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Study of the Limit State of Open Renewable Systems

Anna Tarasenko, Oleksandr Karelin, Oleksandr Barabash and Manuel Gonzalez-Hernandez

***Abstract*—In previous works, cyclic models, in which the initial state of the system coincides with the final state, were considered. Applying the principles of modelling of renewable resource systems based on the separation of the individual and the group parameters and discretization of time, we obtained a balance relation that contained linear functional equations with shift. In this work, an open system, in which the final state does not coincide with the initial state, is considered. We model and study these systems using Neumann series. The study of the models is carried out in weighted Holder spaces. The system's evolution is analyzed.**

***Index Terms*—renewable resources, open systems, limit state, Holder space, operators with shift, norm of operators, Neumann series.**

I. Introduction

SYSTEMS whose state depends on time and whose resources are renewable form an important class of general systems. A great number of works has been dedicated to systems with renewable resources [1], [2]. The core of the mathematical apparatus used for the study of such systems consists of differential equations in which the sought for function is dependent on time [3], [4], [5].

Our approach presupposes discretization of processes with respect to time. We move away from tracking the changes in the system continuously to tracking the changes at fixed time points. This discretization and the identification of the individual parameter and the group parameter lead us to functional equations with shift.

In this work, open models are considered. The study of the models is carried out in weighted Holder spaces. The estimate of the norm of shift operator is produced. The system's evolution is analyzed.

II. Models of systems with a set of renewable resources.

Let S be a system with r resources λ^1, $\lambda^2, ..., \lambda^r$ and let T be a time interval. The choice of T is related to periodic processes taking place in the system and to human interferences.

Let these resources have individual parameters with scales

$$x^1_{min} = x^1_1 < x^1_2 < ... < x^1_{n_1} = x^1_{max},$$

for the first resource,

$$x^2_{min} = x^2_1 < x^2_2 < ... < x^2_{n_2} = x^2_{max},$$

for the second resource

....

$$x^r_{min} = x^r_1 < x^r_2 < ... < x^r_{n_r} = x^r_{max}$$

for the r-th resource.

$$v^1(x^1, t_0), x^1_{min} \le x^1 \le x^1_{max},$$
$$v^2(x^2, t_0), x^2_{min} \le x^2 \le x^2_{max},$$

....

$$v^r(x^r, t_0), x^r_{min} \le x^r \le x^r_{max},$$

We introduce the group parameters by functions

$$v^1(x^1_i, t), v^2(x^2_i, t), ..., v^r(x^r_i, t),$$

which express a quantitative estimate of the elements of resources λ^1, $\lambda^2, ..., \lambda^r$ with the individual parameter

$$x^1_i, i = 1, 2, ..., n_1, x^2_i, i = 1, 2, ..., n_2,$$

....

$$x^r_i, i = 1, 2, ..., n_r$$

at the time t.

As in our previous work [6], [7], [8] on modeling the system, we will hold the following principles:

I. The description of changes that occur on the interval $(t_0, t_0 + T)$ will be substituted by the fixing of the final results at the moment $t_0 + T$;

II. We will separate parameters into individual parameters and group parameters and will study of the dependence of group parameters on individual parameters.

The initial state of system S at time t_0 is represented as density functions of a distribution of the group parameter by the individual parameter for each resource

$$v^1(x^1, t_0), x^1_{min} \le x^1 \le x^1_{max},$$
$$v^2(x^2, t_0), x^2_{min} \le x^2 \le x^2_{max},$$

....

$$v^r(x^r, t_0), x^r_{min} \le x^r \le x^r_{max},$$

which express a quantitative estimate of the elements of resources λ^1, λ^2,...,λ^r with the individual parameter $x^1, x^2, ..., x^r$.

We will omit t_0 and will assume that the initial values of the individual parameters are zero:

$$v^1(x^1, t_0) = v^1(x^1), v^2(x^2, t_0) = v^2(x^2),\ ...,\ v^r(x^r, t_0) = v^r(x^r);$$

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A.Tarasenko, O.Karelin, M.Gonzalez-Hernandez, are with Institute of Basic Sciences and Engineering, Hidalgo State University, C.P. 42184, Pachuca, Hidalgo, Mexico. Tel: +52 771 7172000 ext.6162. Email: anataras@uaeh.edu.mx, karelin@uaeh.edu.mx, mghdez@uaeh.edu.mx

O. Barabash is with Institute of Business, Economics and Information Technology, Department of Enterprise Economics, Odessa National Politechnic University, C.P.65044, Odessa, Ukraine. Email: odessaaa48@gmail.com

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$$x^1_{min}=x^2_{min}=...=x^r_{min}=0.$$

We will now analyse the system's evolution. In the course of time, the elements of the system can change their individual parameter - e.g. fish can change their weight and length.

Modifications in the distributions of the group parameters by the individual parameters are represented by displacements. The state of the system S at the time $t = t_0 + T$ is:

$$v^1(x^1, t_0+T)=\frac{d}{dx^1}\ \alpha^1(x^1)\cdot v^1(\alpha^1(x^1)),$$

$$v^2(x^2, t_0+T)=\frac{d}{dx^2}\ \alpha^2(x^2)\cdot v^2(\alpha^2(x^2)),$$

....

$$v^r(x^r, t_0+T)=\frac{d}{dx^r}\ \alpha^r(x^r)\cdot v^r(\alpha^r(x^r)).$$

Over the period $j_0 = [t_0, t_0 + T]$, extractions might be taken from the system as a result of human economic activity; these are represented by summands $\rho^j(x^j)$. If an artificial entrance of elements into the system has taken place, it shall be accounted for by adding terms $\zeta^j(x^j)$. We take natural mortality into account with coefficients $d^j(x^j), 1 \le j \le r$.

The process of reproduction will be represented by terms

$$\mathcal{P}^j(x^j)\cdot v^j(x^j) + \sum_{i=1}^{n_r} P^j_i p^j_i(x^j), \quad 1 \le j \le r,$$

where

$$P^j_1 = \int_{\nu^j_0}^{\nu^j_1} v^j(x^j)dx^j, \quad ,..., \quad P^j_{n_r} = \int_{\nu^j_{n-1}}^{\nu^j_n} v^j(x^j)dx^j,$$

$$0 = \nu^j_0 < \nu^j_1 < ... < \nu^j_{n_r} = x^j_{max}, 1 \le j \le r$$

are integrals with degererate kernels.

We obtain

$$v^j(x^j, t_0+T) = d^j(x^j)\frac{d}{dx^j}\alpha^j(x^j)v^j(\alpha^j(x^j)) + \rho^j(x^j) +$$

$$\zeta^j(x^j) + \mathcal{P}^j(x^j)\cdot v^j(x^j) + \sum_{i=1}^{n_r} P^j_i p^j_i(x^j).$$

Resources λ^1, λ^2,..., λ^r are not independent. We will account for reciprocal influence by terms

$$\sum_{k=1,k\ne j}^{r}\sum_{i=1}^{l^j} F^{j\ k}_i f_i(x^j),$$

where

$$F^{j\ k}_1 = \int_{\epsilon^j_0}^{\epsilon^j_1} v^k(x^k)dx^k, \quad ,..., \quad F^{j\ k}_l = \int_{\epsilon^j_{l^j-1}}^{\epsilon^j_{l^j}} v^k(x^k)dx^k,$$

$$0 = \epsilon^j_0 < \epsilon^j_1 <, ..., < \epsilon^j_{l^j} = x^j_{max}, 1 \le j \le r,\ 1 \le k \le r$$

are integrals with degererate kernels.

This term shows that the resource j is related to other resources.

Thereby, the final state of the system at the moment $[t_0+T]$ is described as follows:

$$v^j(x^j, t_0+T) =$$

$$d^j(x^j)\frac{d}{dx^j}\alpha^j(x^j)v^j(\alpha^j(x^j)) + \rho^j(x^j) + \zeta^j(x^j)$$

$$+\mathcal{P}^j(x^j)\cdot v^j(x^j) + \sum_{i=1}^{n_r} P^j_i p^j_i(x^j) + \sum_{k=1,k\ne j}^{r}\sum_{i=1}^{l^j} F^{j\ k}_i f_i(x^j). \tag{1}$$

Balance equation of the cyclic model, when $v^j(x^j, t_0 + T) = v^j(x^j, t_0)$, has the form

$$v^j(x^j) = d^j(x^j)\frac{d}{dx^j}\alpha^j(x^j)v^j(\alpha^j(x^j)) + \rho^j(x^j) + \zeta^j(x^j) +$$

$$\mathcal{P}^j(x^j)\cdot v^j(x^j) + \sum_{i=1}^{n_r} P^j_i p^j_i(x^j) + \sum_{k=1,k\ne j}^{r}\sum_{i=1}^{l} F^{j\ k}_i f_i(x^j).$$

If previously special attention has been paid to the study of the density distribution of the group parameters by individual parameters then now we present a study of the dynamics of the individual parameters of resources.

In [9], we presented a study of the dynamics of the individual parameters of resources. We carried out a detailed investigation of the graphics of the function of change of the individual parameter with time. We showed the necessity of introducing the age parameter for elements of the system into consideration. Here, we continue a study on the dynamics of the individual parameters of resources.

Now, we consider a model for the study of the dynamics of the individual parameter

For simplicity, consider a system with r resources $\lambda^1, \lambda^2, ..., \lambda^r$ and with the same individual parameter x. We will interpret the resource $\lambda^k, 1 \le k \le r$, as a fish of a certain type, and the individual parameter x, as weight, the group parameter, as the number of fish individuals with this weight ν^k, or more precisely: the integral $\int_J \nu^k(x)dx$ expresses the total number of species of fish of the type k whose weight is in this interval of integration $J \subset (0, x^k_{max})$.

For every type of fish, the range of weight is different, $x^k_{min} = 0 \le x \le x^k_{max}; 1 \le k \le r$.

The balance relation is described by equation (1).

We will analyze in more detail the origin of derivatives in (1). Let the initial distribution of the resource k at time $t = t_0$ be $\nu^k(x, t_0) = \nu^k(x)$.

We select some interval of weight (ζ, ξ) from the interval $(0, x^k_{max})$.

The integral $\int_\zeta^\xi \nu^k(x)dx$ expresses the total number of species of fish of type k whose weights are in this interval (ζ, ξ). At the expiration of time T, at the moment of time $t = t_0 + T$, the weight $x \in (\zeta, \xi)$ of the fish changes and becomes $\alpha^k(x) \in (\alpha^k(\zeta), \alpha^k(\xi))$. Fish with weight from the interval (ζ, ξ) become fish with weight from the interval $(\alpha^k(\zeta), \alpha^k(\xi))$, however, the number of individuals does not change:

$$\int_\zeta^\xi \nu^k(x)dx = \int_{\alpha^k(\zeta)}^{\alpha^k(\xi)} \nu^k(x, t_0+T)dx$$

Let the function β^k be the inverse of the function α^k: $\beta^k(x) = {\alpha^k}^{-1}(x)$.

After the substitution in the first integral

$$z = \alpha^k(x),\ x = {\alpha^k}^{-1}(z), dx = \frac{d}{dz}{\alpha^k}^{-1}(z)dz;$$

$$x = \zeta \Rightarrow z = \alpha^k(\zeta),\ x = \xi \Rightarrow z = \alpha^k(\xi),$$

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we obtain

$$\int_{\alpha^k(\zeta)}^{\alpha^k(\xi)} \nu[\beta^k(z)]\frac{d}{dz}\alpha^{k-1}(z)dz = \int_{\alpha^k(\zeta)}^{\alpha^k(\xi)} \nu^k(x,t_0+T)dx,$$

taking into account the nonnegativity of the function $\nu^k(x)$, we have

$$\nu^k(x,t_0+T) = \frac{d}{dx}\beta^k(x)\nu^k[\beta^k(x)],\ 0 \le x \le x^k_{max}.$$

We assume that in the relation (1), the functions of changes of the individual parameters

$$\alpha^j(x^j), \quad j = 1,2,...,r$$

are unknown functions and that other functions are known.

We represent the equations (1) in the form

$$d^j(x^j)\frac{d}{dx^j}\alpha^j(x^j)v^j(\alpha^j(x^j)) = \Theta^j(x^j), \qquad (2)$$

where $\Theta^j(x^j)$ are known functions

$$\Theta^j(x^j) = \nu^j(x^j,t_0+T) - \rho^j(x^j) - \zeta^j(x^j) -$$

$$-\mathcal{P}^j(x^j)\cdot v^j(x^j) - \sum_{i=1}^{n_r} P^j_i p^j_i(x^j) - \sum_{k=1,k\neq j}^{r}\sum_{i=1}^{l} F^{j\ k}_i f_i(x^j).$$

For cyclic models $\nu^j(x^j,t_0+T) = \nu^j(x^j,t_0) = \nu^j(x)$.

For open models $\nu^j(x^j,t_0) \neq \nu(x^j,t_0+T) = \Omega^j(x^j)$, where $\Omega^j(x^j)$ is a known function, that can be interpreted as the state to which we want to convert the system.

Balance relations (2) are differential equations.

The application of principles I and II leads us to functional operators with shift.

The study of the models is carried out in the weighted Holder spaces.

III. Open models in weighted Hölder spaces

We recall the definition [6] of the weighted Hölder spaces $H^0_\mu(J,\rho)$.

A function $\varphi(x)$ that satisfies the following condition on $J = [0,1]$,

$$|\ \varphi(x_1)-\varphi(x_2)\ | \le C\,|\ x_1-x_2\ |^\mu,\ x_1\in J, x_2\in J,\ \mu\in(0,\ 1),$$

is called a Hölder's function with exponent μ and constant C on J.

Let ρ be a power function which has zeros at the endpoints $x=0$, $x=1$:

$$\rho(x) = (x-0)^{\mu_0}(1-x)^{\mu_1}, \quad \mu < \mu_0 < 1+\mu,\ \mu < \mu_1 < 1+\mu.$$

The functions that become Hölder functions and valued zero at the points $x=0$, $x=1$, after being multiplied by $\rho(x)$, form a Banach space:

$$H^0_\mu(J,\rho), \quad J = [0,\ 1].$$

The norm in the space $H^0_\mu(J,\rho)$ is defined by

$$\|\ f(x)\ \|_{H^0_\mu(J,\rho)} = \|\ \rho(x)f(x)\ \|_{H_\mu(J)},$$

where

$$\|\ \rho(x)f(x)\ \|_{H_\mu(J)} = \|\rho(x)f(x)\|_C + \|\rho(x)f(x)\|_\mu,$$

and

$$\|\rho(x)f(x)\|_C = \max_{x\in J}|\rho(x)f(x)|,$$

$$\|\rho(x)f(x)\|_\mu = \sup_{x_1,x_2\in J, x_1\neq x_2} |\rho(x)f(x)|_\mu,\ |\rho(x)f(x)|_\mu =$$

$$\frac{|\ \rho(x_1)f(x_1) - \rho(x_2)f(x_2)\ |}{|\ x_1 - x_2\ |^\mu}.$$

We denote by $\mathcal{B}(X)$ a set of all bounded linear operators acting on the Banach space X. The norm of an operator $\mathcal{D} \in X$ will be denoted by $\|\mathcal{D}\|_{\mathcal{B}(X)}$.

Let $\beta(x)$ be a bijective orientation-preserving shift on J: if $x_1 < x_2$, then $\beta(x_1) < \beta(x_2)$ for any $x_1 \in J,\ x_2 \in J$; and let $\beta(x)$ have only two fixed points:

$$\beta(0) = 0, \quad \beta(1) = 1,$$

and $\beta(x) \neq x$, when $x \neq 0,\ x \neq 1$.

In addition, let $\beta(x)$ be a differentiable function with $\frac{d}{dx}\beta(x) \neq 0$ and $\frac{d}{dx}\beta(x) \in H_\mu(J)$.

IV. System's evolution.

Now, we return to the situation when the functions of changes of the individual parameters $\alpha^j(x^j)$, $j = 1,2,...,r$ are known functions and the density distributions of the group parameters by individiall parameters $v^j(x^j)$ are unknown. In order not to overload (clutter) the presentation, consider a system with one renewable resource, $n = 1$.

In this section, we do no require that the final state of the system S coincide with the initial state

$$\nu(x,t_0) \neq \nu(x,t_0+T),$$

but the proposition

$$\nu(x,t_0+T) = d(x)\frac{d}{dx}\beta(x)\nu(\beta(x)) + r(x)\nu(x) - g(x) + p(x) \qquad (3)$$

holds. This allows to use (3) to obtain the consequent states of the system S from the previous states.

Without losing the generality, we will assume $x_{max} = 1$.

At the time t_0 the system is in the state $\nu(x)$,

$$\nu(x) \equiv \nu(x,t_0)$$

At the time $t_1 = t_0 + T$, the system will be in the state $\nu_1(x)$:

$$\nu_1(x) = \nu(x,t_1) \equiv d(x)\frac{d}{dx}\beta(x)\nu(\beta(x)) + r(x)\nu(x) - g(x) + p(x);$$

in terms of the shift operator $(B_\beta\nu)(x) = \nu(\beta(x))$, we have

$$\nu_1(x) \equiv b(x)(B_\beta\nu)(x) + r(x)(I\nu)(x) + f(x),$$

where

$$b(x) = d(x)\frac{d}{dx}\beta(x);$$

in terms of the operator weighted shift

$$(\tilde{B}_\beta\nu)(x) = \beta'(x)(B_\beta\nu)(x),$$

we have

$$\nu_1(x) \equiv d(x)(\tilde{B}_\beta\nu)(x) + r(x)(I\nu)(x) + f(x);$$

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in terms of the operator

$$(A\nu)(x) = r(x)(I\nu)(x) + b(x)(B_\beta \nu)(x),$$

we have

$$\nu_1(x) \equiv (A\nu)(x) + f(x).$$

At the time $t_2 = t_0 + 2T$, the system will be in the state $\nu_2(x)$:

$$\nu_2(x) = \nu(x, t_2) = (A(\nu_1 + f))(x) + f(x)\nu_2 \equiv$$

$$A(A\nu + f) + f = A^2\nu + Af + f.$$

....

At the time $t_k = t_0 + kT$ the system will be in the state

$$\nu_k \equiv A(\nu_{k-1} + f) + f \equiv A^2\nu_{k-2} + Af + f \equiv$$

$$A^k\nu + (A^{k-1} + A^{k-2} + ... + A + I)f.$$

Finally, the limit state is represented by a Neumann series. It is found that over time the state of the system tends toward a limit state

$$\nu_\infty = \lim_{k\to\infty} \left(A^k\nu + (A^{k-1} + A^{k-2} + ... + A + I)f\right). \quad (4)$$

The function ν_∞ presents the limit distribution of the group parameter by the individual parameter.

In order to investigate the convergence of the Neumann series (4) in theHolder spaces, we make an estimate of the norm of operator A.

V. Boundedness of shift operators in the weighted Hölder spaces.

Let us begin with the shift operator $(B_\beta\varphi)(x) = \varphi[\beta(x)]$.

Theorem Operator B_β is bounded on the space $H_\mu(J)$,

$$\|B_\beta\|_{\mathcal{B}(H_\mu(J))} \le \left\|\beta'\right\|_C^\mu.$$

Operator B_β is bounded on the space $H^0_\mu(J,\rho)$,

$$\|B_\beta\|_{\mathcal{B}(H^0_\mu(J,\rho))} \le \left\|\frac{\rho}{\rho[\beta]}\right\|_{H_\mu(J)} \|B_\beta\|_{\mathcal{B}(H_\mu(J))}.$$

Let $\varphi \in H_\mu(J)$,

$$\|B_\beta\varphi\|_{H_\mu(J)} = \|B_\beta\varphi\|_C + \|B_\beta\varphi\|_\mu =$$

$$\|\varphi\|_C + \sup_{x_1\neq x_2} \frac{|\ \varphi[\beta(x_2)] - \varphi[\beta(x_1)]\ | \cdot |\ \beta(x_2) - \beta(x_1)\ |^\mu}{|\ x_2 - x_1\ |^\mu |\ \beta(x_2) - \beta(x_1)\ |^\mu} \le$$

$$\|\varphi\|_C + \sup_{x_1\neq x_2} \left|\frac{\beta(x_2) - \beta(x_1)}{x_2 - x_1}\right|^\mu \|\varphi\|_\mu.$$

From here, it follows that

$$\|B_\beta\|_{\mathcal{B}(H_\mu(J))} \le \max\left\{1, \sup_{x_1\neq x_2}\left|\frac{\beta(x_2) - \beta(x_1)}{x_2 - x_1}\right|^\mu\right\} =$$

$$\sup_{x_1\neq x_2}\left|\frac{\beta(x_2) - \beta(x_1)}{x_2 - x_1}\right|^\mu = \left\|\beta'\right\|_C^\mu.$$

Let $\varphi \in H^0_\mu(J,\rho)$; from

$$\|\frac{\rho}{\rho[\beta]}B_\beta(\rho\varphi)\|_\mu =$$

$$\sup_{x_1\neq x_2}\left|\frac{\frac{\rho(x_1)}{\rho[\beta(x_1)]}(B_\beta(\rho\varphi))(x_1) - \frac{\rho(x_2)}{\rho[\beta(x_2)]}(B_\beta(\rho\varphi))(x_2)}{(x_1 - x_2)^\mu}\right| =$$

$$\sup_{x_1\neq x_2}\left|\frac{\left(\frac{\rho(x_1)}{\rho[\beta(x_1)]} - \frac{\rho(x_2)}{\rho[\beta(x_2)]}\right)(B_\beta(\rho\varphi))(x_1) + (B_\beta(\rho\varphi))(x_1) -}{(x_1 - x_2)^\mu}\right|$$

$$\left|\frac{+((B_\beta(\rho\varphi))(x_1) - (B_\beta(\rho\varphi))(x_2))\left(\frac{\rho(x_2)}{\rho[\beta(x_2)]}\right)}{(x_1 - x_2)^\mu}\right| \le$$

$$\|\frac{\rho}{\rho[\beta]}\|_\mu \|B_\beta(\rho\varphi)\|_C + \|\frac{\rho}{\rho[\beta]}\|_C \|B_\beta(\rho\varphi)\|_\mu$$

and

$$\left\|\frac{\rho}{\rho[\beta]}\right\|_C \le \left\|\frac{\rho}{\rho[\beta]}\right\|_{H_\mu(J)},$$

it follows that

$$\|B_\beta\varphi\|_{H^0_\mu(J,\rho)} = \|\rho B_\beta\varphi\|_{H_\mu(J)} =$$

$$\|\frac{\rho}{\rho[\beta]}B_\beta(\rho\varphi)\|_C + \|\frac{\rho}{\rho[\beta]}B_\beta(\rho\varphi)\|_\mu \le$$

$$\|\frac{\rho}{\rho[\beta]}\|_C \|B_\beta(\rho\varphi)\|_C + \|\frac{\rho}{\rho[\beta]}\|_\mu \|B_\beta(\rho\varphi)\|_C +$$

$$\|\frac{\rho}{\rho[\beta]}\|_C \|B_\beta(\rho\varphi)\|_\mu \le$$

$$\|\frac{\rho}{\rho[\beta]}\|_{H_\mu(J)} \|B_\beta(\rho\varphi)\|_C + \|\frac{\rho}{\rho[\beta]}\|_C \|B_\beta(\rho\varphi)\|_\mu \le$$

$$\left\|\frac{\rho}{\rho[\beta]}\right\|_{H_\mu(J)} \|B_\beta(\rho\varphi)\|_{H_\mu(J)} \le$$

$$\left\|\frac{\rho}{\rho[\beta]}\right\|_{H_\mu(J)} \|B_\beta\|_{\mathcal{B}(H_\mu(J))} \|\rho\varphi\|_{H_\mu(J)} =$$

$$\left\|\frac{\rho}{\rho[\beta]}\right\|_{H_\mu(J)} \|B_\beta\|_{\mathcal{B}(H_\mu(J))} \|\varphi\|_{H^0_\mu(J,\rho)}.$$

Since $\frac{\rho(x)}{\rho[\beta(x)]} = \left|\frac{x}{\beta(x)}\right|^{\mu_0} \left|\frac{1-x}{1-\beta(x)}\right|^{\mu_1} \in H_\mu(J)$, we complete the proof.

VI. Analysis of the system's evolution.

Thus the operator $A = rI + bB_\beta$, with coefficients $r \in H_\mu(J),\ b \in H_\mu(J)$, is bounded on the space $H^0_\mu(J,\rho)$.

The operator

$$A = r[I + \frac{b}{r}B_\beta], \quad r(x) \neq 0$$

has the inverse operator when the norm

$$\|\frac{b}{r}B_\beta\|_{H^0_\mu(J,\rho)} < 1$$

is less than one and the inverse operator has a form [11]

$$A^{-1} = [I + \frac{b}{r}B_\beta]^{-1}\frac{1}{r},$$

$$[I + \frac{b}{r}B_\beta]^{-1} = I + K + K^2 + ... + K^n +,$$

where

$$K = -\frac{b}{r}B_\beta.$$

We continue the study of the limit state of the renewable system S.

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Neumann serie (4) is converged in $H^0_\mu(J,\rho)$, if the norm of the operator A is less than one,

$$\|rI + bB_\beta\|_{H^0_\mu(J,\rho)} < 1,$$

in this case [11]

$$I + A + A^2 + ... + A^n + ... = (I - A)^{-1}$$

and

$$\nu_\infty(x) = (I - A)^{-1} f(x)$$

Condition $\|A\|_{\mathcal{B}(H^0_\mu(J,\rho))} < 1$ is fulfilled, when

$$\|r\|_{H_\mu(J)} + \left\| b\frac{\rho}{\rho[\beta]} \right\|_{H_\mu(J)} \left\|\beta'\right\|^\mu_C < 1.$$

It follows from

$$\|A\|_{\mathcal{B}(H^0_\mu(J,\rho))} \leq \|rI\|_{\mathcal{B}(H^0_\mu(J,\rho))} + \|bB_\beta\|_{H^0_\mu(J,\rho)},$$

$$\|rI\|_{\mathcal{B}(H^0_\mu(J,\rho))} < \|r\|_{H_\mu(J)}, r \neq 0$$

and the theorem:

$$\|bB_\beta\|_{H^0_\mu(J,\rho)} \leq \left\| b\frac{\rho}{\rho[\beta]} \right\|_{H_\mu(J)} \|B_\beta\|_{\mathcal{B}(H_\mu(J))},$$

$$\|B_\beta\|_{\mathcal{B}(H_\mu(J))} \leq \left\|\beta'\right\|^\mu_C.$$

The limit state, when $\|A\|_{H^0_\mu(J,h)} < 1$, does not depend on the initial reserve of the resource at the time $t = t_0$, $\nu(x, t_0)$.

The proposed approach can be applied to the modelling of more complex systems. One example would be systems with several interconnected resources, the modelling of which leads to integral and differential matrix balance equations.

VII. Conclusion

The theory of linear functional operators with shift is the adequate mathematical instrument for the study of renewable systems. In this work, we present models of systems with a set of renewable resources, a mathematical model for the study of the changes of the individual parameters and open models in which the final state does not coincide with the initial state. Applying this theory, the system's evolution is analyzed. Based on the presented models, it is possible to formulate and analyse some problems for the rational use of resources.

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