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Localization, Path integral and Supersymmetry

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***Abstract*—Equivariant geometry involves a group action on a manifold. This is the starting point to consider a super-geometry showing even and odd variables (bosons, fermions) The localization methods that provide source in the symplectic geometry (Duistermaat-Heckman formula), allow in certain cases to compute path integrals in an explicit way by using the concept of localization. Applications are important in topological field theory: They lead to the definition of new symplectics invariants.**

Keywords: Localization, Duistermaat-Heckman formula, Path integral.

I. Introduction

In this article, we discuss a fundamental problem, both in mathematics and physics. This is the problem of localization. Localization is a fundamental idea that makes a problem that is not countable a problem that it is. for example consider a map from a finite dimensional n vector space E to a finite dimensional m vector space F , the problem of finding the image of any vector of E by an application f in F is an uncountable problem because there is an infinite number of vectors in E thus an infinite number of possible images. Now if **we located** on the set of linear applications, to find the image of a vector of E, just know the image of n vectors: namely a basis of E, to know the image of any vector from the starting space. In other words, knowing a table of size nxm: the matrix of the linear map is enough to solve the problem. We will begin by giving some examples of the problem of localization in mathematics, mainly in symplectic geometry, and quantum field theory, before then we recall the approach of the Feynman path integral [1]. We then show how the introduction of fermionic variables can help to locate a problem and compute a path integral by using a localization principle. The main idea of the localization of the integrals comes from the oscillatory integrals, mainly the laplace method on the localization around the critical points of a Morse function.

II. Motivations, simple examples: Quantization problem of Feynman

The idea of localization has a lot of applications in quantum field theory (QFT) . Feynmann has shown that the quantization of a classical field theory led to the computation of a path integral. This integral consists of calculating all the possible trajectories from a point A to a point B, then integrate on this space.

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A. Toy model

: Let's start with a very simple problem: This problem is resolvable by a high school student: Consider a mouse moving over a grid consisting of n rows and m columns. His only possible actions are to advance or step up without ever going back or down. it is well known that counting all the possible trajectories consists of **counting the number of anagrams** consisting of n times the letter A and m times the letter M. in this case the sum of all possible trajectories is a finite number is the combinations of n among $n+m$: $N_T = \begin{pmatrix} n+m \\ n \end{pmatrix}$

B. Path integral

: In the case of path integral,the set of trajectory is in general infinite. Localization problems then make perfect sense. Recall that the path integral introduced by Feynman is given by:

$$K(x(t_i), x(t_f)) = \int_{x(t_i)\to x(t_f)} \mathcal{D}x(t) exp(\frac{iS(x(t))}{\hbar}) \quad (1)$$

In this formula, the measure D is poorly defined: relates to an infinite number of paths from an initial configuration to a final configuration. The fact of considering forced passages by certain points leads to the notion of correlation function . Physicists have postulated that knowledge of all correlation functions makes it possible to understand a quantum field theory. In the case of our toy model, it is very easy to determine all the correlation functions passing through a point, two or more points of the grid).

III. Equivariant cohomology, and localization

Berline and Vergne [2] define equivariant cohomology. This leads to a very useful localization formula: The equivariant geometry localization formula. In the context of symplectic geometry, this formula becomes the formula of Duistermaat-Heckman. Witten was able to give a concrete application of this formula in the framework of supersymmetric field theory: We can then give a dictionary that allows us to translate the vocabulary of equivariant geometry into that of supersymmetric field theories.

A. Equivariant cohomology

Briefly, equivariant geometry consists in making a group G act on a variety. In the same way that the cohomology of de Rham is defined on M, equivariant cohomology can be defined on M_G, if it is necessary to define a differential d_g in $\Omega^{\bullet}_G$ we set:

$$(d_g\alpha)(X) := d(\alpha(X)) - i_X\alpha(X) \quad (2)$$

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The symbol g denotes the Lie algebra associated with the G group (assumed to be a Lie group) We can see link between this differential and the cohomology operator BRST Q physicists.

B. Localization formula in equivariant geometry

Let G a compact Lie group, with Lie algebra g, acting on an oriented compact manifold M of even dimension $2n$. Let α be an equivariantly closed form M. Let $X \in g$ and assume X_M has only isolated 0 then:

$$\int_M \alpha(X) = (-2\pi)^l \sum_{p \in M_0(X)} \frac{\alpha(X)_{[0]}(p)}{det^{\frac{1}{2}}(L_p)} \tag{3}$$

where $M_0(X)$ is the set of $p \in M$ with $X(p) = 0$, $\alpha(X)_{[0]}(p)$ designe f function (0- form) at $p \in M$ and L_p is the transformation matrix of the action $\mathcal{L}_X\xi$ evaluated at $p \in M_0(X)$

C. Localization formula in symplectic geometry

A particularly interesting case of application of the above formula is that of symplectic geometry: given symplectic manifold (M, ω) , hamiltonian function H, and hamiltonian vector field X_H, given G acting in (M, ω), and the moment map $\mu : M \to g^*$, we have The integral of Duistermaat-Heckman [3] theorem:

Theorem

Let M be a compact symplectic manifold of dimension $n = 2l$ and G a compact Lie group acting over it. Let $X \in g$ and let $X_M \in X(M)$ be the hamiltonian vector field generated by the moment map μ, that holds the identity $i_{X_M}\Omega = d\mu(X)$ with Ω be a symplectic 2-form over M. If $M_0(X)$ is the finite set of point on which X vanish, then

$$\int_M e^{i\mu(X)} d\beta = i^l \sum_{p \in M_0(X)} \frac{(e^{i\mu(X)})_{[0](p)}}{det^{\frac{1}{2}}((L_p)(X))} \tag{4}$$

where the square root sign is chosen by canonical orientation on T_pM, we denote $d\beta := (e^{\frac{\Omega}{2\pi}})_{[n]} = \frac{\Omega^l}{(2\pi)^l l!}$

Application

Let $S1S2$ an action given by the rotation with respect to the vertical axis. Let us verify the hypothesis in the theorem of Duistermaat-Heckman for the action before:

1) $(S^2, sin\phi d\phi \wedge d\theta)$ is a symplectic manifold with $dim(S^2) = 2$.
2) S^1 is a compact Lie group.
3) The moment map μ is given by the height function, $\mu = cos\phi$.
4) X The vector field on the sphere is $X = \frac{\partial}{\partial\theta} = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$
5) X The finite set of points where X vanishes, i.e, the fixed points of the action are $N = (0, 0, 1)$ and $S = (0, 0, -1)$.

We can apply the theorem of Duistermaat-Heckman!

1) The equivariant symplectic differential form is $\Omega_e q = \mu(X) + \omega_{S^2} = tcos\phi + sin\phi d\phi \wedge d\theta$.
2) The Lie algebra is given by $g = R = t$. The vector field that this induces is $X_M = tX = t(-y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y})$
3) The transformation matrix A corresponding to the Lie bracket is given by $\mathcal{L}(X_M) : X(S^2) \to X(S^2)$, $\xi \mapsto [X_M, \xi]$: $A = \begin{pmatrix} 0 & t \\ -t & 0 \end{pmatrix}$
4) $det^{\frac{1}{2}}(A) = \pm t$,then the sign of the square root in the formula of Duistermaat-Heckman depends on the orientation coming from T_pS^1: At the point $(0, 0, 1)$ the sign is negative, and positive at $(0, 0, -1)$.
5) the Liouville form is $d\beta := \frac{\Omega^l}{(2\pi)^l l!} = \frac{1}{2\pi} sin\phi \wedge d\theta$

By the theorem of Duistermaat-Heckman we obtain:

$$\int_{S^2} e^{itcos(\phi)} 12\pi sin\phi d\phi \wedge d\theta = i(\frac{e^{itcos(0)}}{-t} + \frac{e^{itcos(\pi)}}{t})$$

multiplying by 2π to both sides of the equality and expressing the exponential terms as a trigonometric function, it becomes to:

$$\int_{S^2} e^{itcos(\phi)} \frac{1}{2\pi} sin\phi d\phi \wedge d\theta = 4\pi \frac{sin(t)}{t} \tag{5}$$

the applications of the concept of localization goes beyond the framework of mathematics. We will now apply this concept to physics and particularly to the topological fields theories.

IV. Become to physic: path integral, supersymmetry

A. Classical fields

The concept of field is fundamental in physics. A field φ is a function of a world sheet (source space) into a target space, M (space physics) with a sufficient number of dimensions. So given a "package" (Σ, M, φ) and a classical action: S where: Σ is the source space, often a manifold: for The classical mechanic of the point is the time axis (world line), for the conformal field theories like strings theories: a Riemann surface ...
The Lagrangian density is a function on one or more fields and its first derivatives:

$$\mathcal{L} = \mathcal{L}(\varphi_1, \varphi_2, ..., \partial_\mu\varphi_1, \partial_\mu\varphi_2 ...).$$

The classical action is the integral of the classical Lagrangian density on the parameter space $S = \int \mathcal{L} d^{n+1}x$
Principle of least action: The minimization of the action ($\delta S = 0$, leads to each field noted just φ to the Euler-Lagrange equation gives the equations of motion of the particle $\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\mu(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}) = 0$

Example of classical fields

The free particle: For a free particle, the field is simply the parameterized curve that describes the trajectory of the particle in free space :$x(t)$. In this case, you can take to Lagrangian density
$\mathcal{L} = \mathcal{L}(x(t), \dot{x}(t)) = \frac{1}{2}m\dot{x}^2$, Euler Lagrange equation is:
$\frac{\partial\mathcal{L}}{\partial x} - \partial_t(\frac{\partial\mathcal{L}}{\partial(\dot{x})}) = -\partial_t(\frac{\partial\mathcal{L}}{\partial(\dot{x})}) = 0$ solution is the uniform motion.

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The free string: For the free string, the field is simply the function that describes the trajectory of the string in the target space: $X(\tau,\sigma)$. Its Lagrangian density is contained in the Nambu-Goto action:
$S = -T\int_{\tau_1}^{\tau_1} d\tau \int_0^l d\sigma \sqrt{\dot{X}^2 X'^2 - (\dot{X}X')^2}$ with $\dot{X}^2 = \dot{X}^\mu \dot{X}^\nu \eta_{\mu\nu}, \dot{X}X' = \dot{X}^\mu X'^\nu \eta_{\mu\nu}$ Euler Lagrange equation is then: $\partial_\tau \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} + \partial_\sigma \frac{\partial \mathcal{L}}{\partial X'^\mu} = 0$ and the solution is the equation of vibrating strings.

B. Noether Symmetries

Symmetry of the action: the role of symmetry in physics is essential. We want, for example, such action invariant through a transformation like translation, rotation ...: if $\varphi \to \varphi + \delta\varphi$ alors $S \to S + \delta S$
Noether's theorem: through any symmetry, the action is the same: $\delta S = 0$

Example translation: $x \to x + \epsilon$ is taken up the free particle, ϵ small, independent of time,
$\delta S = \int (m\dot{x}\dot{\epsilon})dt = \epsilon m\dot{x}|_{t_0}^{t_1} - \int (m\ddot{x})\epsilon dt$
and as $\ddot{x} = 0$ we get: The symmetry by translation is equivalent to the conservation of momentum $p = m\dot{x}$

Notation: In physics, symmetry $x \to x + \epsilon$ is denoted $\delta x = \epsilon$.

C. Quantum fields, QFT

Path integral:

Uncertainty on the position or momentum in quantum mechanic led to replace the classical solution (least action) by the partition function or the set of all possible solutions: It is the ***path integral*** $\mathcal{Z} = \int_{\Sigma \to M} e^{-S(\varphi)} \mathcal{D}\varphi$:

Correlation Functions:

Similarly, one can calculate correlation functions, or functions with n points.
$< \varphi_1(x_1), ..., \varphi_n(x_n) > =$
$\int_{\Sigma \to M} \varphi_1(x_1)...\varphi_n(x_n) e^{-S(\varphi)} \mathcal{D}\varphi$

We can apply this machinery to the supersymmetric sigma model and define a new quantum field theory: the topological field theory TFT .The program developed by Witten is to calculate the correlation functions, by replacing each value by a cohomology class called $BRST$ cohomology. That requires, introducing fermionic variables ***invariant*** under this ***generalized Noether symmetries***. These tools are ***supersymmetry***.

D. Supersymmetry

We can define a supersymmetric field theory $\Sigma \to M$ by adding fermionic variables, that is to say sections of some vector bundle E on Σ. A good image of a fermionic field is $\psi(x) = \Sigma f_i(x) dx_i$, a 1-form equipped with a wedge-product . We have the theorem:
Localization theorem: the path integral is localized around field configurations where fermionic variables stay invariant under supersymmetric transformations. Supersymmetric transformation is infinitesimal transformation of the action, which transforms bosons into fermions and vice versa.

Calculus supersymmetric

We can define a supersymmetric Calculus:

Algebraic Computation: Let ψ_1, ψ_2 two fermions $\psi_1\psi_2 = -\psi_2\psi_1$ we deduce $\psi\psi = 0$ Let a bosonic variable $Xboson$,$\psi X = X\psi$
Calculus: $\int (a + b\psi)d\psi = b$, $\int \psi d\psi = 1$, $\int \psi_1\psi_2...\psi_n)d\psi_1 d\psi_2...d\psi_n = 1$, $\int d\psi = 0$
Change of variables : We have: $\int \widetilde{\psi} d\widetilde{\psi} = \int \psi d\psi = 1$

V. Localization in Physic

A. Example 1 zero-dimensional supersymmetry

A "Toy" model is to make space for starting $\Sigma = \{P\}$ and target $M = R$ the real line. In this context, a field is simply the variable x, the path integral is just $\mathcal{Z} = \int_M e^{-S(x)} dx$
A supersymmetric action is given by:
$S(x, \psi_1, \psi_2) = \frac{h'(x)^2}{2} - h"(x)\psi_1\psi_2$.
hence the partition function:
$\mathcal{Z} = \int e^{\frac{-h'(x)^2}{2} + h"(x)\psi_1,\psi_2} dx d\psi_1 d\psi_2$
by developing in power series fermionic part, we get:
$\mathcal{Z} = \int e^{\frac{-h'(x)^2}{2}} (1 + h"(x)\psi_1\psi_2) dx d\psi_1 d\psi_2$, but $\int d\psi = 0$,
hence the first integral is zero, then:
$\mathcal{Z} = \int_M h"(x) e^{\frac{-h'(x)^2}{2}} dx \int \psi_1 d\psi_1 \int \psi_2 d\psi_2$,
and as $\int \psi d\psi = 1$ (fermionic integration) we get:
$\mathcal{Z} = \int_M h"(x) e^{\frac{-h'(x)^2}{2}} dx$

Supersymmetric transformations

For the example above, we can define supersymmetric transformations that respect this action.
$\delta x = \epsilon_1\psi_1 + \epsilon_2\psi_2$
$\delta\psi_1 = h'(x)\epsilon_2$
$\delta\psi_2 = -h'(x)\epsilon_1$

We show that $\delta S = 0$, the fermionic variables are invariant for supersymmetric transformation iff $h'(x) = 0$. If $h'(x) \neq 0$, the change of variables $(x, \psi_1, \psi_2) \to (x - \frac{\psi_1\psi_2}{h'(x)}, \psi_1, \psi_2)$ shows that the partition function is zero outside the critical points. By expanding to second order near the critical point x_c
$(h(x) = h(x_c) + \frac{h"(x_c)}{2}(x - x_c)^2)$:
$\mathcal{Z} = \int_M h"(x) e^{\frac{-h'(x)^2}{2}} dx$
$\mathcal{Z} = \sum_{h'(x_c)=0} h"(x_c) \int_M exp(-\frac{(h"(x_c)(x-x_c))^2}{2}) dx$,
with change of variables $y = |h"(x_c)|(x - x_c)$:
$\mathcal{Z} = \sum_{h'(x_c)=0} \sqrt{\pi} \frac{h"(x_c)}{|h"(x_c)|}$

Abstract

Supersymmetry: We just define an action for a supersymmetric field theory of dimension 0 by adding fermions, supersymmetry variables.
Invariance: This action is invariant under supersymmetric transformations.
Location: The associated path integral is localized on the fields for which the fermions are invariant under supersymmetry.
Towards a generalization: This suggests defining an operator that vanishes on the fermionic fields. A fermionic field is associated to a differential form, there is an idea of cohomology below.

B. Exemple two: Supersymmetric quantum mechanic (one dimensional TQFT

1) Now we study a supersymmetric field theory in ***one dimension***. This is the model of ***supersymmetric quantum mechanics*** which has allowed Witten [4] to give a new proof of the ***index theorem***[5], [6]. We considÃ¨re the lagrangian:
$L = \frac{\dot{x}^2}{2} - \frac{h'(x)^2}{2} + i(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi) - h"(x)\bar{\psi}\psi$.
$\psi = \psi_1 + i\psi_2$
$\bar{\psi} = \psi_1 - i\psi_2$
2) let $\pi = \frac{\partial L}{\partial \dot{\psi}} = i\bar{\psi}$, $p = \frac{\partial L}{\partial \dot{x}} = \dot{x}$ the conjugate moments
3) Let ***supersymmetric*** relations
$\delta_\epsilon x = \epsilon\bar{\psi} - \bar{\epsilon}\psi$
$\delta_\epsilon \psi = \epsilon(i\dot{x} + h'(x))$ $\epsilon = \epsilon_1 + i\epsilon_2$
$\delta_\epsilon \bar{\psi} = \epsilon(i\dot{x} + h'(x))$
4) <4-> We can show :
$\delta_\epsilon S = \int \delta L dt = \int \frac{d}{dt} L dt = 0$

Localization of supersymmetric quantum mechanic

1) The two operators of supersymmetry, are associated ***supercharge*** Q, $\bar{Q}$ with $Q^2 = \bar{Q}^2 = 0$ and we deduce an elliptic complex:
$\mathcal{H}_F \xrightarrow{Q,\bar{Q}} \mathcal{H}_B \xrightarrow{Q,\bar{Q}} \mathcal{H}_F \xrightarrow{Q,\bar{Q}} ....$
2) ***In hamiltonian*** formalism, $\{Q, \bar{Q}\} = 2H$
3) SQM compactified on S^1 give:
$Tr(-1)^F e^{-\beta H} = dim\mathcal{H}^B_{(0)} - dim\mathcal{H}^F_{(0)}$ with F ***fermion number***.
4) The supertrace giving the index, expressed by : $Tr(-1)^F e^{-\beta H} = \int_{periodicBd} \mathcal{D}X\mathcal{D}\psi\mathcal{D}\overline{\psi}e^{-S}$
5) $\frac{\partial}{\partial\beta} Tr(-1)^F e^{-\beta H} = -\int_{periodicBd} \mathcal{D}X\mathcal{D}\psi\mathcal{D}\overline{\psi}He^{-S}=0$

Limite: From 1-dim TQFT to 0-dim TQFT

The fundamental result is that ***only time-independent contribute***: that reduce calculation to 0-dim TFT:
$\mathcal{Z} = Tr(-1)^F e^{-\beta H} = \sum_{h'(x_c)=0} \sqrt{\pi} \frac{h"(x_c)}{|h"(x_c)|}$

VI. Example III: A model of Witten "A side of the mirror"

L, the supersymmetric lagrangian of a super-string is given by:
$L = 2t \int_\Sigma (\frac{1}{2} g_{IJ} \partial_z \phi^I \partial_{\bar{z}} \phi^J) d^2 z$
$+ 2t \int_\Sigma (i\psi^{\bar{i}}_z D_{\bar{z}} \chi^i g_{\bar{i}i} + i\psi^i_{\bar{z}} D_z \chi^{\bar{i}} g_{\bar{i}i} - R_{i\bar{i}j\bar{j}} \psi^i_{\bar{z}} \psi^{\bar{i}}_z \chi^j \chi^{\bar{j}}) d^2 z$
The beginning of integral is the bosonic part of the action, the last , the fermionic part: fields are sections of bundles on Σ:

Fermionic part
•$\chi(z)$ a section $\mathcal{C}^\infty$ de $f^*TX \otimes C$
•$\psi_z(z)$ a section $\mathcal{C}^\infty$ de $(T^{10}\Sigma)^* \otimes f^*T^{01}X$
•$\psi_{\overline{z}}$, a section $\mathcal{C}^\infty$ de $(T^{01}\Sigma)^* \otimes T^{10}X$

Supersymmetric transformation preserving action

$\delta x^I = \eta\chi^I$ $\qquad \delta\chi^I = 0$
$\delta\psi^i_{\bar{z}} = \eta\partial_{\bar{z}}\phi_i$ $\qquad \delta\psi^{\bar{i}}_z = \eta\partial_z\overline{\phi_i}$

• If $\delta\psi^i_{\bar{z}} = \delta\psi^{\bar{i}}_z = 0$, we recognize the conditions of Cauchy-Riemann!: The instantons of this model are curves "minimum energy" according to Gromov: holomorphic curves [7].

A. BRST Cohomology

At previous fermionic transformations one associates an operator Q (for charge), the terminology come from electromagnetism: charge is the integration of a "current". Mathematically, the operator Q has the properties of ordinary differential form (they will have an isomorphism between $BRST$ cohomology with that of De Rham:
we give now the main properties of this operator

Properties of the operator
• $Q(x^I) = \chi^I$ $Q(\chi^I) = 0$
• Q is a linear operator.
• $Q(fg) = Q(f)g + fQ(g)$: Q is a derivation.

BRST cohomology
• We note that $Q^2 = 0$
• $H^p_{BRST} = \frac{KerQ:\mathcal{H}_p \rightarrow \mathcal{H}_{p+1}}{ImQ:\mathcal{H}_{p-1} \rightarrow \mathcal{H}_p}$ is the p- th cohomology group BRST
endsubsection

B. Correlation functions BRST

In correlation functions fields are replaced by their cohomology classes [4], so they are defined modulo an exact term by:

Correlation Functions
Correlation functions of topological field theory will be given by:
$< [\Phi_1(x_1)], ..., [\Phi_n(x_n)] >= \int_{\Sigma\rightarrow M} \Phi_1(x_1)...\Phi_n(x_n) e^{-S} \mathcal{D}x\mathcal{D}g\mathcal{D}\chi\mathcal{D}\psi$
they do not depend on the selected points on the Riemann surface.

Correlations functions of side A of the mirror
• Let $\omega_1, ...\omega_n$ forms on M,
$< [\omega_1], ..., [\omega_n] >= \int_{\Sigma\rightarrow M} \omega_1...\omega_n e^{-(S_B(f)+S_F(f))} \mathcal{D}x\mathcal{D}g\mathcal{D}\chi\mathcal{D}\psi$
• For the theorem location: the path intgral is localized around holomorphic curves $\tilde{f}$:
$< [\omega_1], ..., [\omega_n] >= \int_{\Sigma\rightarrow M} \omega_1...\omega_n e^{-(S_B(\tilde{f}))} \mathcal{D}x\mathcal{D}g\mathcal{D}\chi\mathcal{D}\psi$
• $e^{-(S_B(\tilde{f}))} = e^{-\int_\Sigma \tilde{f}^*\omega}$ is a topological invariant gives the "degree" of application $\tilde{f}$.

C. Relationship with enumerative geometry

The path integral above can be rewritten:
$< [\omega_1], ..., [\omega_n] >= \sum_{\beta\in H_2(M,Z)} e^{-\int_\Sigma \tilde{f}^*\omega} \int_{\tilde{f}(\Sigma)\in\beta} \omega_1...\omega_n \mathcal{D}x\mathcal{D}g\mathcal{D}\chi\mathcal{D}\psi$
Here $\beta \in H_2(X, Z)$ is a cohomology class, "specifically" the degree of $\tilde{f}$.

Counting curves
we can hope the integral:
$\int_{f(\Sigma)\in\beta} \omega_1...\omega_n \mathcal{D}x\mathcal{D}g\mathcal{D}\chi\mathcal{D}\psi$
taken on a moduli space $\mathcal{M}$ to define properly, can provide an integer. This will be the case if the dimension of this moduli space is related to the number of fields $[\omega_i]$

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Gromov Witten invariants These integrals, which give integers in good cases are Gromov Witten invariants [8] [9] [10].Their knowledge provides a means of calculating correlation functions from a topological viewpoint and hope to understand better the physics!

VII. Conclusion

Localization methods are crucial in mathematical physics. As we have seen, it allow t possible to make certain quantities calculable. It has brought back to the taste of the day some of the algebraic geometry methods as enumerative geometry. Its application to mathematical physics leads to the definition of moduli spaces and, in the best case, to instantons counting. This makes it possible to calculate certain correlation functions, resulting from a path integral. These methods have allowed a better understanding of quantum field theories in physics, they now connect topology, geometry and physic, to the concept of supersymmetry correctly defined from a mathematical point of view.

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