A Study of Unsteady MHD Vertical Flow of an Incompressible, Viscous, Electrically conducting Fluid bounded by Two Non-Conducting Plates in Presence of a Uniform inclined Magnetic Field

MRINMOY GOSWAMI, KRISHNA GOPAL SINGHA, AMARJYOTI GOSWAMI, P.N.DEKA

Abstract— In this article we are considering unsteady magnetohydrodynamic flow of an incompressible, viscous fluid bounded by two no-conducting parallel plates placed vertically in presence of uniform inclined magnetic field. One of the plates is considered to be in motion with constant velocity whereas the other plate is adiabatic. Using transformation associated with decay factor, we have deduced a set of ordinary differential equations which are solved analytically for the flow field, temperature field and induced magnetic field for different values of magnetohydrodynamic flow parameters. The results are presented graphically and corresponding effects have been discussed.

Keywords— MHD fluid flow, Heat transfer, unsteady, Prandtl number, adiabatic, analytical solution.

I. Introduction

The study of MHD flow under the action of a uniform transverse magnetic field has generated much interest in recent years in view of its numerous industrial applications such as the MHD generators and plasma MHD accelerators, pumps, flowmeters, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology etc..

The consequent effect of the presence of solid particles on the performance of such devices has led to the studies of particulate suspensions in conducting fluids in the presence of externally applied magnetic field.

Seth and Ghosh [1] considered the unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid in a rotating channel under the influence of a periodic pressure gradient and of uniform magnetic field, which was inclined with the axis of rotation. An analytical solution to the problem of steady and unsteady hydromagnetic flow of viscous incompressible electrically conducting fluid under the influence of constant and periodic pressure gradient in presence of inclined magnetic field has been obtained exactly by Ghosh [2] to study the effect of slowly rotating systems with low frequency of oscillation when the conductivity of the fluid is low and the applied magnetic field is weak. Yang and Yu [3] have investigated the entrance problem of convective magnetohydrodynamic channel flow between two parallel plates subjected simultaneously to an axial temperature gradient and a pressure gradient. They have also considered both the cases of constant heat flux and constant wall temperature.

The unsteady magnetohydrodynamic flow of an electrically conducting viscous incompressible non- Newtonian Bingham fluid bounded by two parallel non-conducting porous plates with heat transfer considering the Hall Effect has been studied by Attia and Ahmed [4]. Borkakati and Bharali [5] have studied the problem of flow and heat transfer between two infinite horizontal parallel porous plates, where the lower plate is a stretching sheet and the upper one is a porous solid plate in presence of a transverse magnetic field. The heat transfer in an axisymmetric flow between two parallel porous disks under the effect of a transverse magnetic field was studied by Bharali and Borkakati [6]. Shih-I-Pai [7] studied an unsteady motion of an infinite flat insulated plate set impulsively into the uniform motion with velocity in its own plane in the presence of a transverse uniform magnetic field. The problem of combined free and forced convective magnetohydrodynamic flow in a vertical channel has been studied by Umavathi and Malashetty [8]. They had also considered the effect of viscous and ohmic dissipations. It has been observed that the viscous dissipation enhances the flow reversal in the case of downward flow while it countered the flow in the case of upward flow. Singha and Deka[9] considered the problem of two phase MHD flow and heat transfer problem in a horizontal channel. Jordán [10] has investigated the transient free convection MHD flow of a dissipative fluid along a semi-infinite vertical plate with mass transfer, the surface of which is exposed to a constant heat flux. In his paper he also studied the influences of the viscous dissipation, buoyancy ratio parameter, Schmidt number and magnetic parameter on heat and mass transfer and on the time needed to reach the steady-state.

The effects of heat transfer on unsteady hydromagnetic flow in a parallel-plate channel of an electrically conducting, viscous, incompressible fluid have been investigated by Singha [11]. He found that velocity distribution increases near the plates and then decreases very slowly at the central portion between the two plates. The principal numerical results presented in his work showed that the flow field is appreciably influenced by the applied magnetic field. Singha[12] investigated the effect of transversely applied external magnetic field on the unsteady laminar flow of an incompressible viscous electrically conducting fluid in a channel of two horizontal heated plates.

Manuscript received April 10, 2017
Mrinmoy Goswami is with the Department of Mathematics, Kaziranga University,Jorhat-785006,India (e-mail: mrinmoygoswami1977 @yahoo.in).
Krishna Gopal Singha is with the Department of Mathematics, Karanga Girls' Higher Secondary School, Jorhat -785008, India (e-mail : kgsingha@rediffmail.com).
Amarjyoti Goswami is with the Department of Electronics, Kaziranga University, Jorhat -785006, India (e-mail: amarjyoti@kazirangauniversity.in).
P. N. Deka is with Department of Mathematics, Dibrugarh University,Dibrugarh,786004 , India (e-mail: pndeka@yahoo.co.in)

Hence the present study investigates the effect of the unsteady MHD flow of an incompressible, viscous, electrically conducting fluid bounded by two non-conducting parallel plates vertically in presence of uniform inclined magnetic field. one of which is at rest, other moving in its own plane with a velocity u_0. The analytical solutions for the fluid velocities, magnetic field and temperature distributions are obtained. The effects of various parameters on the flow and heat transfer are shown graphically.

II. FORMULATION OF THE PROBLEM

The unsteady laminar flow of an incompressible viscous electrically conducting fluid between two non-conducting parallel plates placed vertically at a distance 2h apart is considered in presence of uniform inclined magnetic field. The flow is assumed to be along the X -axis parallel to vertical direction through the central line of the channel and Y -axis is normal to it.

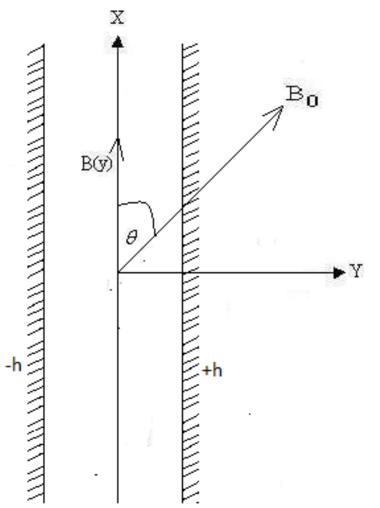

Figure: 1 Geometrical configuration

The plates of the channel are at $y = \pm h$ and that the relative velocity between the two plates is $2u_0$ and also, there is no pressure gradient in the flow field. An external uniform magnetic field of strength B_0 makes an angle θ with the positive direction of X -axis which induces a magnetic field $B(y)$ makes also an angle θ to the free stream velocity. The plate at $y = -h$ is maintained at temperature T_0, while the other plate $y = +h$ is kept at temperature T_1 $(T_1 > T_0)$ and the plates are electrically non-conducting.
The components of the velocities and the magnetic field are given as follows:

$$\vec{V} = \{u, v, w\} = \{u(y,t), 0, 0\},$$

$$\vec{B} = \{B_x, B_y, B_z\} = \{cos\theta\, B(y,t),\ cos(90^0 - \theta)B_0,\ 0\}$$

$$= \{cos\theta\, B(y,t),\ sin\theta B_0,\ 0\}$$

$$= \{\lambda\, B(y,t),\ \sqrt{1-\lambda^2}\, B_0,\ 0\}$$

where p = constant pressure gradient in the flow direction, $\lambda = \cos\theta$ and $'t'$ is the time.

III. ASSUMPTIONS

In order to derive the governing equations of the problem the following assumptions are made.
(i) The fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected.
(ii) Hall effect and polarization effect are negligible.
4 Governing equations
The governing equations of the problem under the above conditions are as follows:

$$\nabla \cdot \vec{V} = 0. \tag{1}$$

$$\rho\left\{\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\cdot\nabla\right)\vec{V}\right\} = -\nabla p + \mu\nabla^2\vec{V} + \left(\vec{J}\times\vec{B}\right) + \vec{Z} \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} - \nabla\times(\vec{V}\times\vec{B}) - \left(\frac{1}{\sigma\mu_e}\right)\nabla^2\vec{B} = 0. \tag{3}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}\right) = \frac{d}{dy}\left(\kappa\frac{dT}{dy}\right). \tag{4}$$

Here the third term in the right hand side of Eq. (2) is the magnetic body force and $\vec{J}$ is the electric current density due to the magnetic field defined by

$$\vec{J} = \sigma(\vec{E} + \vec{V}\times\vec{B}) \tag{5}$$

$\vec{Z}$ is the force due to buoyancy,

$$\vec{Z} = \rho\beta g(T - T_0) \tag{6}$$

It has been taken that $\vec{E} = 0$.
That is, in the absence of convection outside the boundary layer, $\vec{B} = B_0$ and $Curl\,\vec{B} = \mu\vec{J} = 0$, then (5) leads to $\vec{E} = 0$.

The fluid motion starts from rest at $t = 0$, and the no-slip condition at the plates implies that the fluid velocity has neither a z-nor an x-component at $y = \pm h$.

Using the velocity and magnetic field distribution as stated above, the Eq. (1)- (4) are as followed:

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^{\,2}}{\rho}(1-\lambda^2)u + \beta g(T - T_0) \tag{7}$$

$$\frac{\partial B}{\partial t} - \left(B_0\sqrt{\frac{1}{\lambda^2} - 1}\right)\frac{\partial u}{\partial y} - \left(\frac{1}{\sigma\,\mu_e}\right)\frac{\partial^2 B}{\partial y^2} = 0 \tag{8}$$

$$\frac{\partial T}{\partial t} = \left(\frac{\kappa}{\rho\, c_p}\right)\frac{\partial^2 T}{\partial y^2}$$

ISBN: 978-988-14047-9-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\Rightarrow \frac{\partial T}{\partial t} = \alpha_1 \frac{\partial^2 T}{\partial y^2} \tag{9}$$

Let us assume that the other plate is adiabatic i.e. thermally insulated walls, then the boundary conditions are

$$\left.\begin{aligned} &t = 0: u = 0, \quad B = B_0, \; T = T_1 \\ &t > 0: u = u_0, \; B = B_0, \; T = T_1, \text{ at } \quad y = +h, \\ &t > 0: u = -u_0, \; B = B_0, \; \frac{\partial T}{\partial y} = 0, \text{ at } \quad y = -h. \end{aligned}\right\} \tag{10}$$

Introducing the following non-dimensional quantities:

$$u^* = \frac{u}{u_0}, \; y^* = \frac{y}{h}, \; t^* = \frac{t\,u_0}{h}, \; b = \frac{B}{B_0}, \; \overline{T} = \frac{T - T_0}{T_1 - T_0}. \tag{11}$$

In terms of the above non-dimensional variables and parameters, the basic equations (7)-(9) take the form

$$\frac{\partial u}{\partial t} = \left(\frac{1}{R_e}\right)\frac{\partial^2 u}{\partial y^2} - H_a R_e (1 - \lambda^2) u + \left(\frac{G_r}{R_e^2}\right)\overline{T} \tag{12}$$

$$\frac{\partial b}{\partial t} - \left(\sqrt{\frac{1}{\lambda^2} - 1}\right)\frac{\partial u}{\partial y} - \left(\frac{1}{R_e\, R_m\, P_r}\right)\frac{\partial^2 b}{\partial y^2} = 0 \tag{13}$$

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{P_e}\left(\frac{\partial^2 \overline{T}}{\partial y^2}\right) \tag{14}$$

The asterisks have been dropped with the understanding that all the quantities are now dimensionless.
For relation (11) the boundary conditions (10) becomes

$$\left.\begin{aligned} &t = 0: u = 0, \; b = 1, \; \overline{T} = 1 \\ &t > 0: u = +1, \; b = 1, \; \overline{T} = 1, \quad \text{at } \; y = +1, \\ &t > 0: u = -1, \; b = 1, \frac{\partial \overline{T}}{\partial y} = 0, \quad \text{at } \; y = -1. \end{aligned}\right\} \tag{15}$$

In order to solve Eqs. (12) - (14), we consider

$$u = f(y)e^{-nt}, \quad b = g(y)e^{-nt}, \quad \text{and } \; \overline{T} = F(y)e^{-nt}, \tag{16}$$

where ‘ n ’ is the decay constant.
Substituting (16) in Eqs. (12) - (14), we get

$$f''(y) - R_e\left\{H_a R_e (1 - \lambda^2) - n\right\} f(y) + \frac{G_r}{R_e} F(y) = 0 \tag{17}$$

$$g''(y) + (nR_e R_m P_r) g(y) + \left(R_e R_m P_r \sqrt{\frac{1}{\lambda^2} - 1}\right) f'(y) = 0 \tag{18}$$

and

$$F''(y) + nP_e F(y) = 0 \tag{19}$$

For relation (16) the boundary conditions (15) again becomes

$$\left.\begin{aligned} &t = 0: \; f = 0, \qquad g = 1, \qquad F = 1, \\ &t > 0: \; f = e^{nt}, \quad g = e^{nt}, \; F = e^{nt}, \text{ at } \; y = +1, \\ &t > 0: f = -e^{nt}, \quad g = e^{nt}, \frac{\partial F}{\partial y} = 0, \text{ at } \; y = -1. \end{aligned}\right\} \tag{20}$$

The solutions of Eqs. (17) - (19) with the help of the boundary conditions (20) and substituting in the relations (16) are

$$u(y) = A_{13} \cos[(1+y)A_5] + e^{-\sqrt{A_1}y} C_5 + e^{\sqrt{A_1}y} C_6 \tag{21}$$

$$\begin{aligned} b(y) = {} & e^{-\sqrt{A_1}y} A_7 (e^{\sqrt{A_1}y} \sin[(1+y)A_5] A_{14} + A_{12}(A_{15} - e^{2\sqrt{A_1}y} A_{16} + \\ & e^{\sqrt{A_1}y} A_9 (\sin[\sqrt{A_3}y] C_7 + \cos[\sqrt{A_3}y] C_8))) \end{aligned} \tag{22}$$

$$\overline{T} = \frac{\cos[(1+y)A_5]}{\cos[2A_5]} \tag{23}$$

where

$$A_1 = R_e\left\{H_a R_e (1 - \lambda^2) - n\right\},$$

$$A_2 = \left(\frac{G_r}{R_e}\right),$$

$$A_3 = \left(n\, R_e\, R_m\, P_r\right),$$

$$A_4 = \left(R_e\, R_m\, P_r \sqrt{\frac{1}{\lambda^2} - 1}\right),$$

$$A_5 = \sqrt{n\,P_e},$$

$$A_6 = \frac{A_2 \operatorname{Cosec}[2A_5]}{A_1 + A_5^2},$$

$$A_7 = \frac{1}{(A_1 + A_3)(A_3 - A_5^2)},$$

$$A_8 = (A_1 + A_3) A_4 A_5 A_6,$$

$$A_9 = A_1 + A_3,$$

$$A_{10} = \sqrt{A_1} A_4 C_1,$$

$$A_{11} = \sqrt{A_1} A_4 C_2,$$

$$A_{12} = A_3 - A_5^2,$$

$$A_{13} = \frac{A_2 \operatorname{Sec}[2A_5]}{A_1 + A_5^2},$$

$$A_{14} = (A_1 + A_3) A_4 A_5 A_{13},$$

$$A_{15} = \sqrt{A_1} A_4 C_5,$$

ISBN: 978-988-14047-9-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$A_{16} = \sqrt{A_1} A_4 C_6 \,,$$

$$C_1 = -\frac{e^{\sqrt{A_1}}\left(1 + e^{2\sqrt{A_1}} - sin[2A_5]A_6\right)}{e^{4\sqrt{A_1}} - 1},$$

$$C_2 = -\frac{e^{\sqrt{A_1}}\left(-1 - e^{2\sqrt{A_1}} + e^{2\sqrt{A_1}} sin[2A_5]A_6\right)}{e^{4\sqrt{A_1}} - 1},$$

$$C_3 = \frac{e^{-\sqrt{A_1}} cosec[\sqrt{A_3}]\left(-2e^{\sqrt{A_1}} sin[A_5]^2 A_8 + \left(e^{2\sqrt{A_1}} - 1\right)(A_{10} + A_{11})A_1\right)}{2A_9 A_{12}},$$

$$C_4 = \frac{e^{-\sqrt{A_1}} sec[\sqrt{A_3}]\left(2e^{\sqrt{A_1}} + A_7\left(2e^{\sqrt{A_1}} cos[A_5]^2 A_8 - \left(1 + e^{2\sqrt{A_1}}\right)(A_{10} - A_{11})A_{12}\right)\right)}{2A_7 A_9 A_{12}},$$

$$C_5 = -\frac{e^{\sqrt{A_1}}\left(1 + e^{2\sqrt{A_1}} + e^{2\sqrt{A_1}} A_{13} - cos[2A_5]A_{13}\right)}{e^{4\sqrt{A_1}} - 1},$$

$$C_6 = -\frac{e^{\sqrt{A_1}}\left(-1 - e^{2\sqrt{A_1}} - A_{13} + e^{2\sqrt{A_1}} cos[2A_5]A_{13}\right)}{e^{4\sqrt{A_1}} - 1},$$

$$C_7 = \frac{cosec[\sqrt{A_3}]\left(-cos[A_5]\, sin[A_5] A_{14} + sinh[\sqrt{A_1}](A_{15} + A_{16})\right)}{A_9 A_{12}},$$

$$C_8 = \frac{sec[\sqrt{A_3}]\left(\dfrac{2 - sin[2A_5]A_7 A_{14}}{A_7 A_{12}} + 2cos[\sqrt{A_1}](A_{16} - A_{15})\right)}{2A_9},$$

IV. RESULTS AND DISCUSSION

Numerical solutions for the Eqs. (21) - (23) are obtained for different values of λ, where

λ= cosθ which varies as θ = $30^0, 45^0, 60^0, 75^0$.

All plotting for such cases are carried out by using MATHEMATICA.

In fig. 2, the variations in velocity field due to variations in magmatic field inclination with flow direction have been plotted. When the external field is more inclined to flow direction the Hartmann effect is considerably reduced. Near θ is near zero, the flow field behaves like an ordinary Couette flow. With the increase of angle of inclination of magnetic field with flow direction the influence of Hartmann effect become more and more clearer.

In fig.3, it has been observed that, with the increase of Grashof number, there appear sharp changes in velocity field. When the Grashof number is increased beyond the critical value Gr=10, reverse flow phenomena near plate which is in motion with velocity v=-1, is observed. We can conclude that for small Grashof number regimes, velocity field changes almost linearly. It indicates that the dominance of buoyancy force in the flow may have nonlinear influence on flow field.

Plottings in fig.4 indicate that greater the angle of inclination of imposed field with flow direction , more the corresponding changes in velocity field due to increase in Hartmann effect. The increased changed in velocity field intensifies the induced field.

Fig. 5 shows effect of Grashof number on induced magnetic field. With the increase in value of Grashof number it is observed that intensity of induced field in flow in increased and it is more clearer in the central region of channel whereas viscous effect out from the walls diminishes the difference in velocity field. The relative dominance of bouncy force and inertia force over viscous force in the flow may increase the intensity of induced magnetic field.

From the plotting of Fig.6 it has been observed that with the increase magnetic Reyonld number, intensity of induced magnetic field is increased. Field intensity in central flow region of the channel is much higher than near the boundary plates. This increase in intensity in central flow region of the channel is much rapid with increase in magnetic Reynold number. This indicates that stronger magnetic convection effect over magnetic diffusion process may result in increase in induced field intensity in such a flow.

In fig.7 it is observed that temperature field changes much rapidly with increase of Prandtl number. In this vertical channel flow, the temperature variation across the boundary plates is more and more rapid with increase of momentum diffusion process in the flow along with decrease in heat diffusion process in the flow.

Nomenclature

- h half width between parallel plates channel (m)
- B_0 external uniform magnetic field (T)
- κ thermal conductivity ($Wm^{-1}K^{-1}$)
- H_a magnetic Hartmann number, $\dfrac{\sigma B_0^2 \nu}{\rho u_0^2}$
- R_e Reynolds number, $\dfrac{u_0 h}{\nu}$
- P_r Prandtl number, $\dfrac{\nu}{\alpha_1}$
- G_r Grashoff number, $\dfrac{\beta g h^3 (T_1 - T_0)}{\nu^2}$
- P_e Peclet number, $P_e = P_r R_e$
- R_a Rayleigh number, $\dfrac{\beta g h^3 T_0}{\nu \alpha_1}$
- R_m Magnetic Reynolds number, $\alpha_1 \mu_e \sigma$
- $\vec{E}$ electric field intensity (NC^{-1})
- u_0 free stream velocity (ms^{-1})
- T_0 temperature of the lower plate (K)
- T_1 temperature of the upper plate (K)

ISBN: 978-988-14047-9-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

t time (s)

c_p specific heat at constant pressure ($J.kg^{-1}K^{-1}$)

g acceleration due to gravity (ms^{-2})

Greek symbols

α_1 thermal diffusivity, $\frac{\kappa}{\rho c_p}$

σ electrical conductivity ($\Omega^{-1}m^{-1}$)

μ co-efficient of viscosity ($kg\, m^{-1}s^{-1}$)

ρ density of the fluid ($kg\, m^{-3}$)

ν kinematic viscosity ($m^2\, s^{-1}$), $\frac{\mu}{\rho}$

β co-efficient of thermal expansion of the fluid (K^{-1})

μ_e permeability of the medium

υ_m magnetic diffusivity, $\frac{1}{\sigma\mu_e}$

Superscript

* non-dimensional variables defined in Eq. (11)

References

[1] G. S. Seth and S. K. Ghosh, Unsteady hydromagnetic flow in a rotating channel in the presence of inclined magnetic field, International Journal of Engineering Science, 24 (7), (1986), 1183-1193.

[2] S. K. Ghosh, A note on steady and unsteady hydromagnetic flow in a rotating channel in the presence of inclined magnetic field, International Journal of Engineering Science, 29 (3.8), (1991), 1013-1016.

[3] H. K. Yang and C. P. Yu, Combined forced and free convection MHD channel flow in entrance region " International Journal of Heat and Mass Transfer" Vol: 17, No. 6 [1974], pp-681-691.

[4] Hazem Ali Attia and Mohamed Eissa Sayed-Ahmed, Hall effect on unsteady MHD Couette flow and heat transfer of a Bingham fluid with suction and injection "Applied Mathematical Modelling", Vol: 28, No. 12 [2004] , pp 1027-1045

[5] A. Bharali and A. K. Borkakati, Magnetohydrodynamical Flow with Hall Effects, Past an Infinite Porous Plate and Heat Transfer "Journal of the Physical Society of Japan" Vol: 49, No. 5 [1980] , pp. 2091-2092.

[6] A. Bharali and A. K. Borkakati, Magnetohydrodynamic Free Convection Flow with Hall Effects, past a Porous Vertical Plate "Journal of the Physical Society of Japan" Vol: 52, No. 1 [1983], pp-6-7.

[7] S. I Pai "Magnetogasdynamics and Plasma Physics" Springer Verlag [1961] , p-54.

[8] J.C. Umavathi and M.S. Malashetty, Magnetohydrodynamic mixed convection in a vertical channel "International Journal of Non-Linear Mechanics" Vol: 40, No.1 [2005], pp. 91-101.

[9] K.G. Singha and P.N. Deka, Magnetohydrodynamic heat transfer in two-phase flow in presence of uniform inclined magnetic field,"Bull.Cal. Math. Soc., Vol. 101,(1) [2009], pp.25-36.

[10] Joaquín Zueco Jordán, Numerical study of an unsteady free convective magnetohydrodynamic flow of a dissipative fluid along a vertical plate subject to a constant heat flux "International Journal of Engineering Science" Vol: 44, No. 18-1 [2006], pp. 1380- 1393.

[11] K.G. Singha, "The effect of heat transfer on unsteady hydromagnetic flow in a parallel plates channel of an electrically conducting , viscous, incompressible fluid," Int. J. Fluid Mech. Research, vol 35, issue 2, 2008, pp. 172-18 [online] . Available: http://www.edata-center.com/journals/71ca5b40f8f8.html

[12] K.G. Singha 'Analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field" International Journal of Applied Mathematics and Computation, Volume 1(4),pp 183–193, 2009

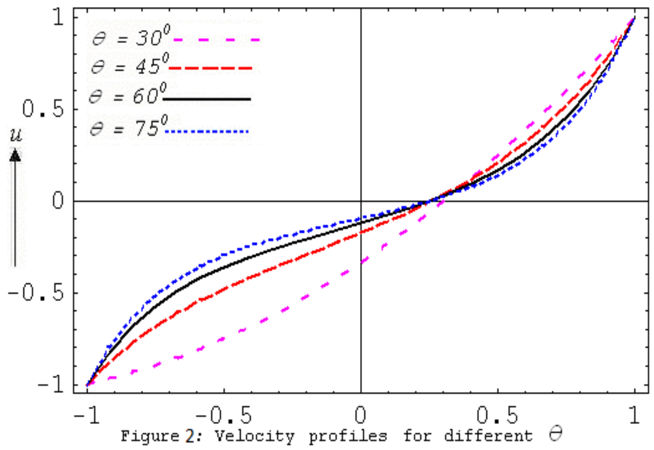

Figure 2: Velocity profiles for different θ
($n = 1.0$, $R_m = 0.2$, $R_e = 1.5$, $H_a = 7.0$, $G_r = 2.0$ and $P_r = 0.71$))

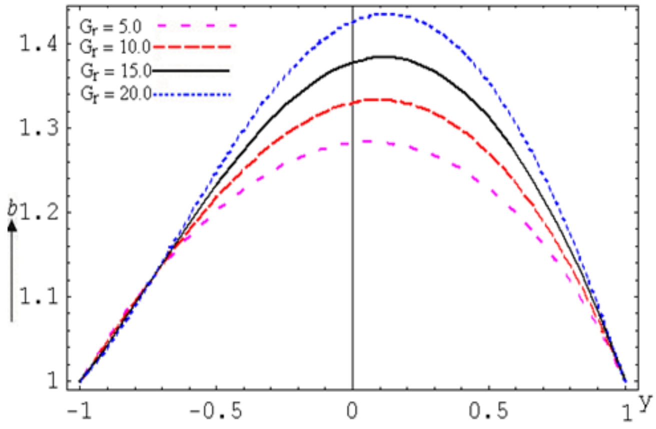

Figure 5: Magnetic field profiles for different G_r
($n = 1.0$, $R_m = 0.2$, $R_e = 1.5$, $H_a = 7.0$, $\lambda = 0.5$ and $P_r = 0.71$)

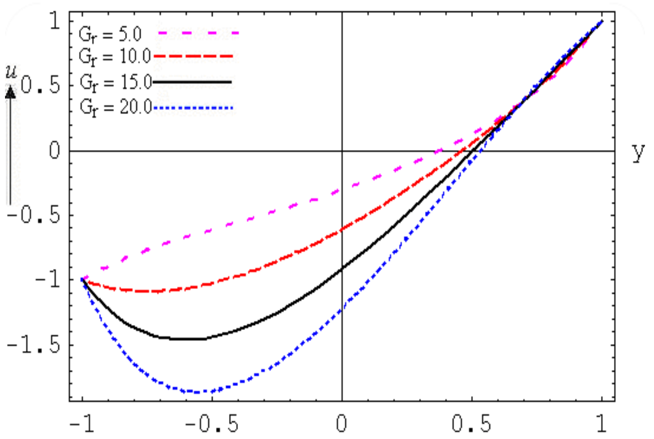

Figure 3: Velocity profiles for different G_r
($n = 1.0$, $R_m = 0.2$, $R_e = 1.5$, $H_a = 7.0$, $\lambda = 0.5$ and $P_r = 0.71$)

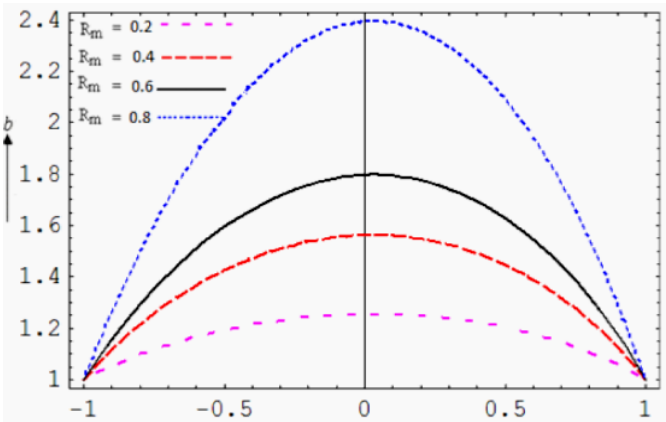

Figure 6: Magnetic field profiles for different R_m

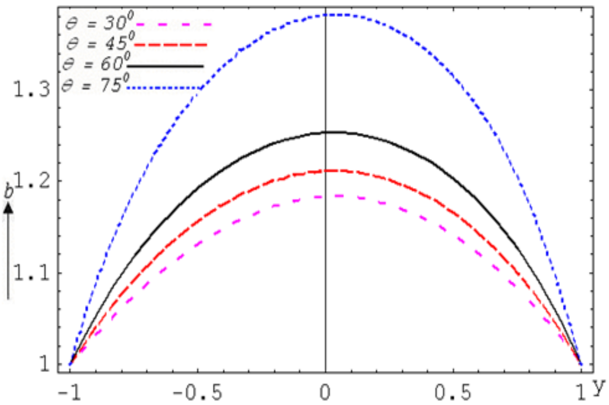

Figure 4: Magnetic field profiles for different θ
($n = 1.0$, $R_m = 0.2$, $R_e = 1.5$, $H_a = 7.0$, $G_r = 2.0$ and $P_r = 0.71$)

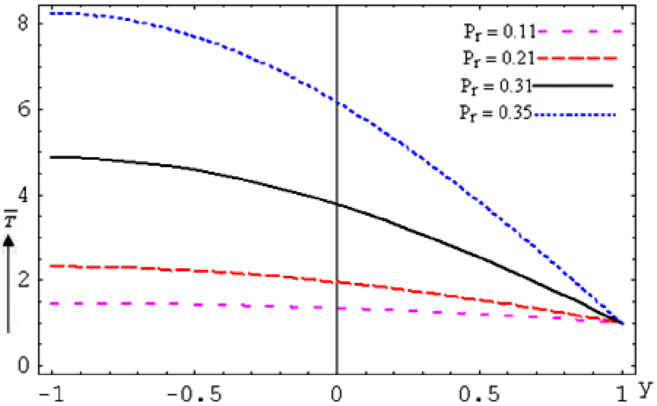

Figure 7: Temperature field profiles for different P_r
($n = 1.0$ and $R_e = 1.5$)