Numerical Solution to Strong Cylindrical Shock Wave in the Presence of Magnetic Field

Narsimhulu Dunna, Addepalli Ramu and Dipak Kumar Satpathi

Abstract- One-dimensional flow of converging shock waves in cylindrical symmetry in MHD, which is propagating into plasma, is analyzed. Due to implosion (by impulse method), plasma is assumed to be generated and is assumed to be non-ideal gas whose equation of state is of Mie-Gruneisen type. Suitable transformations reduce the governing equations into ordinary differential equations of Poincare type. Shock is assumed to be strong and propagating into a medium according to a power law. In the present work, two different equations of state (EOS) of Mie-Gruneisen type have been considered with suitable material constants and the cylindrical case is worked out in detail. The complete set of governing equations is formulated as finite difference problem and solved numerically using MATLAB. The numerical technique applied in this paper provides a global solution to the implosion problem for the flow variables, the similarity exponent α for different Gruneisen parameters. It is

shown that increase in measure of shock strength $\beta\left(\frac{\rho}{\rho_0}\right)$ has

effect on the shock front i.e., the velocity and pressure behind the shock front increases quickly in the presence of the magnetic field and decreases gradually. This phenomenon was found to exist irrespective of the piston is accelerated, is moving at constant speed or is decelerated. In particular we found that numerical solution is shown to exist in the range of cowling number (C_0) $0 < C_0 \leq 1$. These results are presented through the illustrative graphs and tables.

Keywords: Similarity solutions, Magneto hydrodynamics, Shock waves, Rankine-Hugoniot relations, Mie-Grueneisen type, Crout's reduction technique

I INTRODUCTION

The study of magneto hydrodynamic waves (MHD) in nonideal medium is of great scientific interest to many problems, in the areas of astrophysics, geophysics, underground explosions, hypersonic aerodynamics, hypervelocity impact, plasma physics, oceanography, atmospheric sciences, nuclear sciences, etc. Converging shock waves have been a field of growing interest since early 1940s from both mathematical and physical points of view. The study of shock waves produced due to the explosion or implosion in the presence of magnetic field has received much attention in recent times.

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Addepalli Ramu Professor Department of Mathematics BITS-Pilani, Hyderabad Campus, Shameerpet, Hyderabad – Telangana. 500078.INDIA. Member IAENG-129769 (e-mail aramu@hyderabad.bits-pilani.ac.in).

Dipak Kumar Satpathi Associate Professor, Department of Mathematics, BITS-Pilani, Hyderabad Campus, Shameerpet, Hyderabad – Telangana. 500078 INDIA. (e-mail dipak@hyderabad.bits-pilani.ac.in). Many researchers studied the problem of planer, cylindrical, and spherical MHD shock waves, mathematical [5] as well as experimental point of view. Several approaches such as the self-similarity method [6, 12, 13, 17], power series solution method [4, 8, 9, 13, 14, 15, 16] have been used for the theoretical investigations of MHD shock waves in homogeneous and non-homogeneous media.

In this paper, the self-similar solution of imploding cylindrical shock waves in non-ideal medium is presented. The medium generated due to implosion is assumed to be of Mie-Gruesiean type with or without magnetic material. In the implosion process (by impulse method) plasma is assumed to be generated. It is assumed that this plasma has an infinite electrical co nductivity and permeated by an axial magnetic field orthogonal to the trajectories of gas particles. Numerical solution presented here provides a global solution to the implosion problem which is valid for a range of physically meaningful parameters which represent the EOS of Mie-Grueneisen type. Numerical computations are performed to obtain the similarity exponent iteratively using MATLAB. The aim of the present work is to understand the mechanical properties of shock waves in the presence strong magnetic field; to study the behavior of shock characteristics such as shock strength, shock density, shock speed, shock over pressure, and impulse.

II BASIC EQUATIONS

The equations of motion of compressible one-dimensional Euler equations for the motion of a non-ideal gas in the magnetogasdynamics regime with the medium containing no heat conduction, or sinks, viscosity and radiation in non-conservative form can be written as [3, 6, 7, 12,17]

$$F_{vt} + AF_{vr} + D = 0 \tag{1}$$

where F_{v} , D are column vectors of order 4×1 and A is the coefficient matrix of order 4×4 for derivative of state function F_{v} given as follows

$$F_{v} = \begin{bmatrix} \rho \\ v \\ e \\ h \end{bmatrix}, A = \begin{bmatrix} v & \rho & 0 & 0 \\ 0 & v & 1/\rho & 1/\rho \\ 0 & p/\rho & v & 0 \\ 0 & 2h & 0 & v \end{bmatrix} \text{ and}$$
$$D = \begin{bmatrix} \frac{(v-1)\rho v}{r} \\ 0 \\ \frac{(v-1)p v}{\rho r} \\ \frac{2(v-1)hv}{r} \end{bmatrix}$$
(2)

The equations governing the flow are in general form with v = 1, 2, and 3 denoting planer, cylindrical, spherical

geometry respectively. t and r are independent variables representing time and space coordinates, ρ is the density, ρ_0 is the density of unperturbed medium, p is the pressure, v is the velocity of gas particles, e is the specific internal energy, $h = \mu H^2/2$ is the magnetic pressure, H and μ being the magnetic field strength and the magnetic permeability, respectively. The shock position is given by $R_s(t)$ and its speed is $U_s(t) = \frac{dR_s}{dt}$. In this work, the magnetic permeability is assumed to be unity and the electrical conductivity to be infinite. The equation of state under equilibrium condition is of Mie–Gruneisen type [11],

$$p = \rho e \Gamma(\rho/\rho_0) \tag{3}$$

where $\Gamma(\rho/\rho_0)$ is the Mie-Gruenisen coefficient. Shock is assumed to be strong and propagating into a medium according to the power law $R_s(t) \propto (1 - t/t_c)^{\alpha}$, where t_c is the collapse time, α is similarity exponent to be determined by the condition that the similarity variable $\xi = \frac{r}{R_s(t)}$ is 1 at the shock front.

III BOUNDARY CONDITIONS

The strong shock limit of the Rankine-Hugoniot jump conditions may be used to connect the flow just behind the shock to that just ahead of the shock front [3,17]

$$\frac{\rho_1}{\rho_0} = \beta , \quad v_1 = \left(1 - \frac{1}{\beta}\right) U_s(t)$$

$$\frac{p_1}{\rho_0} = \left(1 - \frac{1}{\beta} - \frac{C_0 \beta^2}{2}\right) U_s^2(t) , \quad \frac{h_1}{\rho_0} = \frac{C_0 \beta^2}{2} U_s^2(t) \quad (4)$$

These conditions are derived by the principle of conservation of mass, momentum, and energy across the shock. $C_0 = 2h_0/\rho_0 U_s^2(t)$ is the Cowling number associated with magnetic property and β is the measure of the shock strength. The suffices '1' and '0' refer to conditions just behind and ahead of the shock respectively.

With the above relations (4), the equation (3) can be written as $\left(2 - \frac{c_0 \beta^3}{(\beta - 1)}\right) = (\beta - 1)\Gamma(\beta)$ (5)

Equation (5) is used to determine the measure of shock strength β for various models of EOS, which are provided in this paper.

Also, the boundary conditions for the strong shock at $(\xi=1)$ can be written as

$$G(1) = \beta , \quad V(1) = 1 - \frac{1}{\beta}, \quad Z(1) = 1 - \frac{1}{\beta} - \frac{C_0 \beta^2}{2}$$
$$B(1) = \frac{C_0 \beta^2}{2} \tag{6}$$

IV SOLUTION PROCEDURE

The flow field is bounded by the piston and the shock respectively. In the framework of self-similarity shock is assumed to be strong and propagating into a medium according to the power law where the counter pressure ahead of the shock can be neglected and is the collapse time. The basic equations can be made dimensionless by transforming the independent variables for space and time into new independent variables [17]

$$\rho = \rho_0 G(\xi) ,= U_s(t) V(\xi) , h = \rho_0 U_s^{-2}(t) B(\xi)$$
(7)

where the similarity variable $\xi = \frac{r}{R_s(t)}$, G, V, Z, and B are the functions (known as reduced functions) of the nondimensional variable ξ only. The quantities $U_s(t)$, $\rho_0 U_s^2(t)$ are the velocity scale and pressure scale respectively.

Applying transformations (7), on the system of equations (1) (v = 2 for cylindrical geometry) we obtain reduced system of ordinary differential equations in the dimensionless variables ξ , G (ξ), V (ξ), Z (ξ), and B (ξ). These can be written in the matrix form as

$$\begin{pmatrix} (\xi_{i} - V_{i}) & -G_{i} & 0 & 0\\ 0 & (\xi_{i} - V_{i}) & -1/G_{i} & -1/G_{i} \\ -Z_{i}\phi(G_{i})/G_{i} & 0 & 1 & 0\\ -2B_{i}/G_{i} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} G_{i+1} \\ V_{i+1} \\ Z_{i+1} \\ B_{i+1} \end{pmatrix} = \begin{pmatrix} (\xi_{i} - V_{i})G_{i} + (\frac{h}{\xi_{i}} - 1)G_{i}V_{i} \\ (\xi_{i} - V_{i})G_{i} + (\frac{h}{\xi_{i}} - 1)G_{i}V_{i} \\ (\lambda h + \xi_{i} - V_{i})V_{i} - \frac{1}{G_{i}}(Z_{i} + B_{i}) \\ (1 - \phi(G_{i}))Z_{i} + \frac{2\lambda h Z_{i}}{(\xi_{i} - V_{i})} \\ (\frac{2\lambda h}{(\xi_{i} - V_{i})} - 1)B_{i} \end{pmatrix}$$
(8)

Using Crout's reduction method [1] the system of equations (8) are solved for density (G_{i+1}) , velocity (V_{i+1}) , pressure (Z_{i+1}) , and magnetic pressure (B_{i+1}) functions respectively, where

$$G_{i+1} = \frac{P_i}{1} + \frac{G_i}{(\xi_i - V_i)} Q_i + \frac{1}{(\xi_i - V_i)^2} R_i + \frac{G_i}{[G_i(\xi_i - V_i)^2 - Z_i\phi(G_i)]} S_i(9)$$

$$W_i = \frac{Q_i}{1} + \frac{1}{(\xi_i - V_i)} R_i + \frac{(\xi_i - V_i)}{(\xi_i - V_i)} S_i(9)$$
(10)

$$V_{i+1} = \frac{N}{1} + \frac{1}{(\xi_i - V_i)G_i} R_i + \frac{1}{[G_i(\xi_i - V_i)^2 - Z_i\phi(G_i)]} S_i$$
(10)

$$Z_{i+1} = \frac{R_i}{1} + \frac{Z_i \phi(G_i)}{[G_i(\xi_i - V_i)^2 - Z_i \phi(G_i)]} S_i$$
(11)

$$B_{i+1} = S_i \tag{12}$$

where

$$P_{i} = \frac{\left[G_{i}(\xi_{i}-V_{i})+G_{i}V_{i}\left(\frac{h}{\xi_{i}}-1\right)\right]}{(\xi_{i}-V_{i})}, \qquad Q_{i} = \frac{\left[V_{i}(\lambda h+\xi_{i}-V_{i})-\frac{1}{G_{i}}(Z_{i}+B_{i})\right]}{(\xi_{i}-V_{i})}$$
$$S_{i} = \frac{\left[2B_{i}G_{i}(\xi_{i}-V_{i})M_{i}+\frac{2B_{i}Z_{i}\phi(G_{i})}{\xi_{i}}N_{i}+K_{i}\right]}{G_{i}(\xi_{i}-V_{i})^{2}[G_{i}(\xi_{i}-V_{i})^{2}-Z_{i}\phi(G_{i})-2B_{i}]}$$

$$R_{i} = \frac{\left[\{Z_{i}G_{i}(\xi_{i}-V_{i})(2\lambda h+\xi_{i}-V_{i})\} + \frac{Z_{i}\phi(G_{i})}{\xi_{i}} \{hG_{i}V_{i}(\lambda h+\xi_{i}-V_{i})-(Z_{i}+B_{i})\xi_{i}\} \right]}{\{G_{i}(\xi_{i}-V_{i})^{2}-Z_{i}\phi(G_{i})\}}$$

$$M_{i} = \{G_{i}V_{i}(\lambda h+\xi_{i}-V_{i}) - B_{i}\}(\xi_{i}-V_{i}) + 2\lambda hZ_{i}$$

$$N_{i} = (\xi_{i}-1)(Z_{i}+B_{i}) + (h-\xi_{i})(\lambda h+\xi_{i}-V_{i})$$

$$K_{i} = \{(2\lambda h+\xi_{i}-3V_{i}) + \frac{2h}{\xi_{i}}V_{i}\} [B_{i}G_{i}^{2}(\xi_{i}-V_{i})^{3} - G_{i}Z_{i}B_{i}\phi(G_{i})(\xi_{i}-V_{i})]$$

V NUMERICAL SOLUTION

The solution to equations (9 to 12) involves evaluation of and considering the EOS of Mie-Gruneisen type [2]

(a) Dusty gas
$$\Gamma(G) = \frac{M}{(1-G^{-M})}$$
 (13)

where M is constant parameter and represents material property of the dusty gas particles.

(b) Royce EOS [10]
$$\Gamma(G) = \Gamma_0 - a \left(1 - \frac{1}{G}\right)$$
 (14)

where Γ_0 represents specific heat of the solid particles and 'a' is an arbitrary constant. Along with equation (2.5) equations (13) and (14) we obtain the equation in β as

$$\begin{split} \phi(\beta) &= C_0 \beta^{M+3} + M \beta^{M+2} - 2(M+1)\beta^{M+1} + (M+2)\beta^M - C_0 \beta^3 + 2\beta - 2 = 0 \end{split}$$
(15)
$$\psi(\beta) &= C_0 \quad \beta^4 + (\Gamma_0 - a)\beta^3 + (3a - 2 - 2\Gamma_0)\beta^2 + (2 - 3a + \Gamma_0)\beta + a = 0 \end{aligned}$$
(16)

The equation (16) reduces to perfect gas EOS when $\Gamma_0 = (\gamma - 1)$, a = 0 and along with equation (3) the EOS for perfect gas becomes

$$p(\rho, e) = \rho e(\gamma - 1) \tag{17}$$

where e is the specific internal energy, $\gamma = \frac{C_p}{C_v}$ denotes the specific heat ratio of perfect gas and ρ , p represent density and pressure respectively. Thus along with $\Gamma_0 = (\gamma - 1)$ and a = 0 and equation (17) on substituting in equation (16) the equation for measure of shock strength β for perfect gas can be written as

$$\pi(\beta) = C_0 \beta^3 + (\gamma - 1)\beta^2 - 2\gamma\beta + (\gamma + 1) = 0$$
(18)

and the expression for α as

$$\frac{(1-(\beta-1)\varphi(\beta))}{\beta^2} + \frac{1}{2} C_0 \frac{\alpha_0^2}{\alpha^2} (\varphi(\beta) - 2)\beta = 0$$
(19)

where $0 < \alpha_0 < 1$. Also

$$\varphi(\beta) = 1 + M; \quad \varphi(\beta) = 1 + \Gamma(\beta) - \frac{b}{\beta\Gamma(\beta)} \text{ and } \Gamma(\beta) = \Gamma_0 - a\left(1 - \frac{1}{\beta}\right)$$
(20)

for dusty gas and Royce EOS respectively and β is the root of equations (15, 16 and 19). Starting with a guess values of α , α is evaluated from equation (19) using a bracketing method for various values of C_0 , a, M, α_0 and β . The equations (12 to 15) are solved with boundary conditions (6) in the region of $1 \leq \xi < \infty$ using MATLAB. The numerical evaluation is carried out from the shock front ($\xi = 1$) and proceed inwards until it reaches $\xi = \infty$, indicating $\xi_{min} = 1$, and $\xi_{max} = 5$, where ξ_{max} corresponds to $\xi = \infty$, $h = \Delta\xi$ and $\xi_i = 1 + (i - 1)h$, $i \geq 1$ is a positive integer, where step size is h = 0.1 and the number of grid points were approximately 1000 for the solution procedure.

VI RESULTS and DISCUSSION

In this paper, the entire computational work has been carried out using MATLAB. The computed values of similarity exponent α for various values of measure of shock strength β for the Dusty gas, Royce and perfect gas EOS are presented in (Tables (1) – (4)). The unsteady nature of flow variables with the effect of magnetic field for the EOS under consideration are illustrated through Figs (1) – (6) for cylindrical converging shock waves. Equations (15, 16 and 19) are solved numerically for all roots of β , which depend on the parameters C_0 = 0.02,0.05; a = 1.0,1.2,1.5 and Γ_0 = 1.78(for Royce EOS) with (0 < $C_0 \le 0.1$); and M = 1.4,1.5,1.6,1.8 and 2.0 (for dusty gas); and γ = 1.2, 1.4 and 1.6 (for perfect gas) respectively.

Table I The values of similarity exponent for dusty gas EOS flow:

М	$C_0 = 0.02$		$C_0 = 0.05$	
	β	α	β	α
1.4	1.01841	0.71357	1.05085	0.71152
1.5	1.01887	0.70525	1.05240	0.70247
1.6	1.01935	0.69694	1.05405	0.69330
1.8	1.02040	0.67982	1.05776	0.67421
2.0	1.02157	0.66226	1.06213	0.65409

Table II The values of similarity exponent for Royce EOS when $\Gamma_0 = 1.78$

а	$C_0 = 0.02$		$C_0 = 0.05$	
	β	α	β	α
1.0	2.52063	0.30570	2.26132	0.45097
1.2	2.73109	0.32313	2.39457	0.46735
1.5	3.29655	0.37204	2.70082	0.50691

Table III The values of similarity exponent for Royce EOS when $\Gamma_0 = 2.02$

а	$C_0 = 0.02$		$C_0 = 0.05$	
	β	α	β	α
1.0	2.24026	0.28362	2.05447	0.42712
1.2	2.35998	0.29285	2.13702	0.43636
1.5	2.62946	0.31465	2.30779	0.45661

Table IV The values of similarity exponent for perfect gas $EOS:\gamma=1.2.1.4, 1.6$

γ	$C_0 = 0.02$		$C_0 = 0.05$	
	β	α	β	α
1.2	6.29241	0.64632	4.49743	0.75963
1.4	4.63191	0.49275	3.67781	0.64173
1.6	3.70626	0.40856	3.13229	0.56545

From the Table I, we observe that with the change in the values of *M* and C_0 , measure of shock strength β increases, whereas similarity exponent α decreases. It is observed from Tables II and III that the increase in the similarity exponent α , corresponds to the increase in the values of measure of shock strength β and arbitrary constant a for Royce EOS. It is notable from Table (4), that the specific heats ratio γ has direct impact on the similarity exponent α and the measure of shock strength β i.e., with increase in the value of γ the similarity exponent α and β decreases. Also, we may note that the change in similarity exponent α is more prominent and faster in the perfect gas, as compared to that of dusty gas, because of considerable change in the initial strength of shock. The variations of non-dimensional shock velocity, pressure, magnetic pressure and density with ξ for dusty gas, Royce and perfect gas EOS are presented pictorially and some of the Figs are included in this paper. It is observed that the flow variables density, velocity, pressure and magnetic pressure are high at the shock front (for the dusty gas EOS) and increases with the increase in the non-idealness parameter C_0 and reduce gradually as ξ increases. In particular, it is observed that the shock wave travels very slowly i.e., a low value of similarity exponent α in the dusty gas medium for various values of C_0 which is due to increase in strength of shock density. Figs (1 -6) depict the density, velocity, pressure, and magnetic pressure distributions as functions

of dimensionless variable (i.e., reduced distance) over a subset of the computational domain for Royce EO and perfect gas. Again we note a sharp rise in density, pressure and magnetic pressure distributions behind the shock front, located between $\xi = 1$ and $\xi = 2.0$ and then decrease monotonically along the axis. It may be observed that variation in these peaks is reducing with an increase in the values of the parameter *a*,

which is higher in the case of Cowling number $C_0 = 0.02$, whereas in the case of Cowling number $C_0 = 0.05$ not much variation is observed. It can be seen from Fig (2) for the case (i.e., $C_0 = 0.05$) a sharp rise in velocity behind the shock front, located between $\xi = 1$ and $\xi = 1.5$ and then decrease in velocity monotonically towards the axis. In particular, with change in the value of *a* it is observed from Fig 2 that velocity slowly decreases beyond the region $1 \le \xi < 1.5$ in the both cases. The maximum values of reduced density, velocity, pressure, and magnetic pressure peaks for various values of C_0 in case of Royce EOS, as well as perfect gas for ($\alpha = 0$ & $\Gamma_0 = (\gamma - 1)$) can be seen from the above Tables.



Fig 1Density profiles for Royce EOS when $\Gamma_0 = 1.78$ $C_0 = 0.02$



Fig 2 Velocity profiles for Royce EOS when $\Gamma_0 = 1.78$ $C_0 = 0.05$



Fig 3 Pressure profiles for Royce EOS when $\Gamma_0 = 1.78$ $C_0 = 0.02$



Fig 4: Magnetic pressure profiles for Royce EOS when $\Gamma_0 = 1.78$; $C_0 = 0.05$



Fig 5 Profiles for perfect gas when $\gamma = 1.4$; a) density



Fig 6 Profiles for perfect gas when $\gamma = 1.4$; e, and magnetic pressure

VII CONCLUSION

A problem involving a cylindrical converging strong shock wave has been formulated with a gas of varying density obeying a power law. and shock propagates through a medium characterized by a Mie-Gruneisen EOS. The governing equations are non-dimensionalized using suitable similarity transformations. A finite difference scheme is employed to solve the system of non-linear differential equations. Crout's reduction technique is used to solve the system of algebraic equations. The nature of flow variables with the effect of magnetic field in the respective models of EOS is investigated.

From the present study, we notice that the similarity exponent decreases with an increase in the values of and fixed causes an increase in for dusty gas medium. In perfect gas similarity exponent α decreases with decrease in β with increasing values of γ and fixed C_0 . In case of Royce's EOS the similarity exponent α decreases with an increasing values of Γ_0 and fixed *a* and C_0 due to decrease in β . The decay of shock wave is more prominent and slower in the dusty gas EOS. The effect of magnetic field on flow variables is less pronounced for dusty gas particles, because of lower compression between the gas particles. We conclude that the less compressible medium has higher wave propagation speed. As shown in Figs (1)-(6), the approximate reduced density, velocity, pressure, and magnetic pressure shows largest peaks to the right behind the shock front in Royce EOS, whereas in perfect gas flow variables have small peaks. This is due to fact that the effect of measure of shock strength β , which causes the change in α and also effect of converging geometry or area of contraction of the shock wave.

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