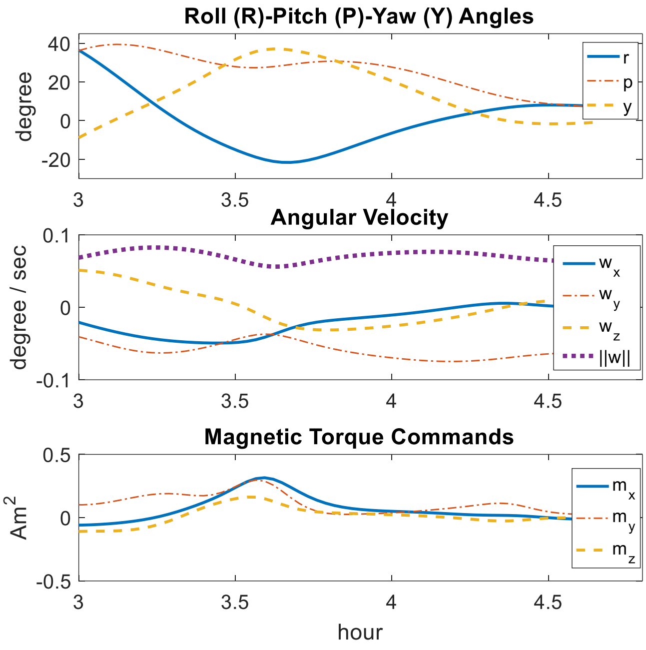

Fig. 3. Detumbling to 3 axis nadir pointing (zoomed)

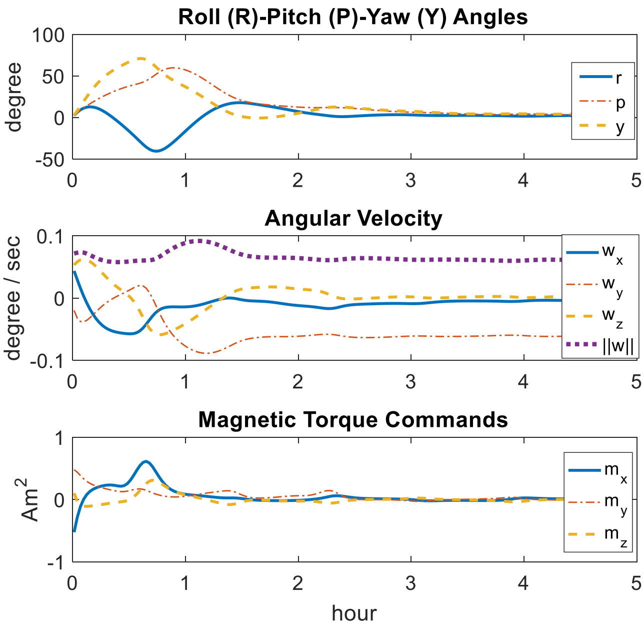

Fig. 4. Nadir pointing simulation with initial rates

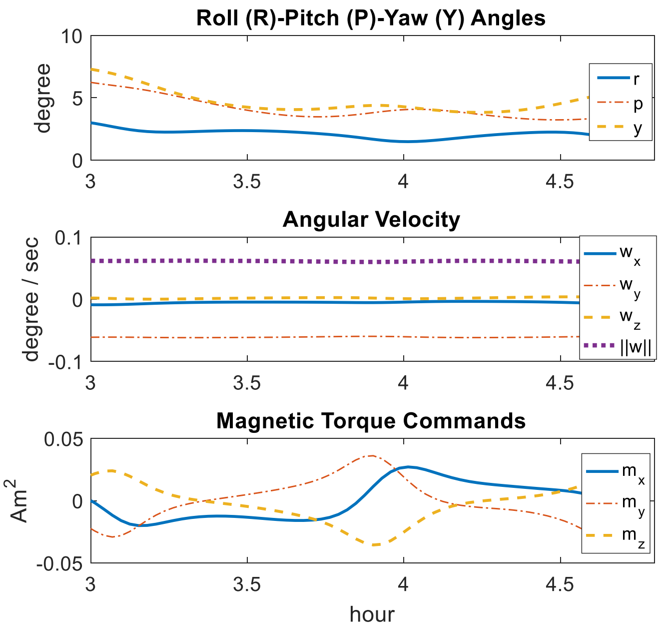

Fig. 5. Nadir pointing simulation with initial rates (zoomed)

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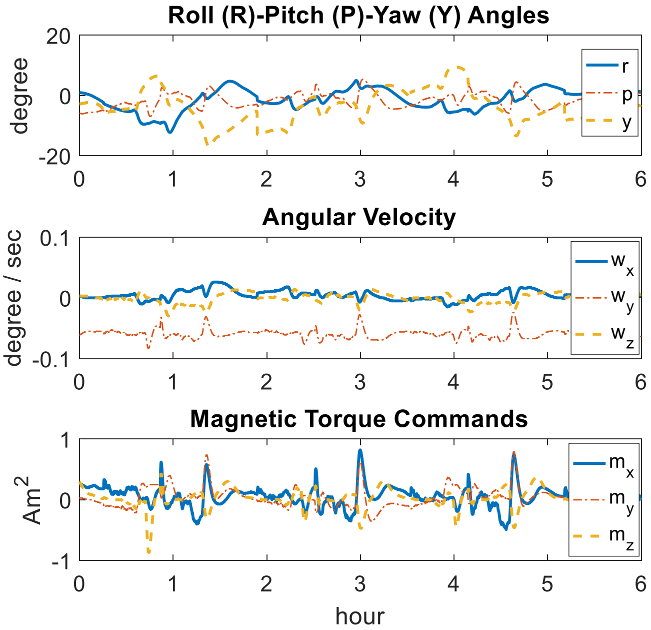

Fig. 6. Nadir pointing to Nadir pointing simulation on DASE

estimator in the flight software. Therefore, the variations of the angle signals can be considered as the attitude estimation error. As a result, the magnetic controller is successful to track the nadir pointing attitude under the real-time calculation and data communication as well.

V. Conclusion

Averaging method based linear quadratic control law is applied to stabilize the satellite. The success of the designed controller is shown through dynamic attitude simulator. However, it still needs to have some improvement on robustness against unmodelled disturbances particularly for magnetic dipole generated by electronics. In real application, unwanted small deviations at attitude are observed. Therefore, residual magnetic dipole estimation would be beneficial to maintain stability.

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