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Residual Stresses Computing in Blood Vessels in virtue of Pathological Growth Processes

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Abstract—**The present study deals with problem of the residual stresses calculations in human blood vessel under pathological growth processes. The vessels are simulated by the thin-walled long circular cylinder. The boundary value problem of the surface growth of an elastic thin-walled vessels is solved. The analytical solution is obtained in terms of velocities of stress strain state parameters. The condition of thinness allows us to study finite displacements of cylinder surfaces by means of infinitesimal deformations. The stress-strain state characteristics are numerically computed and graphically analysed by various mechanical parameters of the pathological growth processes.**

Index Terms—**blood vessel, elasticity, growth, residual stress.**

I. Introduction

ORGANS in the human body and animals morphologically discriminate in two types: tubular and stromal. The first ones are a cavity with walls such as vessels, bronchial tubes, bile ducts, gastrointestinal tract, etc. There are a number of pathological processes leading to obturation of the lumen of tubular organs due to the surface deposition of particles. The present study deals with the two pathological processes atherosclerotic lesions of the arteries and thrombosis of the veins. First process is characterized by infiltration of proteins and lipids into a thin layer of the artery. Second one do by subsidence of blood and plasma proteins on the wall of the vessel. Also the process of artery stenting is separately considered. The wall of a large vessel, both arteries and veins, structurally consists of three shells: outer, middle and inner. The outer shell is a loose connective tissue rich in blood vessels. The middle shell is represented by smooth muscle cells and the arteries also have elastic membranes. The inner membrane is thin and is represented by a flat single-layered epithelium lying on the basal membrane [1], [2].

Infiltration of the inner membrane with lipids, proteins and blood cells (macrophages, leukocytes) occurs during atherosclerotic lesions of the artery resulting in the deposition of atherosclerotic masses in the thin layer, which gradually accumulate form an atherosclerotic plaque that grows predominantly in the lumen of the vessel in virtue of elastic forces acting at the muscular membrane [3]–[6]. The outer diameter of the vessel during an atherosclerotic lesion does not generally increase.

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The thrombus formation is caused by a change in the vascular wall in response to which the platelets adhere to the site of injury. Further, the formation of fibrin with the participation of platelets and consolidation of the protein content of the growing thrombus. At the next stage, there is a seizure and adherence of leukocytes, erythrocytes. The final formation of the thrombus is completed by the settling of the plasma proteins of the blood on the formed convolution and its compaction [7]. As a result of the above-described processes, the thrombus has a non-uniform layered structure. The above processes are characterized mainly by surface growth of the artery wall in a thin layer. On vessels with an atherosclerotic lesion for the resumption of normal blood flow stenting is performed. A stent is a thin metal tube, consisting of wire cells, inflated with a special balloon [8], [9]. It is introduced into the affected vessel and is pressed by expansion into the walls of the vessel increasing its lumen.

Another pathological process is the remodeling of the vessel wall as a result of which a volumetric thickening of the wall occurs. With persistent high blood pressure in the arteries of large and medium diameter elastosis and elastofibrosis is revealed. Elastosis and elastofibrosis are sequential stages of the process and represent hyperplasia and cleavage of the internal elastic membrane, which develops compensatory in response to a persistent increase in blood pressure. In the future, the destruction of elastic fibers occurs and their replacement by collagen fibers, i.e. sclerosis. Thus, the wall of the vessels thickens, the lumen tapers [10], [11]. On Figure 1 changes in the vessel wall (thickness, outer and inner radius) are shown in the first approximation for various pathological processes.

The pathological growth of blood vessel wall can be described in some cases by surface growth mechanics technique. At present study we will focused on the processes of surface growth of thin-walled vessels. We use the ideas of the mechanics of growing solids [12]–[21]. Some one-dimensional problems in frameworks of the thermoelastoplasticity are studied under conditions of axial or central symmetries in [22]–[30]. The principal variables of the boundary value problem for a growing body are the stress rate tensor, the strain rate tensor and the velocity vector. On the surface of growth we set a specific boundary condition depending on the curvature tensor of the growth surface and the tension and inflow rates of the incremented elements.

Some problems for an elastic thin-walled surface-growing cylinder are considered at present work. The condition of thinness allows us to study finite displacements of cylinder points under the condition of small deformations. This, in particular, makes it possible to solve the problem with exact boundary conditions on a moving surface. The behaviour features of the strain–stress state characteristics depending on the pressure on the inner and outer surfaces of the cylinder

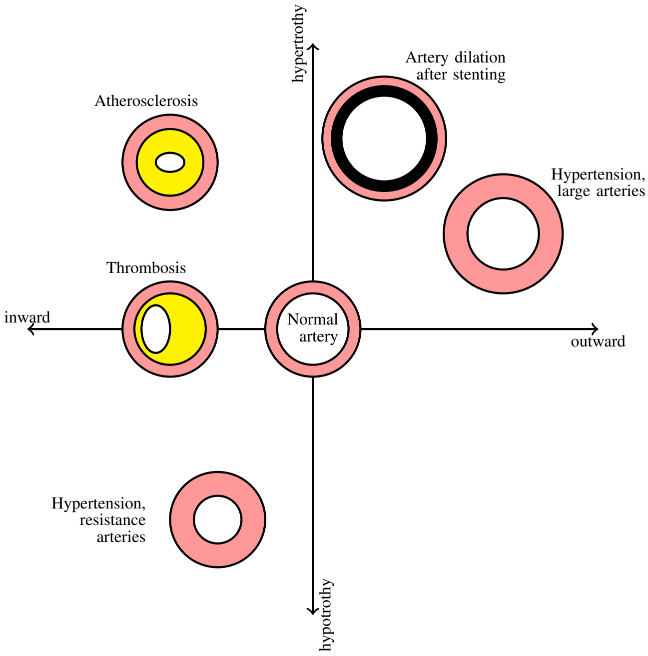

Fig. 1. Abnormal processes in blood vessels wall

is investigated.

II. Constitutive Equations and Boundary Value Problems Statement and Solution

Throughout the paper we will use the conventional linear elastic model [1] generalized on growing body mechanics effects. Consider an infinitely long hollow elastic cylinder with internal and external radii R_1 and R_2, respectively. The relations between the stress tensor σ_{ij} and the infinitesimal deformations e_{ij} for the isotropic elastic material are furnished by the Hooke rule

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}, \tag{1}$$

where λ, μ are the Lame parameters.

Equation (1) in case of cylindrical coordinate system can be rewritten in following form

$$\begin{aligned}
\sigma_{rr} &= (\lambda+2\mu)e_{rr} + \lambda(e_{\varphi\varphi}+e_{zz}),\\
\sigma_{\varphi\varphi} &= (\lambda+2\mu)e_{\varphi\varphi} + \lambda(e_{rr}+e_{zz}),\\
\sigma_{zz} &= (\lambda+2\mu)e_{zz} + \lambda(e_{\varphi\varphi}+e_{rr}).
\end{aligned} \tag{2}$$

The following components of the infinitesimal strain tensor are not vanished by virtue of the hypothesis of a plane strain state and cylindrical symmetry

$$e_{rr} = \frac{\partial u_r}{\partial r}, \qquad e_{\varphi\varphi} = \frac{u_r}{r}, \qquad e_{zz} = 0, \tag{3}$$

where u_r is the radial component of the displacement vector.

The components of the stress tensor satisfy the equation of equilibrium

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr}-\sigma_{\varphi\varphi}}{r} = 0. \tag{4}$$

One can transform the equilibrium equation (4) by determining the deformations from equation (2) in terms of the stresses by

$$\frac{\partial}{\partial r}\left(2\sigma_{rr} + r\frac{\partial \sigma_{rr}}{\partial r}\right) = 0. \tag{5}$$

A general solution of the equation (5) constitutes the radial stress depending on radius and undefined functions A, B

$$\sigma_{rr} = A + \frac{B}{r^2}. \tag{6}$$

The angular stress $\sigma_{\varphi\varphi}$, axis one σ_{zz} and radial displacement u_r define according to (2)–(6)

$$\begin{aligned}
\sigma_{\varphi\varphi} &= A - \frac{B}{r^2},\\
\sigma_{zz} &= \frac{\lambda A}{(\lambda+\mu)},\\
u_r &= \frac{Ar}{2(\lambda+\mu)} - \frac{B}{2\mu r}.
\end{aligned} \tag{7}$$

The lateral surface of the cylinder is subjected to a stationary loading pressure

$$\begin{aligned}
\sigma_{rr}(R_1) &= p_1,\\
\sigma_{rr}(R_2) &= -p_2.
\end{aligned} \tag{8}$$

The relations for an undefined functions A and B are derived by the solution of the boundary conditions system (8) in forms

$$\begin{aligned}
A &= -\frac{p_1R_1^2 + p_2R_2^2}{R_2^2 - R_1^2},\\
B &= \frac{R_1^2R_2^2(p_1+p_2)}{R_2^2 - R_1^2}.
\end{aligned} \tag{9}$$

The equations (6)–(9) determine the stress-strain state parameters before the growth. Suppose at time $t = 0$ on the inner surface of the cylinder the new material is adding with the velocity $v = v(t)$

$$\begin{aligned}
\dot{\sigma}_{rr}(R_1(t)) &= -\frac{v(t)\tau(t)}{R_1(t)},\\
R_1(t) &= R_1 - v(t)t,\\
\dot{\sigma}_{rr}(R_2) &= 0.
\end{aligned} \tag{10}$$

Herein the dot denotes the speed of the considered variable, which can be determined as a partial derivative with respect to time within the frameworks of the infinitesimal deformations approach; $\tau(t)$ is the circumferential tension of the adding layer at the moment of joining to the cylinder surface. For a correct description of the stress-strain state evolution under the conditions of continuous growth $t > 0$ it is necessary to transform governing equations by replacing all variables on its velocities (2)–(4). After that we obtain following solution in terms of the velocities

$$\begin{aligned}
\dot{\sigma}_{rr} &= X(t) + \frac{Y(t)}{r^2},\\
\dot{\sigma}_{\varphi\varphi} &= X(t) - \frac{Y(t)}{r^2},\\
\dot{\sigma}_{zz} &= \frac{\lambda X(t)}{(\lambda+\mu)},\\
\dot{u}_r &= \frac{X(t)r}{2(\lambda+\mu)} - \frac{Y(t)}{2\mu r}.
\end{aligned} \tag{11}$$

Unknown functions $X(t)$ and $Y(t)$ can be computed by the solution of the system (10) in following form

$$\begin{aligned}
X(t) &= \frac{R_1(t)v(t)\tau(t)}{R_2^2 - R_1^2(t)},\\
Y(t) &= -\frac{R_1(t)v(t)\tau(t)R_2^2}{R_2^2 - R_1^2(t)}.
\end{aligned} \tag{12}$$

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The relations for stresses

$$
\begin{aligned}
\sigma_{rr} &= A + \frac{B}{r^2} + \int_0^t \left(X(s) + \frac{Y(s)}{r^2} \right) ds, \\
\sigma_{\varphi\varphi} &= A - \frac{B}{r^2} + \int_0^t \left(X(s) - \frac{Y(s)}{r^2} \right) ds, \\
\sigma_{zz} &= \frac{\lambda}{(\lambda + \mu)} \left(A + \int_0^t X(s) ds \right),
\end{aligned} \tag{13}
$$

and displacements after integrating on time the equations (11) taking into account the boundary conditions (6)–(7) read by

$$
u_r = \frac{Ar}{2(\lambda + \mu)} - \frac{B}{2\mu r} + \left(\int_0^t \frac{X(s) r}{2(\lambda + \mu)} - \frac{Y(s)}{2\mu r} \right) ds. \tag{14}
$$

III. Numerical Results Discussion

The calculations were carried out for various values of the growing layer size $R_1(t)$ and the growth layer tension level $\tau(t)$. For the case of $v(t) = const$ and $\tau(t) = const$ it is established that the growth tension level τ level has a key influence on the formation of the final stress-strain state of the material. Figure 2 and 4 illustrate the radial stress field. On Figure 6 and 7 the circumferential one is shown during the growth. The stresses are vanished on the inner surface $R_1(t)$ for a certain value of growth layer tension τ during the inner cylinder wall growth. On Figure 3 and 5 the radial displacement in the process of the inner cylinder wall growth is shown.

The stresses concentration caused by the influence of internal pressure can be computed in the absence of initial growth tension in the adding layer by formulae (6) and (9). The stresses concentration caused by the high level of the initial growth tension during the narrowing of the vessel lumen are calculated. Thus, it is possible to conclude, that on the one hand, with correct initial parameters of the growing process, it is possible to achieve a minimum effect of the internal pressure in the vessel on its walls deformation. To another hand, a certain growth regime is capable of causing irreversible deformation and fracture vessel wall.

Conclusions

- The constitutive equations of the linear elasticity have been furnished as a tool for mathematical modelling of pathological growth of a blood vessel.
- The boundary value problem of the surface growth of an elastic thin-walled cylinder has been stated and analytically solved.
- The residual stresses caused by the different levels of the initial growth tension in virtue of the narrowing vessel lumen have been calculated.
- The stress-strain state characteristics depending on the mechanical parameters of the biological processes, are numerically computed and graphically analysed.

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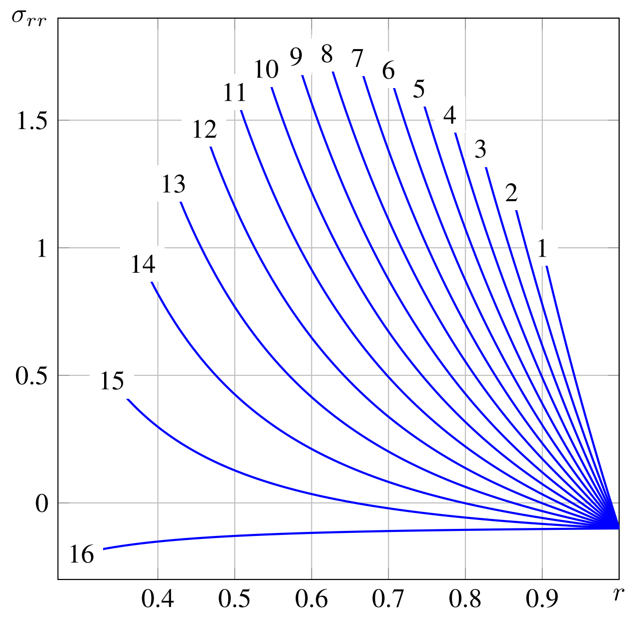

Fig. 2. Radial stress σ_{rr} in the growth cylinder at different times. Line denoted by number i corresponds to the growth time t_i. Material and process dimensionless parameters: $\tau = 6$, $\lambda = 4$, $\mu = 2$, $v = 0.1$, $p_1 = 1$, $p_2 = 0.1$, $R_1 = 0.9$, $R_2 = 1$.

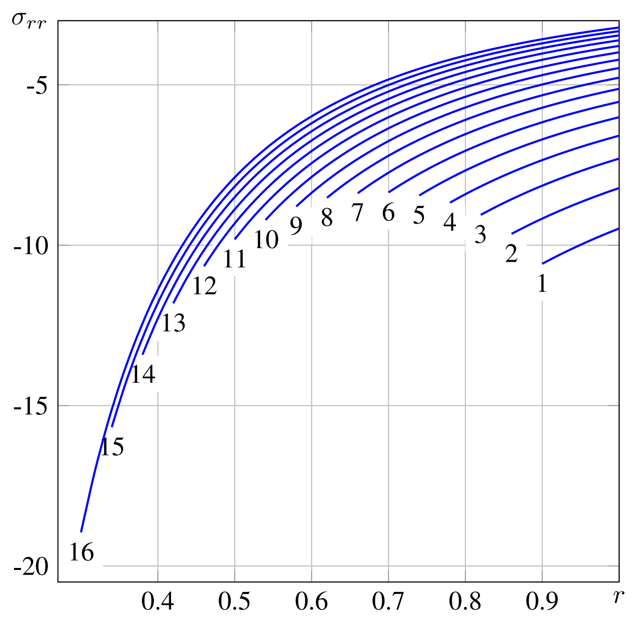

Fig. 3. Radial stress σ_{rr} in the growth cylinder at different times. Line denoted by number i corresponds to the growth time t_i. Material and process dimensionless parameters: $\tau = 3$, $\lambda = 4$, $\mu = 2$, $v = 0.1$, $p_1 = 1$, $p_2 = 0.1$, $R_1 = 0.9$, $R_2 = 1$.

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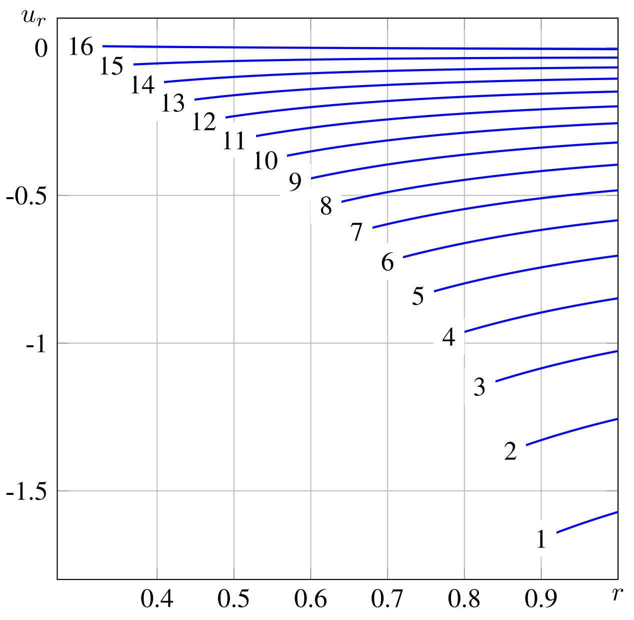

Fig. 4. Radial displacement u_r in the growth cylinder at different times. Line denoted by number i corresponds to the growth time t_i. Material and process dimensionless parameters: $\tau = 6$, $\lambda = 4$, $\mu = 2$, $v = 0.1$, $p_1 = 1$, $p_2 = 0.1$, $R_1 = 0.9$, $R_2 = 1$.

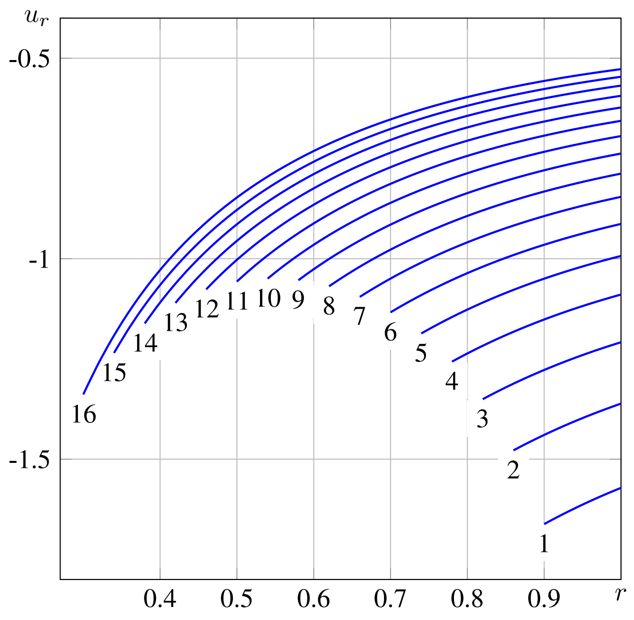

Fig. 6. Radial displacement u_r in the growth cylinder at different times. Line denoted by number i corresponds to the growth time t_i. Material and process dimensionless parameters: $\tau = 3$, $\lambda = 4$, $\mu = 2$, $v = 0.1$, $p_1 = 1$, $p_2 = 0.1$, $R_1 = 0.9$, $R_2 = 1$.

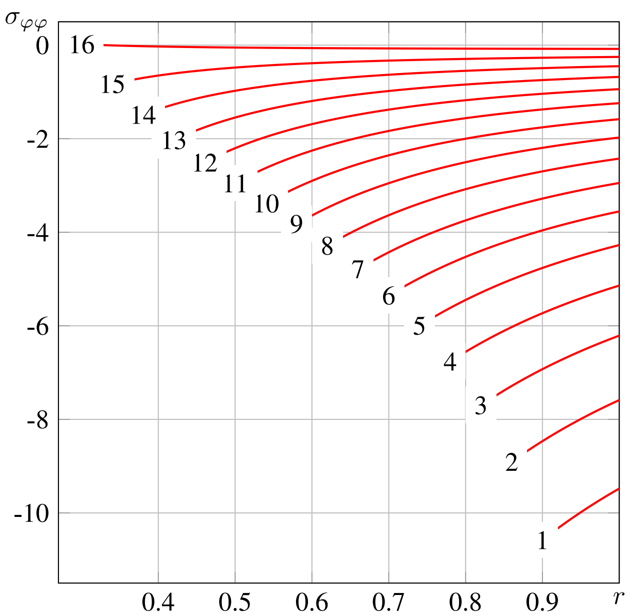

Fig. 5. Circumferential stress $\sigma_{\varphi\varphi}$ in the growth cylinder at different times. Line denoted by number i corresponds to the growth time t_i. Material and process dimensionless parameters: $\tau = 6$, $\lambda = 4$, $\mu = 2$, $v = 0.1$, $p_1 = 1$, $p_2 = 0.1$, $R_1 = 0.9$, $R_2 = 1$.

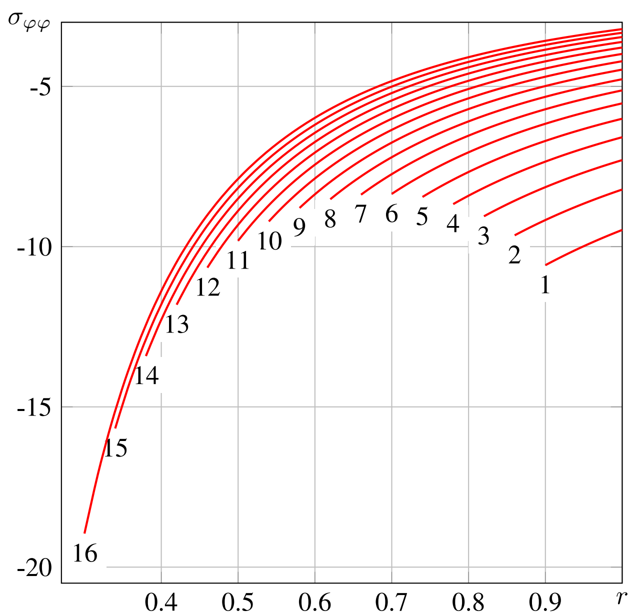

Fig. 7. Circumferential stress $\sigma_{\varphi\varphi}$ in the growth cylinder at different times. Line denoted by number i corresponds to the growth time t_i. Material and process dimensionless parameters: $\tau = 3$, $\lambda = 4$, $\mu = 2$, $v = 0.1$, $p_1 = 1$, $p_2 = 0.1$, $R_1 = 0.9$, $R_2 = 1$.

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