Ferromagnetic Liquid Flow due to Inclined Stretching of an Elastic Sheet

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ABSTRACT—The flow of ferrofluid due to inclined stretching sheet in the presence of a magnetic dipole is studied to provide a window to a general possibility that can be looked into in a stretching sheet problem. The elastic sheet is stretched at an angle to the horizontal into the ferrofluid. The fluid momentum and thermal energy equations are formulated as a six parameter problem. Extensive computation on the velocity and temperature profiles is presented for a wide range of values of the parameters. It was found that the primary effect of the magneto-thermo mechanical interaction is to decelerate the fluid motion as compared to the hydrodynamic case. As the angle of inclination to the horizontal, θ, of the stretching sheet increases, the momentum boundary layer thickness increases and the gravity effects becomes prominent. The inclination reduces the thermal boundary layer thickness. The horizontal and vertical stretching sheet problems are shown to be the limiting cases of the inclined sheet problem.

Index Terms - Ferromagnetic liquid, magnetic dipole, shooting method, stretching sheet

I. INTRODUCTION

Generally stretching sheet problems arise in polymer extrusion processes that involve cooling of continuous strips extruded from a die by drawing them through a stationary cooling liquid. The stretching imparts a unidirectional orientation to the extrudate thereby improving its fluid mechanical properties. The extruded sheet can be a needed product in a plastic or glass industry or in industries dealing with artificial fibre. Ever since the pioneering works of Sakiadis [1], [2] and [3], Tsou et al. [4] and Crane [5] on the stretching sheet problem with boundary layer approximations, several authors have worked on various aspects of the problem as can be seen in Andersson [6], Liao and Pop [7], Magyari et.al.[8], Chen [9], Siddheshwar and Mahabaleshwar [10], Abel and Mahesha [11]. These studies deal with horizontal stretching sheet problems. There are practical situations where the sheet is stretched at an angle to the horizontal. With this viewpoint Abo-Eldahab and El-Aziz [12] have considered the inclined stretching sheet problem and obtained numerical solutions for the same. Mahesha [13] investigated the mixed convection flow due to inclined stretching. Sandeep and Jagadeesh [14] have studied heat and mass transfer behaviour of MHD nanofluid flow embedded with conducting dust particles past an inclined permeable stretching sheet in presence of radiation, non-uniform heat source/sink, volume fraction of nano particles, volume fraction of dust particles and chemical reaction. But the problem pertaining to ferrofluid flow arising due to inclined stretching sheet has not been investigated before. In literature there are papers concerning horizontal and vertical stretching sheet problems as different problems. In reality most of the practical applications would involve the sheet to be stretched at an angle to the gravity which helps in achieving the desired temperature of the stretching sheet [13]. The problem of inclined stretching sheet is the most general case whose limiting cases gives rise to horizontal ($\theta = 0$) and vertical ($\theta = \pi/2$). To the best of the authors knowledge there are no work on inclined stretching sheet problems concerning ferrofluids, hence in this paper we study the same. The problem of stretching sheet is thus a fundamental one and arises in many practical situations that are similar to the polymer extrusion and metallurgical processes. Some of these are listed below:

- Continuous stretching, rolling, manufacturing of polymer sheets
- Drawing, annealing, tinning of copper wires
- Cooling of an infinite metallic plate in a cooling path
- Boundary layer along a liquid film in condensation processes
- Manufacture of materials by extrusion process and heat treated materials travelling between a feed and wind – up rolls or conveyer belts
- Glass blowing, paper production, crystal growing, etc.

Hence, in this paper the problem involving an inclined stretching sheet with ferrofluid as a cooling liquid is explored.

II. MATHEMATICAL FORMULATION

Consider a steady two-dimensional flow of an incompressible, viscous and electrically non-conducting ferrofluid driven by an impermeable sheet in the inclined direction. By applying two equal and opposite forces along the direction of gravity which is taken as the x-axis, and y-axis in a direction normal to the flow, the sheet is stretched with a velocity $u_w(x) = cx$ which is proportional to the distance from the origin, inclined at an angle θ with the

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horizontal. A magnetic dipole is located some distance from the sheet. The centre of the dipole lies on the y-axis at a distance 'a' from the x-axis and whose magnetic field points in the positive x-direction giving rise to a magnetic field of sufficient strength to saturate the ferrofluid. The stretching sheet is kept at a fixed temperature T_w below the Curie temperature T_c, while the fluid elements far away from the sheet are assumed to be at a temperature $T = T_c$ and hence incapable of being magnetized until they begin to cool upon entering the thermal boundary layer adjacent to the sheet.

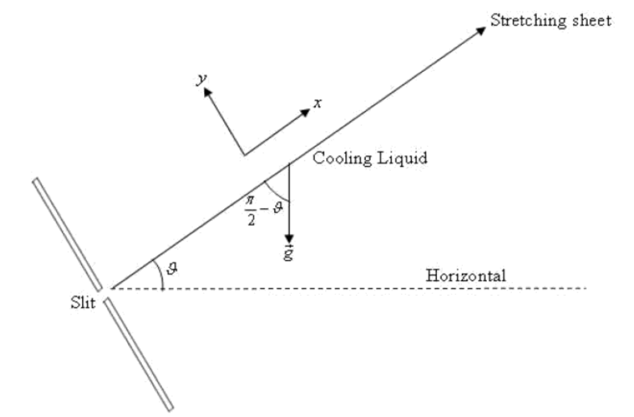

Fig 1 Schematic of inclined stretching sheet problem

The equations governing the motion and heat transfer are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\mu_0}{\rho}M\frac{\partial H}{\partial x}+g\beta^*\left(T_c-T\right)\sin\vartheta, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)+\mu_0 T\frac{\partial M}{\partial T}\left(u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}\,. \tag{3}$$

Here u and v are the velocity components along x and y directions respectively, ρ is the fluid density, μ is the dynamic viscosity, $\nu=\mu/\rho$ is the kinematic viscosity, c_p is the specific heat at constant pressure, k is the thermal conductivity, g is the acceleration due to gravity, β^* is the coefficient of the thermal expansion, μ_0 is the magnetic permeability, M is the magnetization, H is the magnetic field and T is the temperature of the fluid. The assumed boundary conditions for solving the above equations for both the Prescribed Surface Temperature (PST) and Prescribed Heat Flux (PHF) are:

$$\left.\begin{array}{ll} u(x,0)\ =\ cx\,,\ \ v(x,0)\ =\ 0, & \\ T(x,0)\ =\ T_w = T_c - A\left(\dfrac{x}{L}\right) & \text{in PST} \\ -k\dfrac{\partial T}{\partial y}(x,0)=q_w=D\left(\dfrac{x}{L}\right) & \text{in PHF} \\ u(x,\infty)\to 0, \qquad T(x,\infty)\to T_c & \end{array}\right\} \tag{4}$$

Here k is the thermal conductivity of the fluid. A and D are positive constants, and $L=\sqrt{\frac{\nu}{c}}$ is the characteristic length. The flow of ferrofluid is affected by the magnetic field due to the magnetic dipole whose magnetic scalar potential is given by

$$\phi=\frac{\alpha'}{2\pi}\left(\frac{x}{x^2+(y+a)^2}\right), \tag{5}$$

where α' is the magnetic field strength at the source. The components of the magnetic field H are

$$H_x=-\frac{\partial\phi}{\partial x}=\frac{\alpha'}{2\pi}\left\{\frac{x^2-(y+a)^2}{\left(x^2+(y+a)^2\right)^2}\right\}, \tag{6}$$

$$H_y=-\frac{\partial\phi}{\partial y}=\frac{\alpha'}{2\pi}\left\{\frac{2x(y+a)}{\left(x^2+(y+a)^2\right)^2}\right\}. \tag{7}$$

Since the magnetic body force is proportional to the gradient of the magnitude of H, we obtain

$$H=\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]^{\frac{1}{2}}, \tag{8}$$

$$\frac{\partial H}{\partial x}=-\frac{\alpha'}{2\pi}\left(\frac{2x}{(y+a)^4}\right),\ \frac{\partial H}{\partial y}=\frac{\alpha'}{2\pi}\left(\frac{-2}{(y+a)^3}+\frac{4x^2}{(y+a)^5}\right). \tag{9}$$

Variation of magnetization M with temperature T is approximated by a linear equation

$$M=K(T_c-T)\ , \tag{10}$$

where K is the pyro magnetic coefficient.

III. SOLUTION PROCEDURE

We now introduce the non - dimensional variables as assumed by Andersson [6]:

$$(\xi,\eta)=\left(\frac{c}{\upsilon}\right)^{\frac{1}{2}}(x,y)\,,\ (U,V)=\frac{(u,v)}{\sqrt{c\upsilon}}, \tag{11}$$

$$\theta(\xi,\eta)=\frac{T_c-T}{T_c-T_w}=\begin{cases}\theta_1(\eta)+\xi^2\theta_2(\eta) & \text{in PST case}\\ \phi_1(\eta)+\xi^2\phi_2(\eta) & \text{in PHF case}\end{cases}, \tag{12}$$

Where $T_c-T_w=A\left(\frac{x}{L}\right)$ in PST case,

$T_c-T_w=\frac{DL}{k}\left(\frac{x}{L}\right)$ in PHF case.

The boundary layer equations (1) - (3) on using (9) - (12) takes the following form:

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$$\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0 , \tag{13}$$

$$U \frac{\partial U}{\partial \xi} + V \frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial \eta^2} - \frac{2\beta\xi}{(\eta+4)^4}\left(\theta_1+\xi^2\theta_2\right) + Gr\xi\left(\theta_1+\xi^2\theta_2\right)\sin\vartheta , \tag{14}$$

$$\Pr\left[2U\xi\theta_2 + \frac{U}{\xi}\left(\theta_1+\xi^2\theta_2\right) + V\left(\theta_1' + \xi^2\theta_2'\right)\right] + \beta\lambda\left(\varepsilon - \theta_1 + \xi^2\theta_2\right)\left[\frac{-2\xi U}{(\eta+\alpha)^4} - \frac{2V}{(\eta+\alpha)^3} + \frac{4\xi^2 V}{(\eta+\alpha)^5}\right] = \theta_1'' + \xi^2\theta_2'' \tag{15}$$

The boundary condition given by (4) now takes the form:

$$\left.\begin{array}{lll} U(\xi,0) = \xi, & V(\xi,0) = 0, & \\ \theta_1(\xi,0) = 1, & \theta_2(\xi,0) = 0, & \text{(PST)}, \\ \phi_1'(\xi,0) = -1, & \phi_2'(\xi,0) = 0, & \text{(PHF)}, \\ U(\xi,\infty) \to 0, & \theta(\xi,\infty) \to 0 & \end{array}\right\} \tag{16}$$

Introducing the stream function $\psi(\xi,\eta) = \xi f(\eta)$ that satisfies the continuity equation in the dimensionless form (14), we obtain

$$U = \frac{\partial \psi}{\partial \eta} = \xi f'(\eta), \qquad V = \frac{\partial \psi}{\partial \xi} = -f(\eta), \tag{17}$$

where the prime denotes differentiation with respect to η. On using (9), (11) and (17) in (14) and (15) we obtain the following boundary value problem:

(i) PST

$$f''' + ff'' - (f')^2 - \frac{2\beta\theta_1}{(\eta+\alpha)^4} + Gr\theta_1 \sin\vartheta = 0 \tag{18}$$

$$\theta_1'' + \Pr\left(f\theta_1' - f'\theta_1\right) + \frac{2\beta f\lambda}{(\eta+\alpha)^3}(\theta_1 - \varepsilon) - 2\lambda(f')^2 = 0, \tag{19}$$

$$\theta_2'' - \lambda(f')^2 - \Pr(3f'\theta_2 - f\theta_2') + \frac{2\lambda\beta f\theta_2}{(\eta+\alpha)^3} - \lambda\beta(\theta_1 - \varepsilon)\left[\frac{2f'}{(\eta+\alpha)^4} + \frac{4f}{(\eta+\alpha)^5}\right] = 0, \tag{20}$$

$$f = 0, \quad f' = 1, \quad \theta_1 = 1, \quad \theta_2 = 0 \quad \text{at} \quad \eta = 0 \tag{21}$$

$$f' \to 0,\ \theta_1 \to 0,\ \theta_2 \to 0 \quad \text{as } \eta \to \infty . \tag{22}$$

(ii) PHF

$$f''' + ff'' - (f')^2 - \frac{2\beta\phi_1}{(\eta+\alpha)^4} + Gr\phi_1 = 0 , \tag{23}$$

$$\phi_1'' + \Pr\left(f\phi_1' - f'\phi_1\right) + \frac{2\beta f\lambda}{(\eta+\alpha)^3}(\phi_1 - \varepsilon) - 2\lambda f'^2 = 0 , \tag{24}$$

$$\phi_2'' - \lambda(f')^2 - \Pr(3f'\phi_2 - f\phi_2') + \frac{2\lambda\beta f\phi_2}{(\eta+\alpha)^3} - \lambda\beta(\phi_1 - \varepsilon)\left[\frac{2f'}{(\eta+\alpha)^4} + \frac{4f}{(\eta+\alpha)^5}\right] = 0 , \tag{25}$$

$$f = 0,\ f' = 1,\ \phi_1' = -1,\ \phi_2' = 0 \quad \text{at } \eta = 0 , \tag{26}$$

$$f' \to 0, \quad \phi_1 \to 0, \quad \phi_2 \to 0 \quad \text{as } \eta \to \infty . \tag{27}$$

The six dimensionless parameters, which appear explicitly in the transformed problem, are the Prandtl number *Pr*, the viscous dissipation parameter λ, the dimensionless Curie temperature ε, the ferrohydrodynamic interaction parameter β, the Grashof number *Gr* and the dimensionless distance α from the origin to the center of the magnetic pole, defined respectively as

$$\Pr = \frac{\mu C_p}{k} , \quad \lambda = \frac{c\mu^2}{\rho k(T_c - T_w)} , \quad \varepsilon = \frac{T_c}{T_c - T_w} ,$$

$$\beta = \frac{\alpha' \rho}{2\pi\mu^2}\mu_0 K(T_c - T_w) , Gr = \frac{g\beta^* A}{c^2 L} , \quad \text{and}$$

$$\alpha = \left(\frac{c\rho a^2}{\mu}\right)^{\frac{1}{2}} .$$

Equations (18) - (22) and (23) - (27) constitute two sets of nonlinear, two-point boundary value problems, which are solved by means of a standard shooting technique. The higher order ordinary differential equations are decomposed into a set of nine first order equations and integrated as an initial value problem using the adaptive stepping Runge-Kutta-Fehlberg (RKF45) method. Trial values of $f''(0)$, $\theta_1'(0)$, $\theta_2'(0)$ and $\phi_1'(0)$, $\phi_2'(0)$ are adjusted iteratively by Newton-Raphson's method to assure a quadratic convergence of the iterative trial values required in order to fulfil the outer boundary conditions.

IV. RESULTS AND DISCUSSION

In this section we have analysed the effect of magnetic field on the flow of the ferromagnetic liquid due to an inclined stretched sheet. Heat transfer is studied using two different boundary heating conditions, namely, PST and PHF. The numerical results are shown in the graphs.

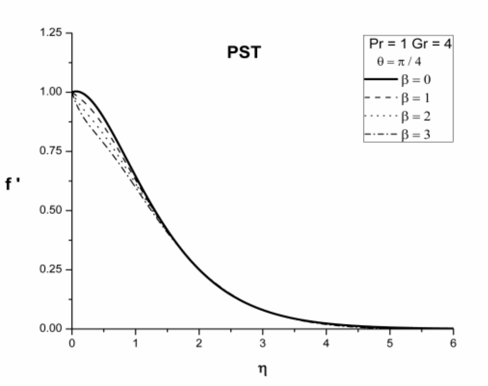

Fig 2a Effect of Ferrohydrodynamic interaction parameter β on velocity profile in the case of PST

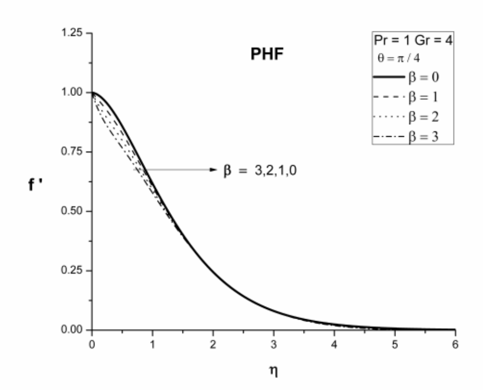

Fig 2b Effect of Ferrohydrodynamic interaction parameter β on velocity profile in the case of PHF

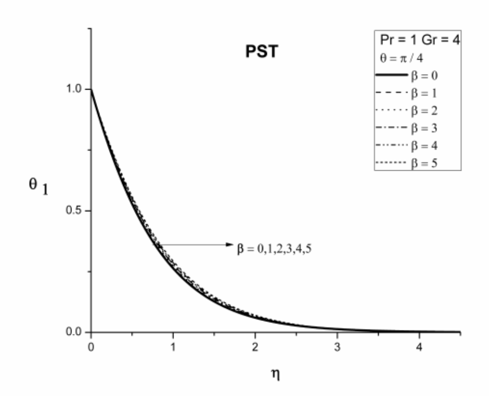

Fig 3a Effect of Ferrohydrodynamic interaction parameter β on temperature profile in the case of PST

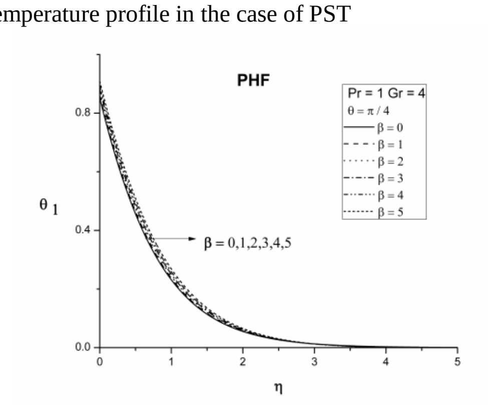

Fig 3b Effect of Ferrohydrodynamic interaction parameter β on temperature profile in the case of PHF

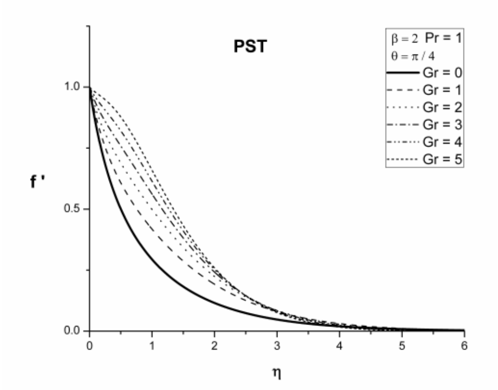

Fig 4a Effect of Grashof number *Gr* on velocity profile in the case of PST

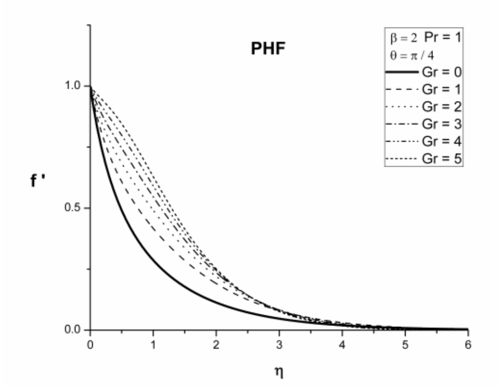

Fig 4b Effect of Grashof number *Gr* on velocity profile in the case of PHF

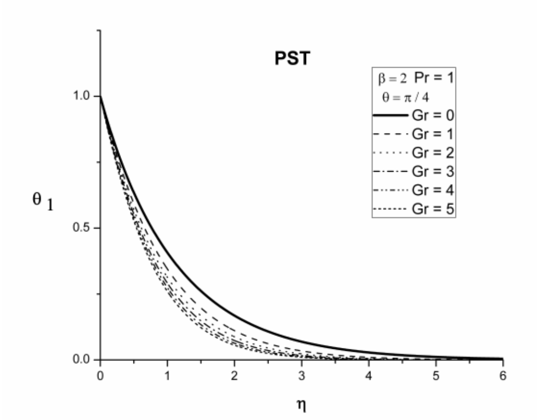

Fig 5a Effect of Grashof number *Gr* on temperature profile in the case of PST

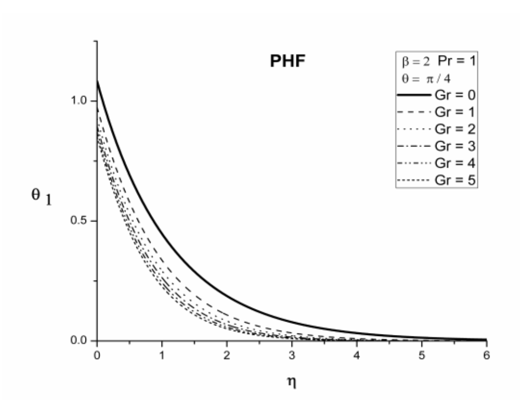

Fig 5b Effect of Grashof number *Gr* on temperature profile in the case of PHF

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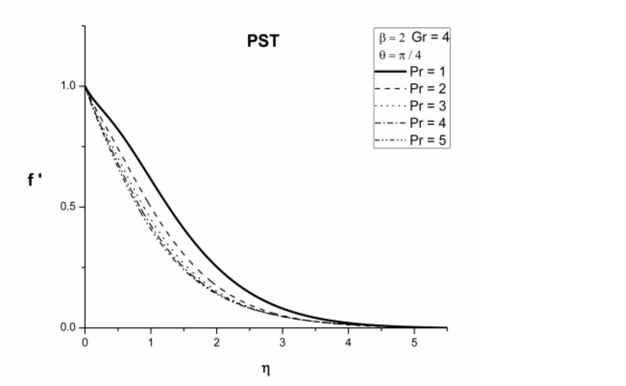

Fig 6a Effect of Prandtl number *Pr* on velocity profile in the case of PST

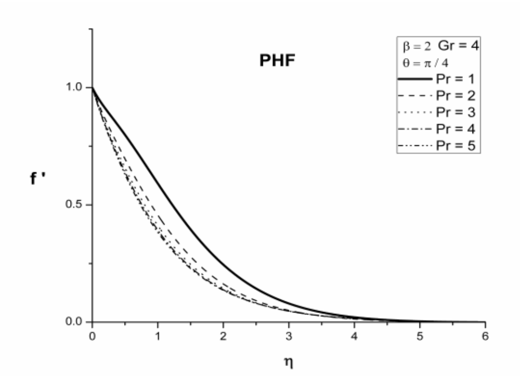

Fig 6b Effect of Prandtl number *Pr* on velocity profile in the case of PHF

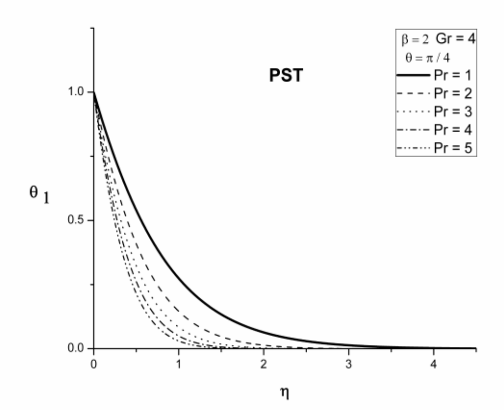

Fig 7a Effect of Prandtl number *Pr* on temperature profile in the case of PST

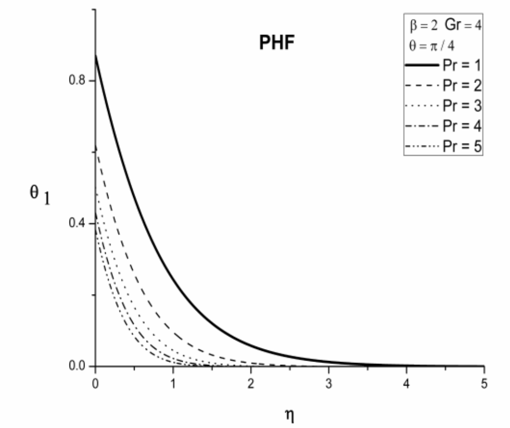

Fig 7b Effect of Prandtl number *Pr* on temperature profile in the case of PHF

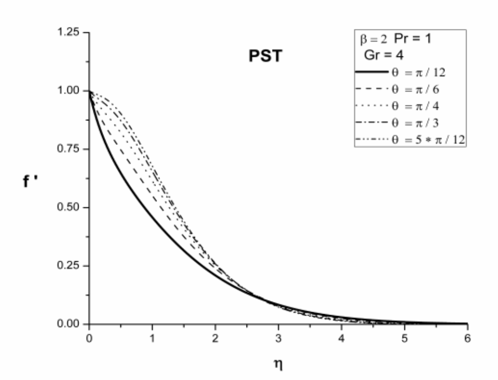

Fig 8a Effect of the inclination ϑ on velocity profile in the case of PST

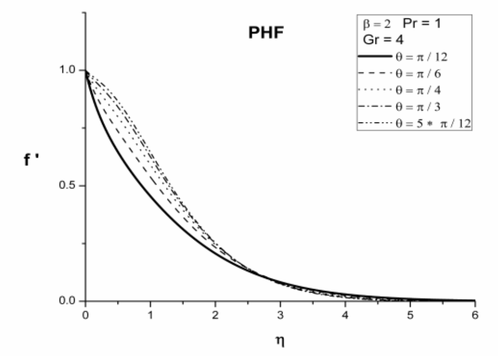

Fig 8b Effect of the inclination ϑ on velocity profile in the case of PHF

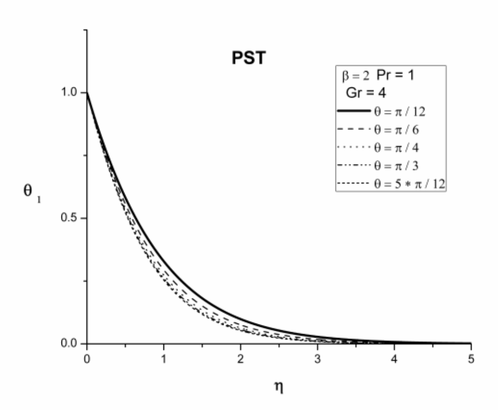

Fig. 9a. Effect of the inclination ϑ on temperature profile in the case of PST

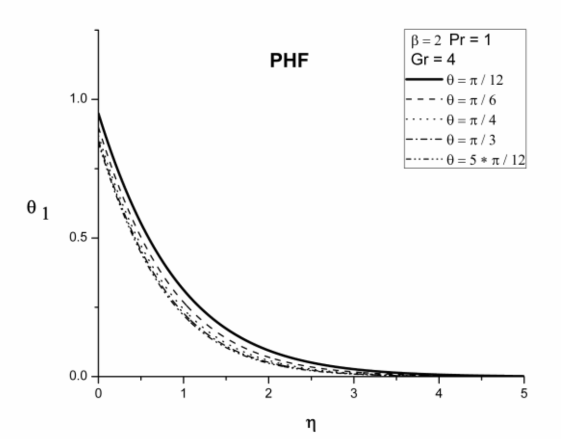

Fig. 9b. Effect of the inclination ϑ on temperature profile in the case of PHF

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Fig. 2a, b shows the interaction of the ferrohydrodynamic parameter β on the velocity profile. As β increases, the presence of the magnetic field induced by the magnetic dipole on the fluid acts as a retarding force, hence decreasing the axial velocity which results in flattening of $f^{'}(\eta)$ whereas, the thermal boundary layer thickens which is seen in Fig. 3a,b.

Fig. 4a, b depicts the increase of velocity as *Gr* increases in both the PST and PHF cases. *Gr* approximates the ratio of buoyancy force to the viscous force acting on the fluid, it also highlights the significance of convection in controlling the axial velocity. As *Gr* increases the momentum boundary layer thickness increases enabling the fluid to flow freely. The buoyancy force evolved as a consequence of the cooling of the inclined stretching sheet acts like a favourable pressure gradient accelerating the fluid in the boundary layer region. Physically $Gr > 0$ means heating of the fluid or cooling of the boundary surface. In Fig. 5a,b we notice that an increase in the value of *Gr* results in thinning of the thermal boundary layer associated with an increase in the wall temperature gradient and hence producing an increase in the heat transfer rate.

From Fig. 6a, b and Fig. 7a, b we observe that increasing the values of *Pr* reduces the horizontal velocity and decreases the thermal boundary layer thickness respectively. This is due to the fact that for small values of the Prandtl number, the fluid is highly thermally conductive. Physically if *Pr* increases, the thermal diffusivity decreases and this phenomenon leads to the decreasing of energy ability which reduces the thermal boundary layer. Thus the viscous boundary layer is thicker than the thermal boundary layer. In this case the temperature asymptotically approaches zero in the free stream region.

As the inclination increases from $\pi/12$ to $5\pi/12$ the momentum boundary layer thickness increases. When the inclination is increased the gravity effect becomes prominent, which helps the liquid to flow freely. The inclination reduces the thermal boundary layer thickness as shown in Fig. 8a, b and Fig. 9a, b respectively. The results clearly reveal that the inclination of the stretching sheet can be effectively used to obtain a desired temperature.

Further work includes, studying the impact of ferrofluid on cooling various stretching sheets of different materials. The authors would also consider studying stretching of filaments that involve cylindrical geometry.

V. CONCLUSIONS

The problem of inclined stretching sheet is analysed in this paper. Numerical solutions of the problem is obtained by the shooting method facilitated with a scientific choice of the missed initial conditions. The important findings of the problem are that the inclination θ together with the ferrohydrodynamic parameter β can be effectively used to have a desired temperature which improves the properties of the stretching sheet. The horizontal stretching sheet problem is a particular case of the inclined stretching sheet problem when θ is zero and the vertical stretching sheet problem is a particular case of the inclined stretching sheet problem when θ is $\frac{\pi}{2}$.

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