A Mathematical Model for the Biological Treatment of Wastewater

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***Abstract*—We investigate a model for the activated sludge process. The reactor configuration consists of a single aerated reactor attached to a settling unit. The biochemical processes occurring within the biological reactor is a slimmed-down version of the activated model number 1. The resulting model contains five state variables. We investigate how the stable steady-state solutions of the model depend upon the residence time. From the steady-state solutions we investigate three important process variables: the chemical oxygen demand, the mixed liquor suspended solids, and the total suspended solids.**

***Index Terms*—activated sludge, modelling, wastewater treatment.**

I. INTRODUCTION

THE activated sludge process is the most widely used biological wastewater treatment method for both domestic and industrial wastewaters [1]. The process revolves around the use of microorganisms, which feed upon the organic pollutants contained in the wastewater to produce more microorganisms. This self-replication ensures that the process can, in principle, contain indefinitely.

At its simplest the reactor configuration consists of a single aerated biological bioreactor, through which wastewater flows, connected to a settling unit. The settling unit 'captures' solids through sedimentation, removing them from the effluent stream and recycling them into the biological reactor at an enhanced concentration. This process is important because the microorganisms are contained within the solids. Hence recycling of the solids maintains a higher concentration of microorganisms within the biological reactor than would otherwise be the case. A schematic of the reactor configuration is shown in figure 1.

Mathematical modelling of the activated sludge process dates back to the late 1960s and early 1970s [2]–[4]. In these models it was assumed that there is one limiting substrate, one limiting microorganism and one rate-limiting biochemical reaction. Although biomass death is included in the model formulation, the decay products are not recycled into the substrate pool. The mathematical model consists of two differential equations, one for the concentration of the microorganisms and one for the concentration of the substrate. This is the 'basic model' for the activated sludge process.

Although highly simplified, models based upon this approach are still widely used to model the processing of industrial wastewaters. The main advantage of the 'basic model' is its simplicity. As it contains few parameters it is comparatively straight-forward to calibrate the model against experimental data. Once calibrated such models can be used as the basis for control schemes or for process design.

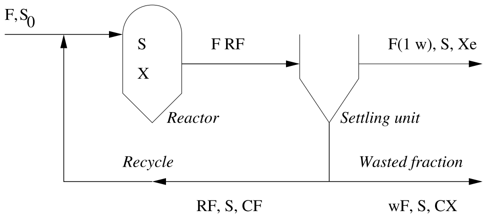

Fig. 1. A bioreactor with recycle and separate wasting of biomass. Figure adapted from [10].

Over the last forty years models containing more detailed biochemistry have been devised for domestic wasters. The most widely known is the sequence of activated sludge models developed by the IWA [5]–[7].

The activated sludge model number 1 (ASM1) [8] is an internationally accepted standard for activated sludge modeling. It describes nitrogen and chemical oxygen demand within suspended-growth treatment processes. The model provides a good description of the activated sludge process provided that the wastewater has been characterised in detail and is of domestic or municipal origin.

The ASM1 model includes eight fundamental biochemical processes. Consequently, it can be used to study in depth the biological processes occurring in both lab scale and full scale waste water treatment plants. The complexity of the ASM1 model is also its primary weakness. In order to use the model the polluted wastewater must be characterised in terms of the thirteen state variables. Furthermore, the model contains twenty parameters, the values of which must be obtained by carefully calibrating the model against experiment data. This is a highly non-trivial task.

In this paper we investigate a simplification of the ASM-1 model which extends the basic model from two to five state variables. The model considers the removal of organic pollutants from a wastewater, processes associated with nitrogen demand have been removed. This presents a more detailed picture of the activated sludge process than the basic model, but remains sufficiently small that it is amenable to computational mathematical analysis. Although this approach was suggested just after the publication of ASM1 [9] the resulting model has not been investigated.

II. THE MODEL

A. Model assumptions

The model contains two biochemical processes. In the first slowly biodegradable particulate substrate (X_S) is hydrolysed by heterotrophic biomass (X_H) to produce soluble substrate (S_S), reaction (8). In the second process heterotrophic

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biomass grow through the consumption of soluble substrate, reaction (6). This reaction is subject to an oxygen 'switch', equation (7). For moderate values of the dissolved oxygen concentration the switch is 'on' and it has minimal effect on the overall reaction rate ($M_{8h} \approx 1$ for $S_O \gg K_{O,H}$). The switch is turned 'off' in the limit that the dissolved oxygen concentration approaches zero. As the growth of biomass requires both a carbon source and an oxygen source it is associated with a decrease in the concentration of both soluble substrate and dissolved oxygen.

The final process modelled is the decay of biomass. This converts heterotrophic biomass into a combination of non-biodegradable particulates (X_P) and slowly biodegradable particulate substrate (X_S). The latter is subsequently hydrolysed to produce soluble substrate, the growth substance for heterotrophic biomass. This cycling of substrate is known as the death-regeneration concept.

Many models for the activated sludge process assume that the dissolved oxygen concentration is constant [11]. This can be justified if the dissolved oxygen concentration is kept over $3\text{mg}\,\text{l}^{-1}$. Under these conditions the dissolved oxygen concentration is not rate limiting [12]. We do not make this assumption. Rather, we assume that the operation of the aeration process is fixed. Consequently, in our model the dissolved oxygen remains a state variable.

Figure 1 shows a schematic of the reactor configuration. The simplest model for a settling unit characterises its behaviour by the value of three parameters: a recycle ratio (R), a wastage fraction (w) and a concentrating factor ($\mathcal{C}$). As the settling unit concentrates particulates we have $\mathcal{C} > 1$. We assume perfect operation of the settling unit. This means that there are no particulates in the effluent stream, i.e. in figure 1 we have $X_e = 0$. A mass balance around the settling unit shows that the concentration factor is given by

$$\mathcal{C} = \frac{1+R}{R+w}.$$

B. Model equations

The rate of change of soluble substrate

$$V\frac{\mathrm{d}S_S}{\mathrm{d}t} = F\left(S_{S,\text{in}} - S_S\right) - \frac{\mu_{\max,H}}{Y_H}\cdot M_2 \cdot M_{8h}\cdot X_{B,H}V + k_h \cdot k_{\text{sat}} \cdot X_{B,H}V. \tag{1}$$

The rate of change of heterotrophic biomass

$$V\frac{\mathrm{d}X_{B,H}}{\mathrm{d}t} = -FX_{B,H} + R\left(\mathcal{C}-1\right)FX_{B,H} - b_H X_{B,H}V + \mu_{\max,H}\cdot M_2 \cdot M_{8h}\cdot X_{B,H}V \tag{2}$$

The rate of change of slowly biodegradable particulate substrate

$$V\frac{\mathrm{d}X_S}{\mathrm{d}t} = F\left(X_{S,\text{in}} - X_S\right) + R\left(\mathcal{C}-1\right)FX_S + \left(1-f_p\right)b_H X_{B,H}V - k_h \cdot k_{\text{sat}} \cdot X_{B,H}V. \tag{3}$$

The rate of chance of soluble oxygen

$$V\frac{\mathrm{d}S_O}{\mathrm{d}t} = F\left(S_{O,\text{in}} - S_O\right) + VK_{L,A}\left(S_{O,\max} - S_O\right) - \frac{\left(1-Y_H\right)}{Y_H}\cdot \mu_{\max,H}\cdot M_2 \cdot M_{8h}\cdot X_{B,H}V. \tag{4}$$

The rate of change of non-biodegradable particulate products

$$V\frac{\mathrm{d}X_P}{\mathrm{d}t} = -FX_P + R\left(\mathcal{C}-1\right)FX_P + f_p b_H X_{B,H}V. \tag{5}$$

The reaction rates are

$$M_2 = \frac{S_S}{K_S + S_S}, \tag{6}$$

$$M_{8h} = \frac{S_O}{K_{O,H} + S_O}, \tag{7}$$

$$k_{\text{sat}} = \frac{X_S}{K_X X_{B,H} + X_S}. \tag{8}$$

The residence time is defined by

$$\tau = \frac{V}{F}. \tag{9}$$

The effective recycle parameter is defined by

$$R^* = R\left(\mathcal{C}-1\right) = \frac{1-w}{R+w}\cdot R. \tag{10}$$

The efficiency of a wastewater treatment plant can be characterised by various combinations of the state variables. We consider three commonly combinations: chemical oxygen demand (COD), mixed liquor volatile suspended solids (MLVSS) and total suspended solids (TSS). The chemical oxygen demand is used to characterise the overall organic content of a wastewater. The mixed liquor volatile suspended solids is a collective parameter defining the particulate organic matter in the reactor. The total suspended solids is used to characterise the amount of excess biological sludge.

The chemical oxygen demand in the influent:

$$\text{COD}_{\text{in}} = S_{S,\text{in}} + X_{S,\text{in}}. \tag{11}$$

The chemical oxygen demand in the reactor:

$$\text{COD} = S_S + X_{B,H} + X_S + X_P. \tag{12}$$

The chemical oxygen demand in the effluent:

$$\text{COD}_e = S_S. \tag{13}$$

The chemical oxygen demand in the wastage stream:

$$\text{COD}_w = S_s + \mathcal{C}\left(X_{B,H} + X_S + X_P\right). \tag{14}$$

A consequence of the assumption that the settling unit captures all particulates is that the chemical oxygen demand in the effluent stream is equal to the soluble substrate concentration inside the bioreactor.

The mixed Liquor Volatile Suspended Solids in the reactor:

$$\text{MLVSS} = X_H + X_P + X_S. \tag{15}$$

The total suspended solids in the reactor:

$$\text{TSS} = c_1 \left(X_\text{S} + X_\text{P} \right) + c_2 X_{\text{B,H}}. \tag{16}$$

Both the mixed liquor suspended solids and the total suspended solids in the effluent stream are zero because the settling unit is assumed to capture all particulates. In this paper we do not report process values in the waste stream.

III. RESULTS

We consider two scenarios. In section III-A we fix the effective recycle parameter (R^*) and vary the residence time (τ). In section III-B we fix the residence time and vary effective recycle parameter (R^*).

A. *Behaviour at constant effective recycle ratio*

By inspection of equations (1)–(5) it can be seen that there is a steady-state solution with no heterotrophic biomass present in the reactor, i.e. $X_{B,H} = 0$. This is known as the washout solution and it corresponds to process failure. The washout steady-state is given by

$$\begin{aligned} (S_S, X_{B,H}, X_S, S_0, X_p) &= (S_S^*, 0, X_S^*, S_O^*, 0)\,, \\ S^* &= S_{S,\text{in}}, \\ X_S^* &= \frac{X_{S,\text{in}}}{1 - R^*}, \\ S_O^* &= \frac{K_{L,A} S_{O,max} \tau + S_{O,\text{in}}}{K_{L,A} \tau + 1}. \end{aligned}$$

A linear stability analysis shows that this solution is stable when $0 \leq \tau \,(\text{day}) < 0.0547$.

Figure 2 shows the variation of the stable steady-state values for four of the dependent variables as a function of the residence time.

Figure 2a shows the concentration of soluble substrate. The behaviour in the post-washout region can be divided into two regions. In the first region there is a very rapid decrease in the soluble substrate concentration in response to small variations in the residence time. For residence times larger than approximately 0.2 (day) the soluble substrate concentration decreases at a much reduced rate.

European Union directive (91/271 'Urban wastewater') fixed the maximum chemical oxygen demand allowed in the effluent of small sized wastewater treatment plants to

$$\text{COD}^{\text{max}} = 125 \, \text{mg l}^{-1}. \tag{17}$$

Recall that as the settling unit is assumed to capture all particulates that the chemical oxygen demand in the effluent stream is equal to the soluble substrate concentration inside the reactor. Figure 2a shows that in the first of the two post-washout regions the soluble substrate inside the reactor is quickly reduced below the target value. For values of the residence time higher than 0.0608 (day) the chemical oxygen demand in the effluent stream is below the target value.

Figure 2b shows the concentration of dissolved oxygen. In the washout region there is no biomass, see figure 2c. Consequently, there is no biological demand for oxygen and the dissolved oxygen concentration increases to a maximum value at $\tau \approx 0.0547$ (day). In the post-washout region the dissolved oxygen concentration decreases rapidly to a minimum value at $\tau \approx 0.08885$ (day) before gradually increasing.

As noted earlier, many models assume that the dissolved oxygen concentration is controlled to be at $3.0 \, \text{mg} \, \text{O}_2 \, \text{L}^{-1}$. The dissolved oxygen concentration is below this value when either $0 \leq \tau \text{ (day)} \leq 0.0015$ or when $0.0603 \leq \tau \text{ (day)} \leq 0.1511$. The first region is not of interest since it corresponds to values at which the washout solution is stable. The value for the residence time at which the chemical oxygen demand in the effluent is equal to its target value is within the second region. Thus we have good removal of chemical oxygen demand even in a region in which oxygen limitations are important.

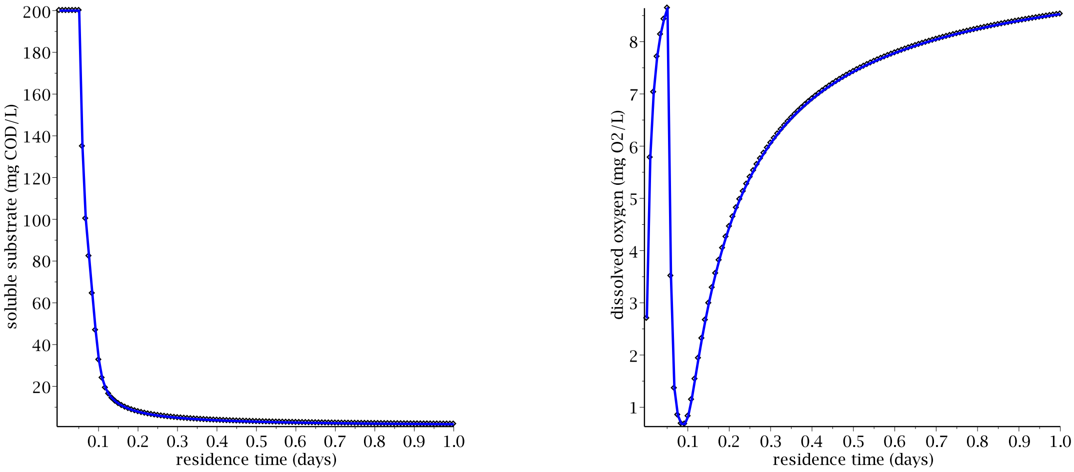
Figure 2c shows the concentration of active heterotrophic biomass. This value is equal to zero along the washout solution branch. As the residence time increases through the critical value, the concentration increases rapidly; reaching a maximum value at $\tau \approx 0.18334$ (day). Thereafter it gradually decreases towards zero. The large and dramatic increase in the concentration of heterotrophic biomass as the residence time increases through its value at the transcritical bifurcation corresponds to the dramatic decrease in both the concentration of soluble substrate and dissolved oxygen shown in figures 2a & 2b respectively.

Figure 2d shows the concentration of readily biodegradable particulate substrate. There are two sources of particulate substrate in the bioreactor. Firstly, it is contained in the feed. Secondly, it is released during the decay of heterotrophic biomass. The behaviour of the readily biodegradable particulate substrate concentration in the post-washout region can be divided into two regions. In the first there is a very rapid decrease in its concentration in response to small variations in the residence time. For residence times larger than approximately 0.2 (day) the readily biodegradable concentration decreases at a much reduced rate. This behaviour is analogous to that of the soluble substrate shown in figure 2a.

Figure 3 shows the variation of the stable steady-state values for the chemical oxygen demand and the total suspended solids inside the bioreactor as a function of the residence time. Note that the values of both process variables is nonzero along the washout solution branch ($0 \leq \tau \text{ (day)} < 0.0547$). As the residence time is increased through the transcritical bifurcation the values of both process variables increase to a maximum. For higher values of the residence time the values decrease towards limiting values.

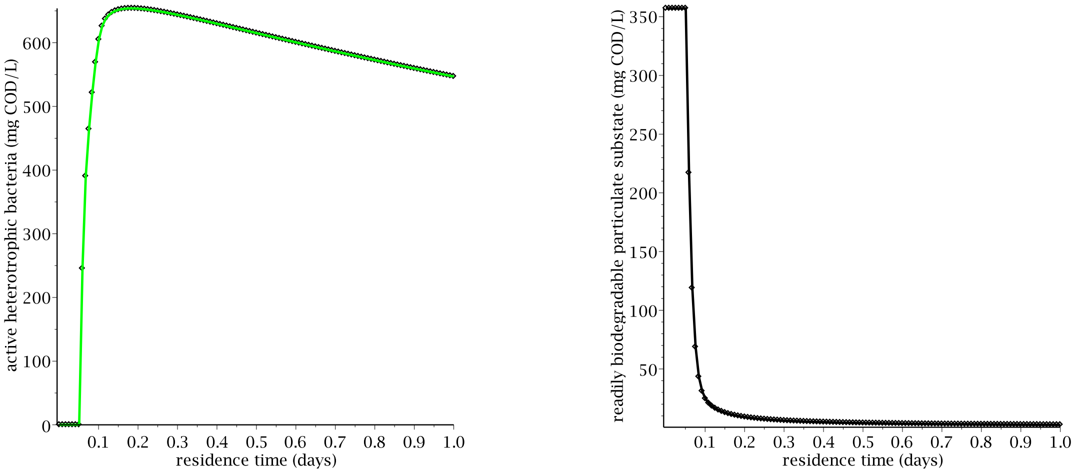
Figure 3a shows the chemical oxygen demand. The chemical oxygen demand inside the bioreactor is maximised when $\tau \approx 0.14452$ (day). Note that the chemical oxygen demand inside the bioreactor is much greater than that in the effluent stream, compare with figure 2a. This is a consequence of the action of the settling unit, which is assumed to remove all particulates from the effluent stream.

Figures 3b shows the total suspended solids. This is maximised when $\tau \approx 0.17591$ (day). The large value of this process variable is correlated to the large concentration of heterotrophic biomass, compare with figure 2c. The formation of large concentrations of solids within the bioreactor is one of the disadvantages of the activated sludge process,



(a) Soluble Substrate.

(b) Dissolved oxygen.



(c) Heterotrophic biomass.

(d) Readily biodegradable substrate.

Fig. 2. Variation of four steady-state variables with residence time. (The steady-state value for non-biodegradable particulate products is not shown).

since their disposal is associated with significant costs. An obvious solution to the problem, namely to increase the residence time at which the plant is operated is infeasible.

Figure 4 shows the variation of the mixed liquor suspended solids as a function of the residence time. This is maximised when $\tau \approx 0.17409\,(\mathrm{day})$. As with the total suspended solids, figure 3b, the large values of the mixed liquor suspended solids are correlated with large concentrations of heterotrophic biomass, compare with figure 2c.

B. Behaviour at constant residence time

In this section we fix the residence time, $\tau = 0.15\,(\mathrm{day})$, and vary the effective recycle ratio (R^*). We have process failure when $0 \leq R^* \leq 0.232$.

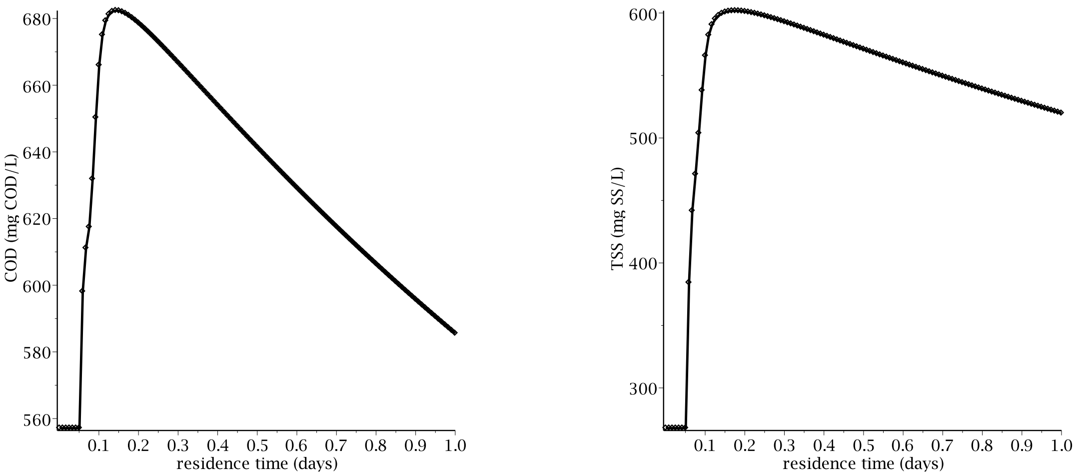
Figure 5a shows that as the value of the effective recycle ratio is increased through the critical value the soluble substrate concentration decreases. The soluble substrate concentration decreases through the value $125\,\mathrm{mg\ COD\,L^{-1}}$ when $R^* = 0.28$. Figure 5b shows that the mixed liquor suspended solids is an increasing function of the effective recycle ratio. As the value of the latter approaches one, the value of the former starts to quickly asymptote. These figures show that, for sufficiently high values of the effective recycle ratio, the marginal reductions in the chemical oxygen demand in the effluent stream obtained at high values of the effective recycle ratio are more than offset by the corresponding increase in the mixed liquor suspended solids within the reactor.

IV. CONCLUSION

In this paper we have provided an initial investigation of a model for the activated sludge process. The model is a ‘slimmed down’ version of the ASM1 containing five state variables — alternatively it is a ‘souped-up’ version of the ‘basic’ activated sludge model. We have investigated the behaviour of this model as a function of the residence time and the effective recycle ratio.

One of the main disadvantages of the activated sludge process is the production of excessive amounts of sludge. The expense for treating excess sludge can account for 50–60% of the total operating costs in a wastewater treatment plant [13].

In the future we intend using our ‘slimmed-down’ ASM1 as a test bed for investigating two mechanisms which have been investigated as means to reduce sludge formation: ozonation and predation. It is hoped that insights gained from the use of the ‘slimmed-down’ ASM1 will provide a stepping stone in the formulation and analysis of these approaches within the full ASM1.



(a) Chemical Oxygen Demand.

(b) Total Suspended Solids

Fig. 3. Variation of the chemical oxygen demand and the total suspended solids inside the bioreactor with residence time.

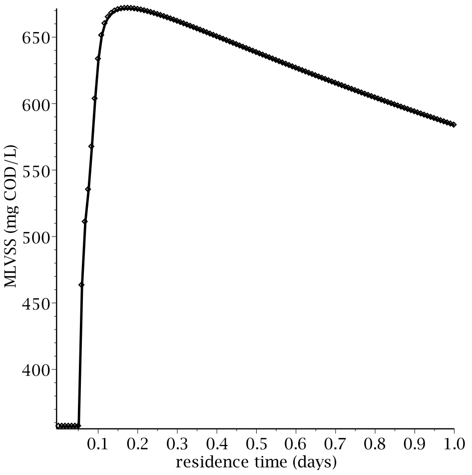


Fig. 4. Variation of the mixed liquor suspended solids inside the bioreactor with residence time.

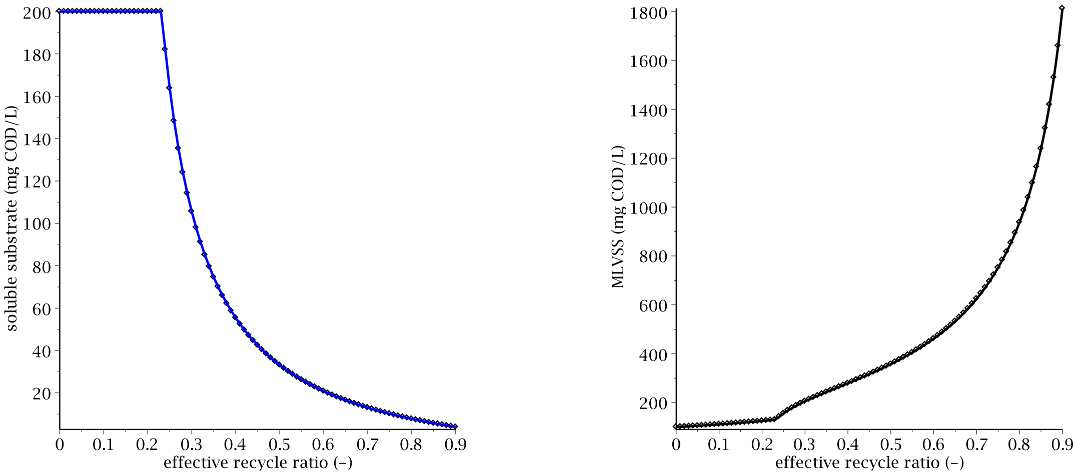
APPENDIX A
NOMENCLATURE

$\mathcal{C}$	Concentration factor.	–
COD	Chemical oxygen demand in the reactor.	mg COD L^{-1}
F	Flow rate through bioreactor.	L day^{-1}
$K_{L,A}$	Oxygen transfer coefficient.	day^{-1}
$K_{O,H}$	Oxygen half-saturation coefficient.	mg O$_2$L^{-1}
K_S	Monod constant for biomass.	g COD L^{-1}
K_X	Contois coefficient for hydrolysis of particulate biodegradable substrate.	–
M_2	Monod kinetics for readily biodegradable soluble substrate.	–
M_{8h}	Monod kinetics for the component S_0 with respect to biomass.	–
MLVSS	Mixed liquor suspended solids.	mg COD L^{-1}
R	Recycle ratio.	–
R^*	Effective recycle parameter.	–
S_O	Concentration of soluble oxygen.	mg O$_2$L^{-1}
$S_{O,\text{in}}$	Soluble oxygen concentration in the feed	mg O$_2$L^{-1}
$S_{O,\max}$	Maximum concentration of soluble oxygen.	mg O$_2$L^{-1}
S_S	Soluble substrate concentration.	mg COD L^{-1}
$S_{S,\text{in}}$	Concentration of soluble substrate in the feed.	mg COD L^{-1}
TSS	Total suspended solids.	g SS L^{-1}
V	Bioreactor volume.	L
$X_{B,H}$	Concentration of heterotrophic biomass.	mg COD L^{-1}
X_P	Concentration of particulate products arising from biomass decay.	mg COD L^{-1}
X_S	Concentration of slowly biodegradable particulates.	mg COD L^{-1}
$X_{S,\text{in}}$	Concentration of slowly biodegradable particulates in the feed.	mg COD L^{-1}
Y_H	Heterotrophic yield coefficient.	–
b_H	Heterotrophic decay coefficient.	day^{-1}
b_1	Recycle concentration factor.	–
c_1	Conversion factor from COD to TSS for X_S and X_S.	g SS (g COD)$^{-1}$
c_2	Conversion factor from COD to TSS for solution $X_\text{B,H}$.	g SS (g COD)$^{-1}$
f_p	The fraction of dead biomass converted to particulate products.	–
k_h	Maximum hydrolysis rate.	day^{-1}
k_sat	Saturation kinetics for hydrolysis.	–
t	Time.	day^{-1}
$\mu_{\max,H}$	Maximum specific growth rate for biomass.	day^{-1} –
τ	Residence time.	day

APPENDIX B
PARAMETER VALUES

The following are typical values for a domestic wastewater at neutral pH and 20^{o}C [14, Table 5].

$K_{O,H}$	0.2	mg O$_2$L^1	[15, Table 6]
K_S	20.0	mg COD L^{-1}	[15, Table 6]
K_X	0.03	g COD (g COD)$^{-1}$	[15, Table 6]



(a) Soluble substrate.

(b) Mixed Liquor Volatile Suspended Solids

Fig. 5. Variation of the soluble substrate concentration and the mixed liquor suspended solids inside the bioreactor with the effective recycle ratio. Note that the chemical oxygen demand in the effluent stream is equal to the soluble substrate concentration in the reactor. Parameter value: $\tau = 0.15$ (day).

Y_H	0.67	–	[15, Table 6]
b_H	0.22	day^{-1}	[15, Table 6]
f_p	0.08	–	[15, Table 6]
k_h	3.0	day^{-1}	[15, Table 6]
$\mu_{\max,H}$	6.0	day^{-1}	[15, Table 6]

The parameter values associated with dissolved oxygen are

$K_{L,A}$	96	day^{-1}	[16, page 855]
$S_{O,\text{in}}$	2.0	mg O$_2$L^1	
$S_{O,\max}$	10.0	mg O$_2$l^1	[16, page 856]

Conversion factors from units of COD to units of TSS are:

c_1	0.75	g SS (g COD)$^{-1}$	[17]
c_2	0.90	g SS (g COD)$^{-1}$	[17]

Typical values for the operation of the settling unit are $R = 0.4$ and $w = 0.1$.

The influent composition is given by

$S_{S,\text{in}}$	200	mg COD L^{-1}	[15, table 9]
$X_{S,\text{in}}$	100	mg COD L^{-1}	[15, table 9]

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