Proceedings of the World Congress on Engineering 2019
WCE 2019, July 3-5, 2019, London, U.K.

A Mathematical Model for the Table Tennis Ma Lin Ghost Serve

Hameez Mohammed, Rajesh Lakhan, and Donna M. G. Comissiong

***Abstract*—In the sport of table tennis, the overall success of any given player is significantly correlated to the points gained on serves. Perfection of the serve is therefore an essential component of the training regime of all professional players. The Ma Lin ghost serve is a particular type of backspin short serve which is quite advantageous. In this paper, we propose a simple two-dimensional mathematical model to describe the behaviour of the ball during this short serve from initial toss to the third board bounce. For this purpose, we subdivide the entire serve into eight phases: four of which were points of contact between the ball and the racket or board, and four airborne trajectories. The first airborne phase is the toss which leads to the backspin serve which is the first contact phase. We account for the Magnus Lift generated by the spin throughout the next three airborne trajectories, and the effect of air resistance. We also consider velocity and rotational changes due to the respective coefficients of restitution and surface grip on the spinning ball as it bounces on both sides of the board to fulfill the requirements of a legal short serve and develop into a ghost serve.**

***Index Terms*—Ma Lin, backspin serve, table tennis, mathematical model.**

I. INTRODUCTION

THE sport of table tennis has become increasingly competitive in recent years. While undergoing intense training regimes to hone their reflexes, professional players seek to expand their arsenal of attacking shots while simultaneously boosting their defensive plays. The psyche of a given competitor fluctuates according to the evolution of the match or the response of a given opponent. Individual levels of dexterity and the tendency or preference for various strokes contribute towards making every table tennis player unique. The success rate of the tactics employed by a player is dependent on the efficient implementation of different strokes with subtle variations in incident racket-ball angles, velocities and points of contact.

The overall performance of a player is significantly correlated to the points gained from serves [1]. The serve is initiated when the ball is tossed vertically upwards from rest in the open palm of the server's hand (without imparting spin). The ball rises at least 16cm and then falls without making contact with anything, before being struck by the racket for the serve. This neutral beginning quickly morphs into a battle of wit, skill and technique. The initial strategy of attack – aimed at immediately or ultimately securing the point – should therefore commence at this stage. Our aim is to develop a mathematical model for a particular type of serve that, when executed well, would severely hamper an aggressive return by the receiver. For a given service to be legal, the ball must bounce once on the server's court and subsequently at least once on the opponent's side. A serve is said to be short when a legal serve then bounces for a third time on the board.

The backspin serve attributed to the Olympic and four-time World champion Ma Lin is a short service that is commonly referred to as the Ghost Serve. It is produced by imparting a heavy (high revolution) backspin imparted on the ball with minimal translational velocity. This allows the ball to have a much lower height when clearing the net in comparison to a serve with no backspin and to bounce twice on the receiver's side of the table unless it is returned in time. When enough backspin is imparted, after bouncing for the first time on the opponent's end, the ball may reverse direction and spin towards the net, away from the opponent. This complex behaviour of the ball severely restricts the choice of possible returns from the opponent, which is to the server's advantage.

Our two-dimensional mathematical model for the Ma Lin ghost serve utilizes the well-known kinematic equations of motion. We also incorporate linear and angular momentum conservation principles. In what follows, we present a step by step trajectory analysis for a given incident racket velocity and angle. We consider the effect of quadratic backspin, air resistance (drag) and Magnus lift on the ball. We also incorporate horizontal and vertical coefficients of restitution with a surface grip analysis for board-ball contact. We utilize finite difference techniques to solve the resulting equations numerically at each stage of motion in order to generate ball trajectories that resemble the Ma Lin ghost serve.

II. THE MATHEMATICAL MODEL

The Ma Lin ghost serve for a table tennis ball is analysed up to the third board-ball collision point. Assuming that there is no sideways motion, we investigate the two-dimensional trajectory of the ball by partitioning the serve into separate phases [2]. This subdivision divides the serve into four airborne and four contact phases, as depicted in Figure 1.

The serve is initiated when the ball is tossed (without spin) vertically upwards from the open palm of one hand. The ball reaches a maximum height, which we refer to as point A. Our analysis of the serve begins when the ball falls from rest at the maximum height A. The first airborne phase therefore represents the vertical drop from the maximum height at A to the point B as indicated in Figure 1. The first contact phase takes place at B when the server hits the ball with the racket to impart the necessary backspin. The second airborne phase BC is the trajectory of the ball after impact with the racket at B until the ball lands for the

Manuscript received February 18th, 2019.

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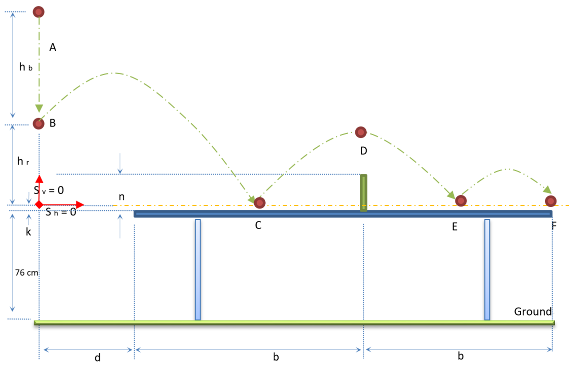

Fig. 1. A basic short serve with four airborne phases and four contact phases

first time on the board at point C. The collision between the ball and the board at C is referred to as the second contact phase. It should be noted that for the serve to be legal, point C must be located on the server's side of the board. The ball subsequently rebounds from C and travels until it lands for a second time on the receiver's board at point E. The serve is said to be illegal if the second bounce at E does not take place on the receiver's end of the board. The third contact phase of the serve considers the impact that the bounce at position E has on the subsequent motion of the ball. During the third airborne phase CE, the ball must have cleared the table tennis net. The position of the ball when it is directly above the net is indicated by the point D. Phase EF investigates the rebound of the ball from E until it lands again at F.

The position of F is instrumental in rating the success of the serve. Point F must be located on the table tennis board for the serve to be classified as short. The Ma Lin ghost serve is a short serve where the spin on the ball reverses the direction of its trajectory after the second board-ball contact at E. It follows that if point F is located closer to the server than point E, the ball would have effectively reversed direction.

The individual style of each player is unique, and this is taken into account in our investigation. Key factors that may vary from player to player include the height of the ball toss, the serve position and the spin on the ball. The system also has fixed standards for regulated table tennis equipment. The radius of the ball k is $0.02m$, the upper surface of the board must be $0.76m$ above the ground, the length of half of the board b is $1.37m$, the height of the net is $0.1525m$ and the width of the board is $1.525m$ [3].

We adopt a two-dimensional Cartesian coordinate system for our analysis. The horizontal axis originates in the line of the ball toss AB and proceeds in a positive direction towards the receiver's end. The vertical axis commences one ball radius above the board's surface and takes the positive direction as vertically upwards. The values of velocities and displacements with respect to time are investigated via perpendicular horizontal and vertical axes. In generating the results of each phase, the final values of velocity and displacement in one phase are the initial values of the consecutive phase. The corresponding ball trajectories are calculated and displayed graphically.

TABLE I
NOMENCLATURE

Symbol	Units	Description
v_r	ms^{-1}	velocity of serve
θ	degrees	angle of elevation to horizontal
N_{BC}	revs per sec	backspin imparted by the racket
W	N	weight of the ball
A_R	N	force due to air resistance
M	N	force due to Magnus Lift
SP	-	spin parameter
C_D, C_L	-	coeff of drag, lift
e_{H_C}, e_{V_C}	-	coeff of horizontal, vertical restitution
K	m	radius of the ball
$\omega_{1_{BC}}$, $\omega_{2_{BC}}$	rad. per sec	initial, final angular velocity BC
$u_{h_{BC}}$, $u_{v_{BC}}$	ms^{-1}	initial horizontal, vertical velocity BC
$v_{h_{BC}}$, $v_{v_{BC}}$	ms^{-1}	final horizontal, vertical velocity BC
F_{H_C}	N	horizontal force at C
F_{R_C}	N	frictional force at C
R_C	N	reaction force at C
G	ms^{-2}	acceleration due to gravity
h_b	m	height of ball toss above board at A
h_r	m	height of serve above board at B
m_b, m_r	kg	mass of ball, racket
Re	-	Reynold's number
ρ_a	kgm^{-3}	density of air
q_d	$Nm^{-2}s^2$	proportional coeff of drag
q_m	$Nm^{-2}s^2$	proportional coeff of Magnus lift

A. Phase AB

We assume that the the ball is dropped vertically downwards from the maximum height A, to the point of contact with the racket B. Since the ball has no spin from the initial toss, no Magnus effect is considered during this phase of motion. We also assume that energy is conserved throughout this period. This allows us to calculate the horizontal and vertical components of velocity prior to impact with the racket in equations 1 and 2.

B. Phase B

In the racket-ball collision phase, the racket moves from left to right with a velocity v_r . The racket strikes the ball at position B with an angle of elevation θ to the horizontal in a counter-clockwise direction. The impact is at the base of the ball and this results in changes in its vertical, horizontal and rotational motions. For simplicity, we assume that the racket and ball coalesce, and that momentum is conserved. The initial velocities of phase B are the final velocities of the previous phase. Horizontal and vertical components of velocity are calculated directly after impact with the racket via equations 3 and 4. We assume the ball leaves the racket with a backspin of N_{BC} revolutions per second.

C. Phase BC

The ball is considered to be a light spherical weight W in flight which loses a significant percentage of its energy due to air resistance A_R [4]. This air resistance or drag force acts on the airborne trajectory phases BC, CE and

ISBN: 978-988-14048-6-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

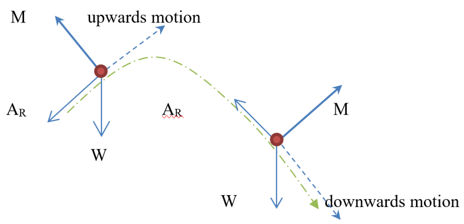

Fig. 2. Magnus lift of table tennis ball

EF. As the Reynolds number is proportional to velocity, it follows that different serve velocities will correspond to a range of possible Reynolds numbers. The drag coefficient is approximately constant for the range of Reynolds numbers that we consider [5]. We may therefore conclude that the drag force is proportional to the square of the velocity, and this is reflected in equation 5.

The Ma-Lin serve requires a significant amount of back-spin on the ball. This produces a variation of speed and pressure difference between the lower and upper surfaces of the ball. The effect of backspin is taken into account during the airborne phases BC, CE and EF. The pressure difference (Bernouilli's effect) generates a perpendicular force to motion usually referred to as the Magnus Lift - denoted by M - as seen in Figure 2.

Rotation produces a factor known as the spin parameter SP which is defined as the ratio of the velocity of a peripheral point of the ball to the velocity of the centre of gravity of the ball [6]. We assume for this system that the spin parameter SP is 3.0 and that the racket imparts an angular velocity $\omega_{i_{BC}}$ on the ball. For this spin parameter, the coefficient of drag C_D is 0.73 and the coefficient of lift C_L is 0.35 [7]. The numerical analysis generates displacement and speed values from equations 7 to 13, with similar analyses for motion in phases CE and EF. The terminal velocity is also calculated to ascertain any effects it has on the velocity.

D. Phase C

The main factors that affect the board-ball contact phases are the change in bounce and grip. The vertical energy loss parameter (i.e. coefficient of vertical restitution) is only considered at the first two points of board-ball contact at C and E. There is an established standard for regulated table tennis boards to generate a fixed rebound from a particular height [3]. We use this standard to establish the coefficient of vertical restitution for our analysis. This parameter is represented by e_C and e_E at points C and E respectively, where $e_C = e_E$.

As the spinning ball grips the surface at C and E, changes in velocity and spin occur at these points [8]. This is illustrated in Figure 3, where a ball of radius k is incident at an angle on a horizontal surface (board) with angular velocity $\omega_{2_{BC}}$, vertical velocity $v_{v_{BC}}$ and horizontal velocity $v_{h_{BC}}$. The ball rebounds off the surface with angular velocity $\omega_{1_{CE}}$, vertical velocity $u_{v_{CE}}$ and horizontal velocity $u_{h_{CE}}$.

The ball bends, grips and deforms upon impact with the surface, which affects the coefficient of horizontal restitution [8]. This coefficient is defined as the negated ratio of the horizontal speed of a point on the bottom of the ball with respect to the board after impact to the horizontal speed of

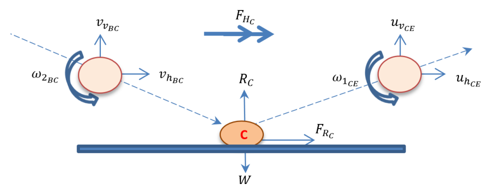

Fig. 3. Table tennis ball grips the surface at C

a point on the bottom of the ball with respect to the board before impact. The coefficient of horizontal restitution has values ranging from -1 to 1, and can lead to the ball slowing down or even reversing direction. We assume that the net reaction force RC acts through the centre of the ball and thus did not affect the torque. It should however be pointed out that a torque is created when the frictional force F_{R_C} acts in a positive horizontal direction at the base of the ball. We also assume conservation of angular momentum about the point of contact between the ball and surface of the board. The ball is considered to be a thin hollow sphere when calculating its moment of inertia. Based on these assumptions for the rotating ball, equations 17 to 20 are generated. A similar analysis is done for phase E, which is the other board-ball contact point.

E. Phase F

The displacement at point F at the end of the serve is instrumental in determining whether the serve is short, long, or qualified as a Ghost serve.

III. Results and Calculations

We consider vertical motion ($\uparrow$) as positive and horizontal motion to the right ($\rightarrow$) as positive.

A. Phase AB

The assumption of no losses in this phase allows the use of the Principle of Conservation of Energy. The final vertical velocity is

$$v_{v_{AB}} = -\sqrt{2gh_b} \tag{1}$$

where acceleration due to gravity is taken to be $g = 9.81ms^{-1}$. The toss was completely vertical, so the final horizontal velocity of the ball during this phase of motion is

$$v_{h_{AB}} = 0 \tag{2}$$

B. Phase B

We assume that the racket and ball coalesce and that momentum is conserved, with the racket striking the ball at a tangent. The equations for the horizontal and vertical motion of the ball in this phase are

$$u_{h_{BC}} = \frac{m_r}{(m_r + m_b)} v_r \cos\theta \tag{3}$$

$$u_{v_{BC}} = \frac{m_r}{(m_r + m_b)} v_r \sin\theta + \frac{m_b}{(m_r + m_b)} v_{v_{AB}} \tag{4}$$

where mass of a racket is taken as $m_r = 90g$ and the mass of the ball is given as $m_b = 2.7g$.

ISBN: 978-988-14048-6-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

C. Analysis of the Airborne Phases

We assume that there is no rotational decrease during each airborne phase of motion, so that $\omega_i = \omega_f$. We also utilize $2 \leq v_r \leq 12ms^{-1}$, which is a realistic range of velocities for a legal serve with no spin. The corresponding range for Reynolds number is $5478 \leq Re \leq 32868$. This is well within the Newton region which ranges from $103 \leq Re \leq 2.5 \times 10^5$, which in turn suggests that the drag coefficient in this region is approximately constant [5] at $C_D = 0.4$. It follows that air drag is

$$A_R = \frac{1}{2} C_D \rho_a \pi k^2 v^2 = q_d v^2 \tag{5}$$

The spinning ball (with angular velocity of 90 $rads^{-1}$) has a force acting on it due to the Magnus lift defined as

$$M = \frac{1}{2} C_L \rho_a \pi k^2 v^2 = q_m v^2 \tag{6}$$

since C_L is constant.

We assume that there is no rotational decrease during each airborne phase of motion, so that $\omega_i = \omega_f$. In our numerical approach, the initial values of horizontal distance $s_{h_0}$and velocity $\dot{s}_{h_0}$are known. Using Forward Euler and Central Differences to discretize the equations, we get

$$s_{h_1} = s_{h_0} + \dot{s}_{h_0} \Delta t \tag{7}$$

$$s_{h_{i+1}} = \ddot{s}_{h_i} \left(\Delta t\right)^2 + 2 s_{h_i} - s_{h_{i-1}} \tag{8}$$

$$\dot{s}_{h_i} = \frac{s_{h_{i+1}} - s_{h_i}}{\Delta t} \tag{9}$$

where $\ddot{s}_{h_i}$ is the discretized horizontal acceleration.

For the horizontal motion in Phase BC, the total horizontal force is given by

$$F_{h_{BC}} = -M_{h_{BC}} - A_{R_{h_{BC}}}$$

hence

$$m_b a_{h_{BC}} = -q_M v_{v_{BC}}^2 - q_D v_{h_{BC}}^2$$

$$\ddot{s}_{h_i} = \frac{-q_M \dot{s}_{v_i}^2 - q_D \dot{s}_{h_i}^2}{m_b} \tag{10}$$

and using (10) in (8)

$$s_{h_{i+1}} = \left(\frac{-q_M \dot{s}_{v_i}^2 - q_D \dot{s}_{h_i}^2}{m_b} \right) \left(\Delta t\right)^2 + 2 s_{h_i} - s_{h_{i-1}} \tag{11}$$

For the vertical motion in Phase BC

$$F_{v_{BC}} = M_{v_{BC}} - A_{R_{v_{BC}}} - W$$

hence

$$m_b a_{v_{BC}} = q_m v_{h_{BC}}^2 - q v_{v_{BC}}^2 - m_b g$$

and we obtain the discretized equations

$$\ddot{s}_{v_i} = \frac{q_m v_{h_{BC}}^2 - q v_{v_{BC}}^2 - m_b g}{m_b}$$

$$s_{v_{i+1}} = \left(\frac{-q_m \dot{s}_{h_i}^2 - q_d \dot{s}_{v_i}^2 - m_b g}{m_b} \right) \left(\Delta t\right)^2 + 2 s_{v_i} - s_{v_{i-1}} \tag{12}$$

$$\dot{s}_{v_i} = \frac{s_{v_{i+1}} - s_{v_i}}{\Delta t} \tag{13}$$

Without loss of generality, the same procedures are repeated for the other two airborne phases with spin: Phases CE and EF.

D. Consideration of Terminal Velocity

Terminal velocity is calculated only with regards to the downward vertical motion from Newton's Second Law. Recall

$$F_{v_{BC}} = A_{R_{v_{BC}}} - W = q_d v_{v_{BC}}^2 - m_b g$$

Since $a_{v_{BC}} = 0$ at terminal velocity, it follows that $v_{t_{BC}}^2 = \frac{m_b g}{q_d} = 2.96ms^{-1}$. From the numerical solution generated, the velocity of the Ma Lin ghost serve does not approach the terminal velocity. As such, this does not affect the solution.

E. Ball-Board Contact Analysis

Energy is lost vertically at the point of ball-board contacts at C and E [6]. The coefficient of vertical restitution [3] is $e_{v_C} = e_{v_E} = 0.77$. We also know that

$$|u_{v_{CE}}| = |e_{v_C} v_{v_{BC}}| \tag{14}$$

hence C and E

$$u_{v_{CE}} = -e_{v_C} v_{v_{BC}} \tag{15}$$

$$u_{v_{EF}} = -e_{v_B} v_{v_{CE}} \tag{16}$$

The ball deforms (slightly) and grips the surface at C and E, with the coefficient of horizontal restitution

$$e_{h_C} = -\frac{u_{h_{CE} - k\omega_{1CE}}}{v_{h_{BC} - k\omega_{i_{BC}}}}, \quad -1 < e_{h_C} < 1 \tag{17}$$

We assume that there is no rotational decrease during each airborne phase of motion, and $\omega_{i_{BC}} = 2\pi N_{BC}$. We also assume conservation of angular momentum about the point of contact with the ball and surface, hence

$$I\omega_1 + m_b k v_{h_{BC}} = I\omega_2 + m_b k u_{h_{CE}} \tag{18}$$

where $I = \alpha m_b k^2$ and $\alpha = \frac{2}{3}$ for a thin spherical sphere. We also assume that the frictional force $F_{R_C} = -m_b \frac{dv_h}{dt}$ and $F_{R_C} k = -I \frac{d\omega}{dt}$. Taking k to be constant in the bounce phase,

$$\int F dt = m_b \left(v_{h_{BC}} - u_{h_{CE}}\right) = m_b \alpha k \left(\omega_{1_{CE}} - \omega_{2_{BC}}\right)$$

These equations for spin are solved to obtain at C

$$\frac{u_{h_{CE}}}{v_{h_{BC}}} = \frac{1 - \alpha e_{h_C}}{1 + \alpha} + \frac{\alpha \left(1 + e_{h_C}\right)}{1 + \alpha} \frac{k \omega_{2_{BC}}}{v_{h_{BC}}} \tag{19}$$

$$\frac{\omega_{1_{CE}}}{\omega_{2_{BC}}} = \frac{\alpha - e_{h_C}}{1 + \alpha} + \frac{1 + e_{h_C}}{1 + \alpha} \frac{v_{h_{BC}}}{k \omega_{2_{BC}}} \tag{20}$$

Without loss of generality, there are similar equations generated at E. We consider the system for

$$e_{h_C} = e_{h_g} = 0.6$$

Thus, using the initial velocities and rotational speed at the beginning of each ball-board contact phase, the final values are generated.

ISBN: 978-988-14048-6-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

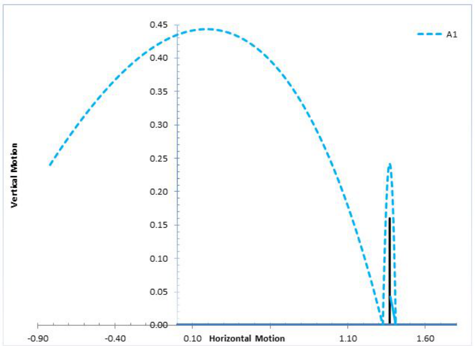

Fig. 4. Numerical solution of ball trajectory $A1$ depicting a Ma Lin Ghost Serve. Trajectory is measured in metres (m)

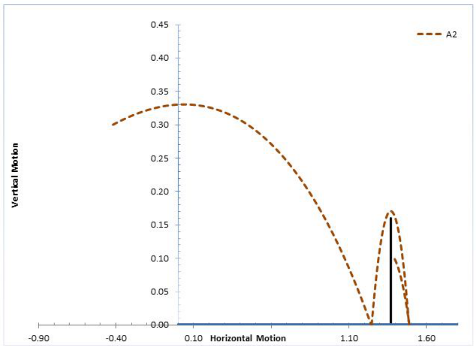

Fig. 5. Numerical solution of ball trajectory $A2$ depicting a Ma Lin Ghost Serve. Trajectory is measured in metres (m)

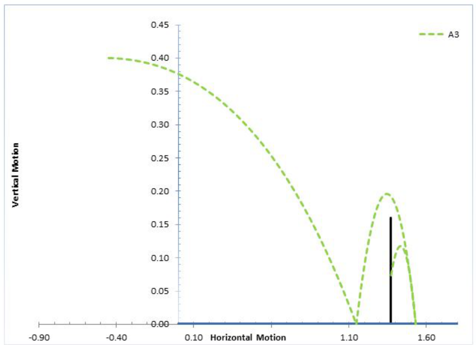

Fig. 6. Numerical solution of ball trajectory $A3$ depicting a Ma Lin Ghost Serve. Trajectory is measured in metres (m)

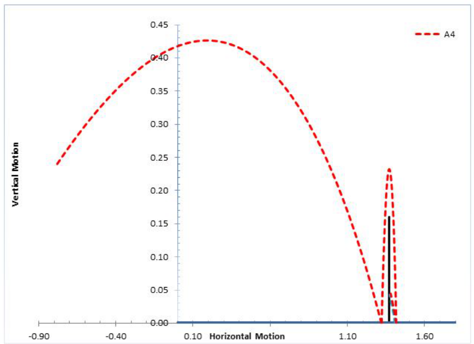

Fig. 7. Numerical solution of ball trajectory $A4$ depicting a Ma Lin Ghost Serve. Trajectory is measured in metres (m)

IV. Discussion

Modern table tennis rackets are constructed with individual variations in adhesives, weight and surface textures which all have an impact on the ball trajectory. For our purposes, we considered an ideal case where the principle of conservation of momentum could be used to determine the velocity after the racket-ball collision. We also assumed that the ball obtained a fixed rotational speed after impact with the surface of the racket.

In the first airborne phase BC, and we incorporated the effect of air resistance and Magnus lift. The coefficients of drag and lift for these effects are held constant since the spin of the ball is assumed to be constant throughout this phase. The air resistance is proportional to the square of the velocity while the Magnus lift term is proportional to the square of the perpendicular velocity. Finite difference methods we utilized to generate numerical values for the ball displacement and velocity. This procedure was repeated for phases CE and EF.

The ball deforms (slightly) at both board-ball contact points at C and E. The resulting vertical and horizontal losses are incorporated into the analysis accordingly. The torque created upon impact of the ball with the board surface (as it grips the moving ball) converts some of the ball's translational energy into rotational energy. This significantly decreases the horizontal velocity of the ball, while simultaneously increasing the magnitude of the backspin at each of the board-ball contact points at C and E. These effects are cumulative, and may decrease the velocity of the ball to the point of reversing its trajectory at the second board-ball contact point E. In such cases, the Ma Lin short ghost serve is observed.

As players have individual playing techniques and styles of delivery, it is possible to generate Ma Lin ghost serves with widely ranging characteristics. We utilized our model to illustrate this by calculating the ball trajectories of four possible Ma Lin ghost serves by having one manipulated variable while keeping the backspin constant (as a controlled variable). The four resulting ball trajectories are illustrated in Figures 4-7.

The Ma Lin ghost serve labelled $A1$ is representative of the original analysis done in the previous sections of this paper. Serve $A2$ was generated by reducing the serve angle (manipulated variable) previously adopted for serve $A1$. In order for the Ma Lin serve to be achieved with this variation, the height of toss, the height of contact and the distance of serve from the board were modified (responding variables). Serve $A3$ has a higher initial serve velocity (manipulated variable) than serve $A1$. In order to obtain the characteristics of a Ma Lin ghost serve for $A3$, alterations (responding variables) were made for the serve angle, height of racket contact and distance of serve from the board. Serve $A4$ varies from serve $A1$ by an increase in the height of the ball toss manipulated variable). The correction made to still generate a Ma Lin serve was an alteration in the serve position (responding variables). Table II summarizes the values of the parameters utilized to calculate the ball trajectories for serves $A1$ to $A4$.

V. Conclusion

We successfully formulated a two-dimensional mathematical model for the Ma Lin ghost serve. We utilized

ISBN: 978-988-14048-6-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE II
PARAMETERS USED FOR BALL TRAJECTORIES IN FIGURES 4-7.

	Serve A_1	Serve A_2	Serve A_3	Serve A_4
v_r	5.5	5.5	6	5.5
θ	20	8	0	20
h_b	0.16	0.64	0.16	1.0
h_r	0.24	0.3	0.4	0.24
d	-0.82	-0.42	-0.45	-0.78

the kinematic equations of motion with linear and angular momentum conservation principles. A step by step trajectory analysis was performed for varying incident racket velocity's and angles. The effect of quadratic backspin, air resistance (drag) and Magnus lift were successfully incorporated. We also included horizontal and vertical coefficients of restitution and a basic surface grip analysis for board-ball contact. Finite difference techniques were adopted to solve the equations numerically at each stage of motion, and graphical illustrations of the ball trajectories for the Ma Lin ghost serve were generated.

This paper represents a first attempt in the creation of a mathematical model for a relatively complicated backspin serve that is commonly employed by professional players in the sport of table tennis. The authors acknowledge that it would be greatly beneficial if experimental work was done in support of the model. This would allow us to obtain accurate values for all the required parameters. In the absence of this, we have estimated the necessary parameter values using established theoretical reference sources that have been duly cited in the body of the paper. It should be noted however that the existence of a theoretical mathematical model is still of great value, as it represents an important first step in the development of a holistic model to accurately trace the trajectory of ball during a Ma Lin ghost serve.

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