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Basic Numerical Study about the Stochastic Advantage on Ultimate Bending Capacity of Hybrid Composite Girder

Kyosuke Yamamoto and Ryunosuke Chino

Abstract— Comparing probability density functions of ultimate bending capacity of a "hybrid composite girder" with those of "composite girder", it can be found that the hybrid composite girder shows higher reliability. In this paper, the numerical experiments based on Monte Calro simulation is carried out and the effect of the prior information is examined. The prior information easily given from testing is the average and variance of the strength, elasticity and other parameters of materials and of the dimensions of members. The result says that these information gives same capacity even under different assumptions for the probability density functions, while the reliability considerably changes. However, it is also found that the reliability of the hybrid composite girder is always much higher than the normal composite girder.

***Index Terms*—Limit State Design Method, Hybrid Composite Girder, Monte Carlo Simulation, reliability**

I. Introduction

Hybrid composite girder, shown in **Fig. 1**, is a kind of composite girder, which is composed of a hybrid steel main girder and a concrete slab. They are rigidly connected to each other. The hybrid steel main girder is a steel girder of which only the bottom flange is made of high performance steel while the other parts, the upper flange and web, are made of normal steel. Because the bottom flange is an important member for mostly resisting tensile stress, and because the high performance steel has higher yield stress, it can be said that the hybrid composite girder is a rationalized structure having higher bending capacity.

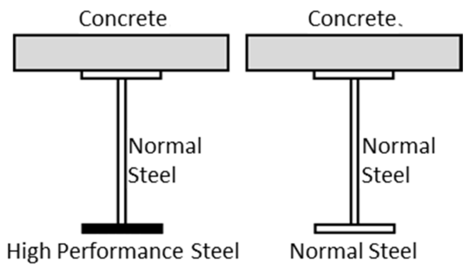

Fig. 1. The hybrid and normal composite girders

It is said that the construction cost of the hybrid composite girders is less than steel girders [1]. Because the concrete slab restrain buckling of hybrid steel girder, the thickness of steel plate can be set smaller. However, the restraint effect of the concrete slab can work in composite girders as well as the hybrid ones. Many advantages of hybrid composite girder are common in composite girders. Considering their reliabilities, the motivation for realization of hybrid composite girder is still low actually.

For bridge structures, reliability, which can be defined as probability that the system satisfies its required performance, is very subjective, or Bayesian in other words. Bridges are usually "one-off" structures so that the uncertainty indicates just the lack of available information. The probabilities about the performance of bridge structures depend on available information. Since we have a great deal of experience of construction of composite girders, the reliability of composite girders can be evaluated higher than that of hybrid ones.

To encourage the construction of hybrid composite girders, it is necessary to find its advantage that normal ones do not have. In this study, numerical experiment using Monte Carlo simulation is carried out to compare the ultimate bending capacities of composite girder and hybrid composite girder.

II. Numerical Experiments

A. Constitutive Equations

The stress-strain relationships of concrete material can be described as Eq. (1):

$$\begin{aligned}&(\varepsilon_c \le 0.002)\\ &\quad \sigma_c = 0.85 f_c \left(\frac{\varepsilon_c}{0.002}\right)\left(2-\frac{\varepsilon_c}{0.002}\right)\\ &(0.002 < \varepsilon_c \le 0.0035)\\ &\quad \sigma_c = 0.85 f_c\end{aligned} \tag{1}$$

where σ_c, ε_c and f_c are stress, strain and strength of concrete. The stress-strain relationship of steel material can be also given by Eq. (2):

$$\begin{aligned}&(\varepsilon_s \le \varepsilon_y)\\ &\quad \sigma_s = E\varepsilon_s\\ &(\varepsilon_y < \varepsilon_s \le \varepsilon_{st})\\ &\quad \sigma_s = f_y\\ &(\varepsilon_{st} < \varepsilon_s)\\ &\quad \sigma_s = \frac{f_y}{\xi}\frac{E_{st}}{E}\left[1-exp\left\{-\xi\left(\frac{\varepsilon_s}{\varepsilon_y}-\frac{\varepsilon_{st}}{\varepsilon_y}\right)\right\}\right]+1\end{aligned} \tag{2}$$

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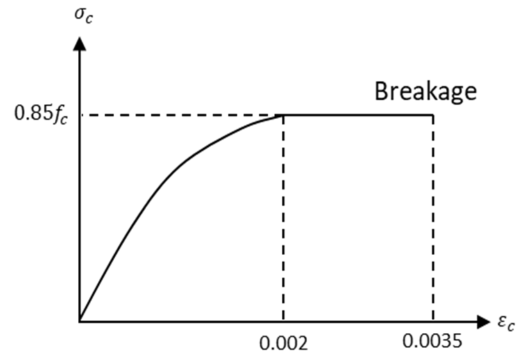
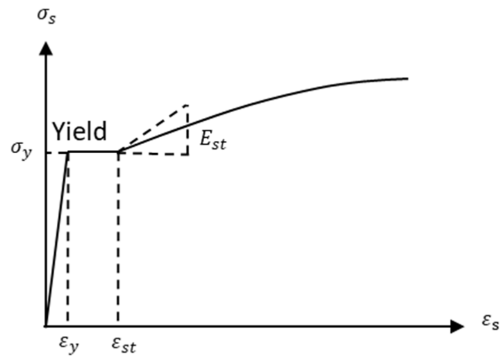
Kyosuke Yamamoto is an assistant professor of University of Tsukuba, Japan (e-mail: yamamoto_k@kz.tsukuba.ac.jp).
Ryunosuke Chino is a master student of University of Tsukuba, Japan.

where σ_s, ε_s, E, ε_y, f_y, ε_{st}, E_{st} and ξ are stress, strain Young's modulus, yield strain, yield strength, hardening strain, hardening elasticity and hardening curvature of steel, respectively. The constitutive equation is common in the normal steel and high performance steel. These curves of Eqs. (1) and (2) are shown in **Fig. 2**.



(a) Concrete



(b) Steel

Fig. 2. The stress-strain curve of materials

The average and standard variance of each variable of concrete, steel and high performance steel used in Eqs. (1) and (2) are shown in **TABLE I**, **II** and **III**, respectively.

TABLE I
THE PARAMETERS OF CONCRETE

Parameters	Average μ	Standard Variation σ	Variation Coefficient CV
Compressive Strength: f_c (N/mm^2)	30	1.2	0.04
Ultimate Strain	0.0035	0	0

TABLE II
THE PARAMETERS OF STEEL

Parameters	Average μ	Standard Variation σ	Variation Coefficient CV
Young's Modulus: E (N/mm^2)	200000	2000	0.010
Yield Strength: f_y (N/mm^2)	293.75	23.5	0.080
hardening strain: ε_{st}	0.0185	0.0049	0.265
Hardening Coefficient: E_{st} (N/mm^2)	4156	1342	0.323
Hardening Curvature: ξ (N/mm^2)	0.049	0.027	0.550

TABLE III
THE PARAMETERS OF HIGH PERFORMANCE STEEL

Parameters	Average μ	Standard Variation σ	Variation Coefficient CV
Young's Modulus: E (N/mm^2)	200000	2000	0.01
Yield Strength: f_y (N/mm^2)	549	36	0.0656
hardening strain: ε_{st}	0.0082	0.0041	0.5
Hardening Coefficient: E_{st} (N/mm^2)	2000	1170	0.585
Hardening Curvature: ξ (N/mm^2)	0.02	0.025	1.25

In this study, steel I-girder is considered. The ultimate bending capacity is usually non-linear so that full plastic moment is often used as the design capacity for ease in computation. Stress distributions of full plastic capacity of cross-section of a hybrid composite girder is shown in **Fig. 3**. The bending capacity of the hybrid composite girder is obviously larger than that of the normal one, if the dimensions are same. Thus, in this study, each capacity distribution is normalized by each full plastic capacity. The member measurements are also given as **TABLE IV**.

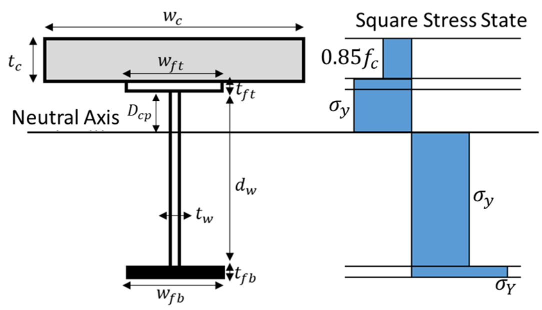


Fig. 3. The hybrid and normal composite girders

TABLE IV
THE PARAMETERS OF MEMBERS

Parameters	Average μ	Standard Variation σ	Variation Coefficient CV
Slab Width: w_c (mm)	1500	6	0.0040
Slab Thickness: t_c (mm)	160	6	0.0375
Upper Flange Width: w_{ft} (mm)	300	4.38	0.0146
Upper Flange Thickness: t_{ft} (mm)	15	0.219	0.0146
Web Thickness: t_w (mm)	15	0.219	0.0146
Web Height: d_w (mm)	1500	21.9	0.0146
Bottom Flange Width: w_{fb}	300	4.38	0.0146
Bottom Flange Thickness: t_{fb} (mm)	40	0.584	0.0146

In this study, the fiber method is adopted as the scheme for calculating ultimate bending capacities. The flow of this method is shown in **Fig. 4**. The ultimate state is defined as

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the point that the strain at the upper edge of the concrete slab reaches the concrete's crushing strain ε_c.

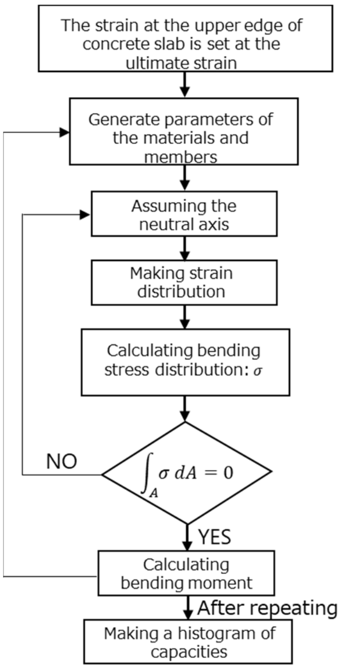


Fig. 4. Flow of the fiber method

To statistically generate values for each parameter, it is necessary to assume approximate function for the probability function. In this study, three most popular functions are considered: normal distribution, log-normal distribution and Weibull distribution. Their probability density functions are shown in Eqs. (3) to (5), respectively.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{3}$$

$$f(x) = \frac{1}{\sqrt{2\pi}\zeta x} \exp\left[-\frac{1}{2}\left(\frac{\ln x-\lambda}{\zeta}\right)^2\right] \tag{4}$$

$$f(x) = \frac{1}{\gamma^m} m x^{m-1} \exp\left[-\frac{1}{\gamma^m} x^m\right] \tag{5}$$

where ζ, m and γ are the parameters of log-normal and Weibull distributions. They can be given as Eqs. (6) to (9):

$$\lambda = \ln\mu - \frac{1}{2}\zeta^2 \tag{6}$$

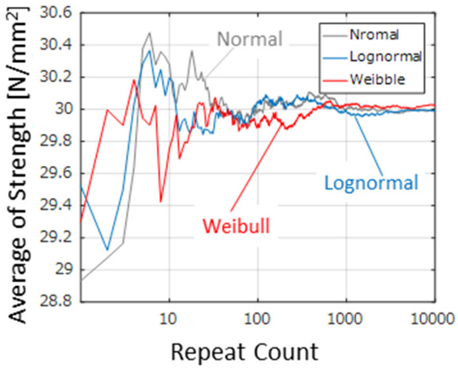
$$\zeta = \sqrt{\ln\left(1+\frac{\sigma^2}{\mu^2}\right)} \tag{7}$$

$$\mu = \eta\Gamma\left(\frac{1}{m}+1\right) \tag{8}$$

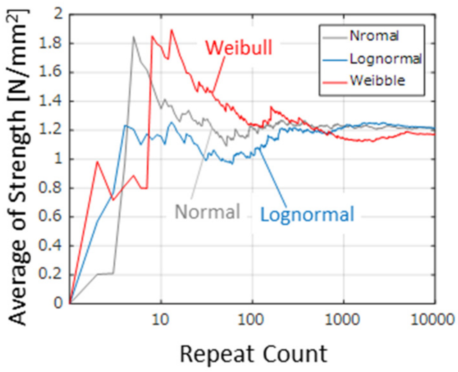
$$\sigma = \eta\sqrt{\Gamma\left(\frac{2}{m}+1\right) - \Gamma^2\left(\frac{1}{m}+1\right)} \tag{9}$$

where$\Gamma(\ )$ denotes Gamma function.

In this simulation, the repeat count is set at 10,000. **Fig. 5** shows the relationship between the repeat count and the average and variance of the compressive strength of the concrete, for each probability density function. According to this figure, it is confirmed that 10,000 is enough repeat number, because the average and variance is convergent after 1000. **Fig. 6** shows the histograms of generated concrete strength for each function.

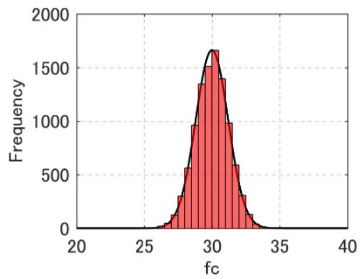


(a) Average

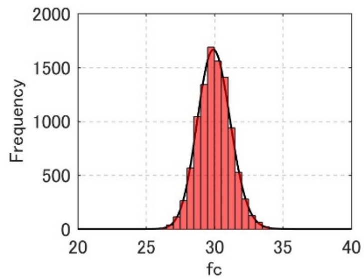


(b) Standard variance

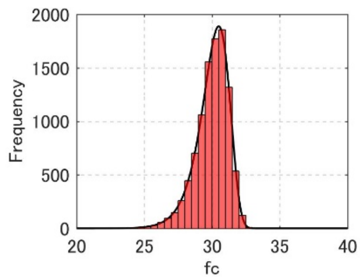
Fig. 5. The convergence of the concrete strength



(a) Normal distribution



(b) Log-normal distribution



(c) Weibull distribution

Fig. 6. The generated concrete strength

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B. Results and Discussion

The obtained histogram of normalized ultimate bending capacities are shown in **Fig. 7**. The statistics are also shown in **TABLE V**. If the value equals to 1, the capacity is same with full plastic moment. According to the result, because the average is very close to 1, it is confirmed that the full plastic moment is a proper feature quantity for design capacity of ultimate limit state of composite/hybrid composite girders. The average and standard variance of the results do not change, as the assumed probability function changes. If material and member data samples are available, it is difficult to determine the best fitting curve for the probability density function. This result shows that it is not necessary to think it so serious, if only average and variance is required to seek.

However, what designers want to know the most is always the “reliability” of the structure. In this case, the reliability would be the probability under the load. If the load is fixed at 0.78, the fault probabilities of this composite girder are 0.0009, 0.0001 and 0.0050, for normal, log-normal and Weibull distributions, respectively. Each value is very small, while this difference is considerable and affective to design. The safest function, which is Weibull distribution in this case, must be adopted unless the best fitting function is determined. On the other hand, the standard variance of hybrid composite girders is much improved from that of the composite girder. If the load is set at 0.78, regardless of assumed probability distributions for materials and members, all results about the fault probabilities are 0. This tendency of hybrid composite girder can be a positive advantage.

III. Conclusions

In this study, the histogram of ultimate bending capacities of the composite/hybrid composite girders are generated by Monte Carlo simulation, assuming three probability density functions for materials and members. The difference of the assuming functions does not affect on the average and variance of the capacity, but much on the reliability. To implement this problem, it is efficient to adopt hybrid composite girder, because the variance of its capacity is much less than that of composite girder.

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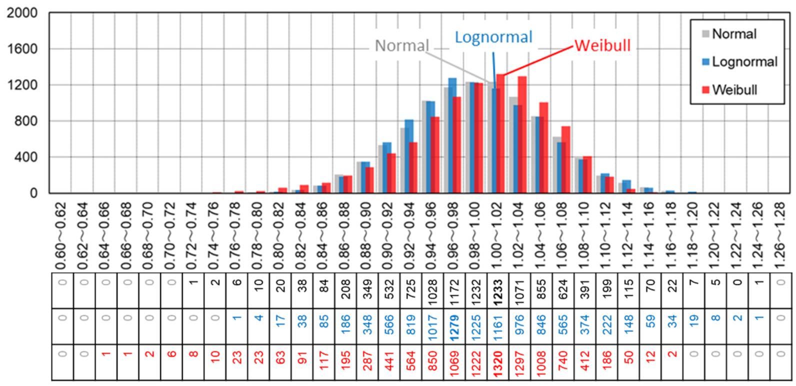
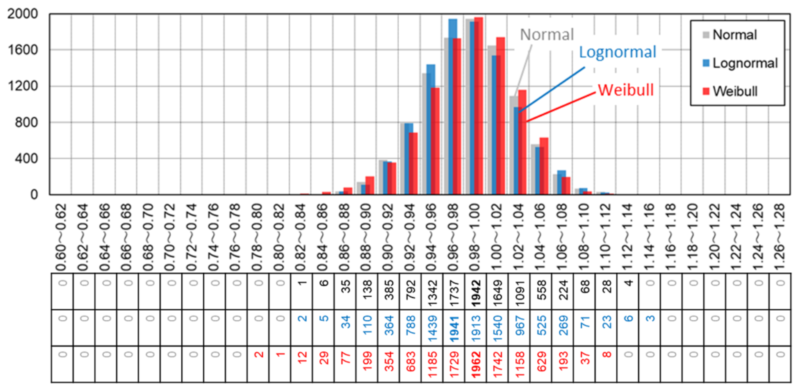


(a) Composite girder



(b) Hybrid composite girder

Fig. 7. The histogram of ultimate bending capacities

TABLE V The Statistics of Obtained Capacity Distributions

(A) Composite Girder

Parameters	Normal Distribution	Log-Normal Distribution	Weibull Distribution
Average	0.993	0.992	0.993
Standard Variance	0.0630	0.0634	0.0645
Skewness	-0.0345	0.14778	-0.696
Kurtosis	0.05971	0.0263013	0.87918

(B) Hybrid Composite Girder

Parameters	Normal Distribution	Log-Normal Distribution	Weibull Distribution
Average	0.985	0.985	0.985
Standard Variance	0.0408	0.0405	0.0417
Skewness	0.00734	0.12656	-0.3685
Kurtosis	-0.0427	0.0957261	0.27994

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