Numerical Study of the Effect of Measurement Noise on the Accuracy of Bridge Parameter Estimation in VBI System Identification

Shin Ryota, Inoue Jun, Okada Yukihiko and Yamamoto Kyosuke, *Member, IAENG*

***Abstract*— In this study, we propose a method to simultaneously estimate a vehicle's parameters and a bridge by estimating the road surface roughness of the front and rear wheels from the vibration data of a running vehicle and minimizing the difference between them. It is possible to estimate the vehicle, bridge, and road surface unevenness simultaneously from vehicle vibration. In this study, we focused on the shape of the objective function of the bridge parameters. We found that the objective function of the bending stiffness is always downward convex. In contrast, the unit weight of the bridge is multimodal.**

***Index Terms*— structure health monitoring , vehicle response analysis, vehicle-bridge Interaction System**

I. Introduction

DUE to the aging of infrastructure structures related to logistics networks, the development of maintenance techniques for road bridges has become an urgent issue. Therefore, bridge monitoring methods using signals have been proposed. They can be classified into bridge response analysis and vehicle response analysis. Bridge response analysis uses sensors mounted directly on the bridge, enabling accurate investigation of the bridge's health, but is expensive. In vehicle response analysis[1], sensors are mounted only on the vehicle. The acceleration response obtained when the vehicle runs over an infrastructure structure can be used for indirect damage detection. Therefore, it can be implemented at a relatively low cost.

Nagayama et al.[2] proposed a method for estimating the road profiles of the front and rear wheels by measuring the rigid behavior of the vehicle using a smartphone and combining the Kalman filter, RTS (Rauch-Tung-Striebel) smoothing, and the Robbins-Monro algorithm. Assuming that the same road profile is input to the front and rear wheels, the genetic algorithm is used to simultaneously identify the vehicle's parameters so that the difference between the estimated road profiles of the front and rear wheels is minimized.

Murakami[3] proposes a method for simultaneously estimating the vehicle, bridge, and road surface from the vibration data of multiple vehicles. The road profile input to each of the multiple vehicles is estimated from the vehicle vibration alone. The combination of vehicle and bridge parameters that minimizes the error in the obtained road profile is searched. Since the shape of the objective function is unknown, the vehicle and bridge parameters and the road surface roughness are estimated based on particle swarm optimization(PSO). This method's features are: 1) only vehicle vibration is used without bridge vibration, and 2) not only vehicle parameters, and road surface profile but also bridge parameters are estimated simultaneously. However, the actual vehicle vibration is greatly influenced by the engine vibration. Besides, since the time that multiple vehicles travel on a bridge simultaneously is short, it is more practical to use a single vehicle as in Nagayama et al.[2]

In this study, we improve the Murakami model's estimation method by constructing a model that considers the effects of external disturbances such as engine vibration. Also, we clarify the shape of the objective function of the bridge parameters estimated by this method. The effect of measurement noise on each parameter's estimated values is determined, and its influence on the objective function is clarified. The experiments were conducted using numerical experiments.

II. BASIS THEORY OF THE PROPOSED METHOD

A. *Vehicle-Bridge Interaction (VBI)*

VBI is a phenomenon that occurs when a vehicle runs over a bridge. It is caused by the vehicle and bridge systems' equations of motion, the contact force between the vehicle and the bridge, and the unevenness of the road surface. When a vehicle enters a bridge, the unevenness of the road surface first shakes the vehicle. Then, the vehicle shakes the bridge. In addition, the shaken bridge and the uneven road surface shake the vehicle in succession. The conceptual diagram of VBI is shown in Fig. 1.

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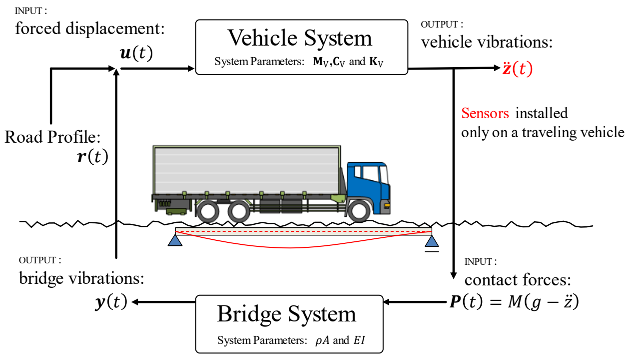

Fig.1 Conceptual image of Vehicle-Bridge Interaction

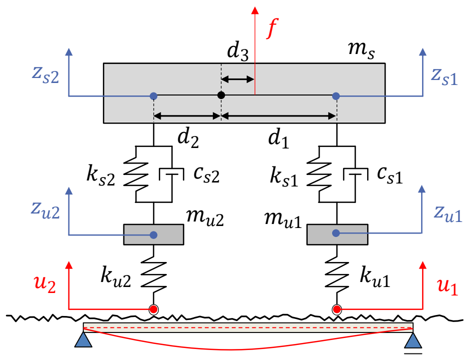

Fig. 2. Dynamical model diagram assumed in this study

B. *Vehicle System*

In this study, the vehicle model is the half-car model shown in Fig. 2. A rigid body of mass m_s is used as the vehicle body, connected to the ground by a suspension modeled as a spring and dampers (spring constant: k_{si}, damping: c_{si}) and tires modeled as a single spring (spring constant: k_{ui}).Here, i represents the axle, and the front wheels are 1, and the rear wheels are 2. The excitation force due to engine vibration is represented by f. Between the suspension and the tires, there is a mass m_{ui}, which is called the unsprung mass, while the car's body is called the sprung mass. Since the sprung mass is a rigid body, it can be expressed by two equations of motion, one for translation and one for rotation. The vertical displacement vibration at the center of gravity on the spring is $z_G(t)$, the rotation is $\theta_G(t)$, the displacement vibration at the front and rear wheel positions is $z_{si}(t)$. The displacement vibration under the spring is $z_{ui}(t)$.The first-order time derivative is denoted by $(\dot{\ })$ and the second-order time derivative by $(\ddot{\ })$.The distance from the center of gravity to the front and rear wheels is d_i, and the distance to the engine is d_3.The equations of motion for the vehicle model are then as follows.

TABLE I
THE VEHICLE SYSTEM PARAMETERS

Mass: m_s	9.00×10^3	[kg]
Unsprung-Mass: m_{u1}, m_{u2}	5.00×10^2	[kg*m^2]
Damping (Sprung-mass): c_{s1}, c_{s2}	2.00×10^3	[kg/s]
Stiffness(Sprung-mass): k_{s1}, k_{s2}	4.50×10^3	[kg/s^2]
Stiffness(Upsprung-mass): k_{u1}, k_{u2}	6.00×10^4	[kg/s^2]
Distance between axles: d_1+d_2	3.00	[m]

$$m_s\ddot{z}_G(t) = f - c_{s1}\left(\dot{z}_{s1}(t) - \dot{z}_{u1}(t)\right) - c_{s2}\left(\dot{z}_{s2}(t) - \dot{z}_{u2}(t)\right) - k_{s1}\left(z_{s1}(t) - z_{u1}(t)\right) - k_{s2}\left(z_{s2}(t) - z_{u2}(t)\right) \tag{1}$$

$$I_s\ddot{\theta}_G(t) = d_3 f - d_1 \times c_{s1}\left(\dot{z}_{s1}(t) - \dot{z}_{u1}(t)\right) + d_2 \times c_{s2}\left(\dot{z}_{s2}(t) - \dot{z}_{u2}(t)\right) - d_1 \times k_{s1}\left(z_{s1}(t) - z_{u1}(t)\right) + d_2 \times k_{s2}\left(z_{s2}(t) - z_{u2}(t)\right) \tag{2}$$

$$m_{ui}\ddot{z}_{ui}(t) = c_{si}\left(\dot{z}_{si}(t) - \dot{z}_{ui}(t)\right) + k_{si}\left(z_{si}(t) - z_{ui}(t)\right) - k_{ui}\left(z_{ui}(t) - u_i(t)\right) \tag{3}$$

Here, the displacement and rotation at the center of gravity of the rigid body are expressed using the displacements at the front and rear wheel positions.

$$z_G(t) = \frac{d_2 z_{s1}(t) + d_1 z_{s2}(t)}{d_1 + d_2} \tag{4}$$

$$\theta_G(t) = tan\left(\frac{z_{s1}(t) - z_{s2}(t)}{d_1 + d_2}\right) \cong \frac{z_{s1}(t) - z_{s2}(t)}{d_1 + d_2} \tag{5}$$

Equation (1) through Equation (5) can be summarized as follows.

$$\mathbf{M}_\mathrm{v}\ddot{\boldsymbol{z}}(t) + \mathbf{C}_\mathrm{v}\dot{\boldsymbol{z}}(t) + \mathbf{K}_\mathrm{v}\boldsymbol{z}(t) = \boldsymbol{f}_\mathrm{v}(t) \tag{6}$$

$$\mathbf{M}_\mathrm{v} = \begin{bmatrix} \frac{d_2 m_s}{d_1 + d_2} & \frac{d_1 m_s}{d_1 + d_2} & & \\ \frac{I}{d_1 + d_2} & \frac{I}{d_1 + d_2} & & \\ & & m_{u1} & \\ & & & m_{u2} \end{bmatrix} \tag{7}$$

$$\mathbf{C}_\mathrm{v} = \begin{bmatrix} c_{s1} & c_{s2} & -c_{s1} & -c_{s2} \\ d_1 c_{s1} & -d_2 c_{s2} & -d_1 c_{s1} & d_2 c_{s2} \\ -c_{s1} & & c_{s1} & \\ & -c_{s2} & & c_{s2} \end{bmatrix} \tag{8}$$

$$\mathbf{K}_\mathrm{v} = \begin{bmatrix} k_{s1} & k_{s2} & -k_{s1} & -k_{s2} \\ d_1 k_{s1} & -d_2 k_{s2} & -d_1 k_{s1} & d_2 k_{s2} \\ -k_{s1} & & k_{s1} + k_{u1} & \\ & -k_{s2} & & k_{s2} + k_{u2} \end{bmatrix} \tag{9}$$

$$\boldsymbol{z}(t) = [z_{s1}(t), z_{s2}(t), z_{u1}(t), z_{u2}(t)]^T \tag{10}$$

$$\boldsymbol{f}_\mathrm{v}(t) = [f(t), d_3 f(t), k_{u1}u_1(t), k_{u2}u_2(t)]^T \tag{11}$$

$\mathbf{M}_\mathrm{v}$, $\mathbf{C}_\mathrm{v}$ and $\mathbf{K}_\mathrm{v}$ are the mass, damping, and composite matrices of the vehicle. $\boldsymbol{f}_\mathrm{v}$ is the external force term, which consists of the excitation force f due to engine vibration and the input displacement $\boldsymbol{u}$ to the tire. The parameters of the vehicle are shown in TABLE I.

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C. Bridge System

The bridge is assumed to be a simple one-dimensional beam. Suppose the vibration of the bridge is $y(x,\ t)$. In that case, the bending stiffness is EI, the mass per unit length is ρA, and the external force is p, the equation of motion of the bridge is expressed by equation (12).

$$\rho A\ddot{y}(x,\ t)+\frac{\partial^2}{\partial x^2}EI\left(\frac{\partial^2}{\partial x^2}y(x,\ t)\right)=p \tag{12}$$

$$p=\sum_{i=1}^{2}\delta\big(x-x_i(t)\big)P_i(t) \tag{13}$$

Here, the external force p is the concentrated external force in the vertical direction at $x_i(t)$ for each axle position of the front and rear wheels, and $P_i(t)$ is the concentrated external force in the vertical direction. However, $\delta\ x$ is Dirac's delta function and satisfies the following conditions.

$$\delta\ x\ =\begin{cases}0 & \text{at } x\neq 0\\ \infty & \text{at } x=0\end{cases} \tag{14}$$

$$\int_{-\infty}^{\infty}\delta\ x\ f\ x\ \ \mathrm{d}x=f\ 0 \tag{15}$$

The weighted residue equation in Eq. (12) is given by Eq. (16) below.

$$\int_0^L\omega\left(\rho A\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}-p\right)\mathrm{d}x=0 \tag{16}$$

Let ω denote the weights. We change equation (16) to the weak formulation.

$$\int_0^L\left(\rho A\omega\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^2\omega}{\partial x^2}\frac{\partial^2 y}{\partial x^2}-p\right)\mathrm{d}x=0 \tag{17}$$

A one-dimensional finite element beam model with Hermite interpolation function is applied to numerically calculate the bridge vibration. The Hermite interpolation function can be defined for an element coordinate system s (normalized to $-1\leq s\leq 1$).

$$\begin{cases}\phi_1\ s\ =\dfrac{1}{4}\ s-1\ ^2\ s+2\\ \phi_2\ s\ =\dfrac{1}{4}\ s-1\ ^2\ s+1\\ \phi_3\ s\ =\dfrac{1}{4}\ s+1\ ^2\ s-2\\ \phi_4\ s\ =\dfrac{1}{4}\ s+1\ ^2\ s-1\end{cases} \tag{18}$$

When the whole coordinate system x is inside a beam element j consisting of nodes x_j and x_{j+1}, where $\Delta x=x_{j+1}-x_j$, the components of the basis function vector $\boldsymbol{N}\ x$ of the interpolation are expressed as follows.

$$\begin{cases}N_{2j-1}\ x\ =\phi_1\ s\\ N_{2j}\ \ \ x\ =\phi_2\ s\\ N_{2j+1}\ x\ =\phi_3\ s\\ N_{2j+2}\ x\ =\phi_4\ s\end{cases} \tag{19}$$

Both components are assumed to be zero outside the element. The component of the deformation vector $\boldsymbol{y}\ t$ at the node x_j is given by equation (20) using the deflection $y(x_j,t)$ and the deflection angle $\theta(x_j,t)$.

$$\begin{cases}y_{2\ j-1\ +1}\ t\ =y(x_j,t)\\ y_{2\ j-1\ +2}\ t\ =\theta(x_j,t)\end{cases} \tag{20}$$

In this case, the approximate solution of the solution y(x,t) is expressed by equation (21).

$$y\ x,t\ =\boldsymbol{N}\ x\ \cdot\boldsymbol{y}\ t \tag{21}$$

The weights can be similarly transformed into equation (22) by substituting $\omega(x)=\boldsymbol{N}(x)\cdot\boldsymbol{\omega}$ into the weighted residue equation.

$$\boldsymbol{\omega}^{\mathrm{T}}\big(\mathbf{M}_{\mathrm{B}}\ddot{\boldsymbol{y}}(t)+\mathbf{K}_{\mathrm{B}}\boldsymbol{y}(t)-\boldsymbol{F}(t)\big)=0 \tag{22}$$

$\mathbf{M}_{\mathrm{B}}$ and $\mathbf{K}_{\mathrm{B}}$ refer to the mass and composite matrix of the bridge, respectively. $\boldsymbol{y}(t)$ is a vector of deformations with components of deflection and deflection angle at each node. $\boldsymbol{F}(t)$ is a vector of external forces with concentrated external force components and moment load of force at each node. The equality condition of equation (15) is obtained for any $\boldsymbol{\omega}$, and the finite element equation shown below is obtained by introducing a damping term.

$$\mathbf{M}_{\mathrm{B}}\ddot{\boldsymbol{y}}+\mathbf{C}_{\mathrm{B}}\dot{\boldsymbol{y}}+\mathbf{K}_{\mathrm{B}}\boldsymbol{y}=\boldsymbol{F}(t) \tag{23}$$

The parameters of the bridge are shown in TABLE II.

TABLE II
THE BRIDGE SYSTEM PARAMETERS

Length	30	[m]
Number of Elements	6	
Mass per unit length values of all elements: ρA	3000	[kg/m]
Flexual Rigidities of all elements: EI	1.56×10^{11}	[N$\times$m^2]

D. Vehicle Bridge Interaction System

In general, the response of a vehicle and a bridge is modeled by their outputs with each other as inputs. First, the vehicle vibration is determined using only the road profile. Then, the bridge vibration is calculated by determining the input load to the bridge from the vehicle vibration and the vehicle model. The bridge vibration is then added to the road profile to obtain the input displacement to the vehicle. By repeating this process, the input displacements to the vehicle and bridge are obtained. The input profile and ground force, which are the vehicle and bridge inputs, are described below.

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D1. Input profile

The input profile u(t) in this study is given by the sum of the road surface profile r(t) and the bridge profile $\tilde{y}(t)$ and is expressed by equation (24).

$$\boldsymbol{u}(t) = \boldsymbol{r}(t) + \widetilde{\boldsymbol{y}}(t) \tag{24}$$

Here, the road profile is the unevenness of the road surface at the axle position. When the road surface roughness is $R(x)$, and the axle position is $x_j(t)$, equation (25) is obtained.

$$r_j(t) = R\left(x_j(t)\right) \tag{25}$$

$r_j(t)$ is a component of $\boldsymbol{r}(t)$.On the other hand, the bridge profile is the bridge vibration $\tilde{y}_j(t) = y\left(x_j(t), t\right)$ at the axle position $x_j(t)$. The bridge vibration $\boldsymbol{y}(t)$ is the vector of deformations at each node. Therefore, the basis functions used in the finite element method are used to obtain the bridge displacement $\tilde{y}_j(t)$ at the axle position $x_j(t)$. Using the transformation matrix $\boldsymbol{L}(t)$, the bridge vibration at the axle position is represented by Equation (26), and $\boldsymbol{L}(t)$ is shown in Equation (27).

$$\widetilde{\boldsymbol{y}}(t) = \boldsymbol{L}^{\mathrm{T}}(t)\boldsymbol{y}(t) \tag{26}$$

$$L_{kj}(t) = \begin{cases} N_k\left(x_j(t)\right) & (k = 2i-1) \\ N_k\left(x_j(t)\right) & (k = 2i \qquad) \\ N_k\left(x_j(t)\right) & (k = 2i+1) \\ N_k\left(x_j(t)\right) & (k = 2i+2) \end{cases} \tag{27}$$

D2. Contact force

The ground force, which is the bridge's input, corresponds to the elastic force acting on the spring with tire stiffness k_{uj}. However, in the equation of motion of the vehicle (Equation (6)), the gravity term disappears because the equation is based on a balanced position. However, in calculating the elastic force, the effect of gravity must be taken into account because it is set to zero when the vehicle is at its natural length. Noting that the center of the vehicle's rotation is the center of gravity, the front and rear wheels' ground forces can be written as equation (28).

$$\begin{aligned} V_1(t) &= \frac{d_2 m_s}{d_1 + d_2}(g - \ddot{z}_{s1}) + m_{u1}(g - \ddot{z}_{u1}) \\ V_2(t) &= \frac{d_1 m_s}{d_1 + d_2}(g - \ddot{z}_{s2}) + m_{u2}(g - \ddot{z}_{u2}) \end{aligned} \tag{28}$$

The external force vector acting on the bridge is Equation (30), where $\boldsymbol{H}(t)$ represents the fulcrum reaction force.

$$\boldsymbol{F}(t) = \boldsymbol{L}(t)[V_1(t) \quad V_2(t)] + \boldsymbol{H}(t) \tag{30}$$

E. Implementation of numerical simulation

In this study, vehicle vibration data during driving is numerically reproduced to investigate whether the mechanical parameters can be estimated from vehicle vibration alone by the proposed method. The VBI system is modeled using a rigid-Bodies-Spring Model (RBSM) vehicle and a one-dimensional Finite Element Method bridge. In the numerical experiments, the vehicle model and the bridge model are separated. Their respective inputs are obtained using the Newmark-β method and the iterative method.

E1. Newmark-β method

The vehicle vibration $\boldsymbol{z}(t)$ and bridge $\boldsymbol{y}(t)$ vibration are obtained by applying the Newmark-β method to the respective equations of motion (Equations (6) and (23)). The equations of motion are expressed in the MCK system as follows.

$$\mathbf{M}\ddot{\boldsymbol{s}}(t) + \mathbf{C}\dot{\boldsymbol{s}}(t) + \mathbf{K}\boldsymbol{s}(t) = \boldsymbol{Q}(t) \tag{31}$$

The time function $\boldsymbol{s}(t)$ is discretized to $\boldsymbol{s}_k$, where $\boldsymbol{s}_k$ is the displacement response of the bridge or vehicle, and Δt is the time increment.

$$\boldsymbol{s}_k = \boldsymbol{s}(k\Delta t) \tag{32}$$

In this study, the update equations of the Newmark-β method are assumed to be equations (33) and (34).

$$\dot{\boldsymbol{s}}_k = \dot{\boldsymbol{s}}_{k-1} + (1-\gamma)\Delta t\ddot{\boldsymbol{s}}_{k-1} + \gamma\Delta t\ddot{\boldsymbol{s}}_k \tag{33}$$

$$\begin{aligned} \boldsymbol{s}_k = \boldsymbol{s}_{k-1} + \Delta t\dot{\boldsymbol{s}}_{k-1} + \left(\frac{1}{2} - \beta\right)\Delta t^2\ddot{\boldsymbol{s}}_{k-1} \\ + \beta(\Delta t)^2\ddot{\boldsymbol{s}}_k \end{aligned} \tag{34}$$

The vehicle and the bridge's vibrations are calculated from their respective equations of motion using the following equations.

$$\ddot{\boldsymbol{s}}_k = \mathbf{A}^{-1}\boldsymbol{b}_k \tag{35}$$

$$\mathbf{A} = \left[\mathbf{M} + \frac{\Delta t}{2}\mathbf{C} + \frac{(\Delta t)^2}{4}\mathbf{K}\right] \tag{36}$$

$$\boldsymbol{b}_k = \boldsymbol{Q}_k + \mathbf{C}\boldsymbol{b}_1 + \mathbf{K}\boldsymbol{b}_2 \tag{37}$$

$$\boldsymbol{Q}_k = \boldsymbol{Q}(k\Delta t) \tag{38}$$

$$\boldsymbol{b}_1 = -\dot{\boldsymbol{s}}_{k-1} - \frac{\ddot{\boldsymbol{s}}_{k-1}}{2}\Delta t \tag{39}$$

$$\boldsymbol{b}_2 = -\boldsymbol{s}_{k-1} - \dot{\boldsymbol{s}}_{k-1}\Delta t - \frac{\ddot{\boldsymbol{s}}_{k-1}}{4}(\Delta t)^2 \tag{40}$$

The vehicle's speed is constant at 10 [m/s], and it runs on the road surface for 8 seconds. If the starting point of the vehicle's frontal area position is -20[m], the bridge is between 0[m] and 30[m]. Fig. 3 and Fig. 4 show the behavior of the vehicle and the bridge obtained from the numerical simulation.

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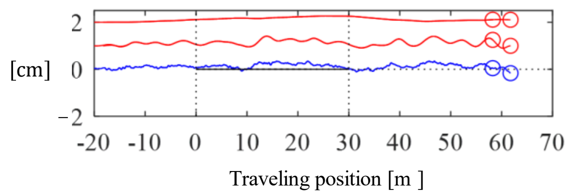

Fig.3 Unsprung, sprung displacement and input profile

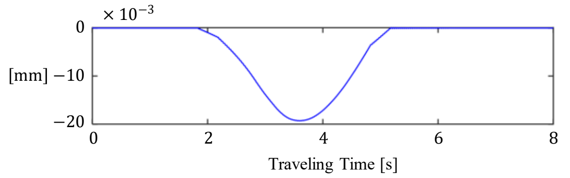

Fig.4 Displacement of the bridge (center section)

F. *VBI System Identification*

Assume that the vehicle vibration $\ddot{\boldsymbol{z}}(t)$ has been obtained as measured data. By substituting the vehicle vibration data and randomly assumed initial values of the vehicle and bridge parameters into the VBI system, the road profile $\boldsymbol{r}(t)$ can be obtained. First, the measured vehicle vibration data, $\ddot{\boldsymbol{z}}(t)$, is substituted into the Newmark-β method to obtain $\dot{\boldsymbol{z}}(t)$ and $\boldsymbol{z}(t)$. Assuming that the vehicle's system parameters are random, $\mathbf{M}_{\mathrm{v}}$, $\mathbf{C}_{\mathrm{v}}$, and $\mathbf{K}_{\mathrm{v}}$ can also be obtained. At this point, all the left-hand sides of Equation (1), the equation of motion of the vehicle, have been obtained, and $\boldsymbol{f}_{\mathrm{v}}(t)$ can be obtained. In equation (11), which expresses $\boldsymbol{f}_{\mathrm{v}}(t)$, d_1, d_2, k_{u1}, and k_{u2} in the equation have already been assumed, so f, d_3, u_1, and $u_2$can be obtained. In other words, the excitation force f and the input profile $\boldsymbol{u}(t)$ due to engine vibration are estimated.

Next, the vehicle vibration data $\ddot{\boldsymbol{z}}(t)$ and the assumed vehicle system parameters are substituted into Equation (28) to obtain the ground forces $V_1(t)$ and $V_2(t)$. If the system parameters of the bridge are also assumed to be random, the bridge vibration $\boldsymbol{y}(t)$ can be obtained using Equation (23), which is the equation of motion of the bridge, and the Newmark-β method. The obtained bridge vibration $\boldsymbol{y}(t)$ can be substituted into equation (26) to obtain the bridge profile $\tilde{\boldsymbol{y}}(t)$.

Now, subtracting $\tilde{\boldsymbol{y}}(t)$ from $\boldsymbol{u}(t)$ obtained earlier, $\boldsymbol{r}(t)$ can be estimated. Furthermore, the road surface unevenness $r_i(t)$ is position-synchronized with the axle position $x_i(t)$, and the road surface unevenness $R_i(x)$ $\left(R_i\big(x_i(t)\big) = r_i(t)\right)$. When the vehicle goes straight, the front and rear wheels follow the same path, so the road surface irregularities $R_1(x)$ and $R_2(x)$ are equal. However, in this estimation process, the vehicle and bridge parameters are assumed randomly. The values of $R_1(x)$ and $R_2(x)$ are expected to be different.

F1. *The error function*

Consider the optimization problem of minimizing the squared error of $R_1(x)$ and $R_2(x)$. The objective function is shown in Equation (30).

$$J = \sum |R_1(x) - R_2(x)|^2 \tag{30}$$

If all the randomly assumed parameters are correct, the two calculated road surface roughness will match. Therefore, if we can update the parameters so that the calculated road surface roughness matches, we can expect that the parameters will eventually approach the correct values. In this study, we refer to such a parameter identification method as "VBI system identification". However, the shape of the objective function is not known. Besides, it is not always the case that there is only one combination of system parameters for the vehicle and the bridge when the road surface roughness estimated for the front and rear wheels coincides. Therefore, the shape of the objective function is checked by selecting one parameter and varying its value. This method is equivalent to estimating the optimization parameters in a brute force fashion in system identification. Although this method is computationally expensive, it is a reliable way to search for the correct combination of values. Next, the VBI system identification is also performed using the gradient descent method, which is the simplest and least computationally expensive method. Finally, the results are compared and discussed.

F2. *Gradient descent method*

In this study, the gradient descent method is used to search for the optimal solution of the parameters. The gradient descent method is one of the gradient method algorithms for the continuous optimization problem to find the minimum value of a function. In the gradient descent method, the parameters are brought closer to the solution using an iterative method; when the solution is at h_k in the k th iteration, the position of the parameters in the $k+1$ th iteration is expressed by equation (31).

$$h_{k+1} = h_k - J_{k-1}\frac{h_k - h_{k-1}}{J_k - J_{k-1}} \tag{31}$$

III. Results and Discussion

Numerical simulations are performed based on TABLE I and TABLE II parameters to obtain the vehicle vibration. Vary EI from 0.5 to 1.5 times the correct value, and vary ρA from 1 to 2 times the correct value, to find the value of the objective function J and examine the shape of each parameter's error function. The shape of the error function for each parameter is examined. The model in this study is a multivariate function. However, for the sake of clarity, the results are presented as a single variable.

A. *Shape of the error function without noise*

The results of the error functions obtained for different bridge parameters (EI, ρA) are shown in Fig. 5 to Fig. 6. The red dotted line is the correct value for each parameter. The red circle is the minimum value of the error function, i.e., the optimal solution when calculated by the brute force method. EI has six parameters, but there is no significant difference in each of them, so the objective function of onlyEI_1 is visualized as a representative value. It can be seen that the stiffness EI of the bridge is convex downward and there is no multimodality. Furthermore, the optimal solution (the minimum value of the error function) was consistent with the correct value. However, the mass per unit length of the bridge, ρA, was convex near the correct solution and resembled a parabola but was multimodal in a broader range. Therefore, the optimal solution for the

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stiffness EI of the bridge can be found by a simple method such as the gradient descent method. In order to find ρA, other optimization methods may need to be combined.

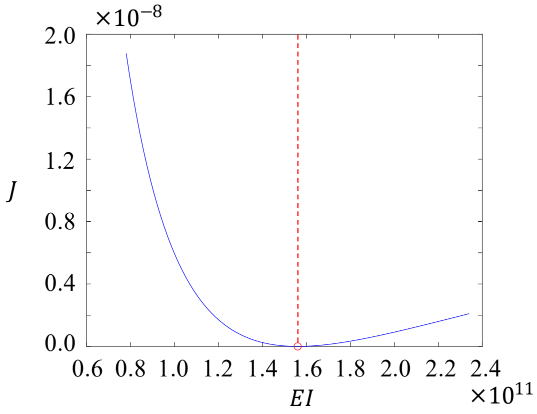

Fig.5 Objective function shape of EI_1

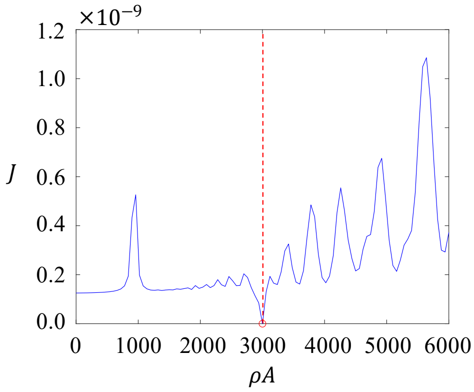

Fig.6 Objective function shape of ρA

B. *Effect of added noise on the error function*

Next, we examine the case where noise is added to the vehicle vibration. Since this study's noise is supposed to be the engine vibration and observation noise, it is assumed that all sensors are subjected to the same magnitude of noise. Therefore, we used a brute force equation to find the optimal solution when 0.01 percent of noise (uniform random number) is added to the vehicle vibration's maximum amplitude under the spring. For each parameter (EI, ρA), the VBI parameter identification results for each noise are shown in Fig. 7 to Fig. 8. It was found that the optimal solution obtained by adding noise deviated from the correct value. However, even with the addition of noise, the shape of EI's objective function is convex.

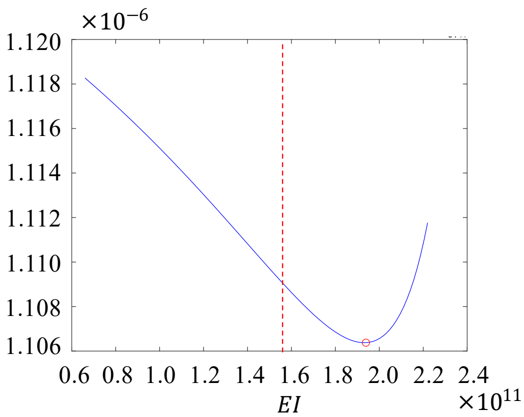

Fig.7 Objective function shape of EI_1 (noise is added)

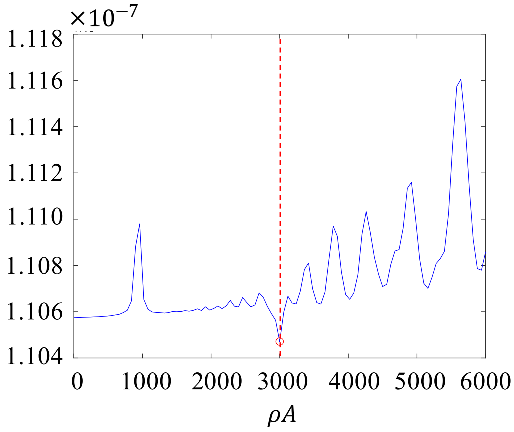

Fig.8 Objective function shape of ρA (noise is added)

On the other hand, the estimated value of ρA was the correct value even when noise was added.

IV. Summary and Future Issues

In this study, we developed a model that took into account the disturbance and clarified the effect of measurement noise on the bridge parameter estimates in VBI system identification from the objective function's shape. The parameters were obtained in a brute force fashion. The results showed that the shape of the EI was convex downward. Also, the vertices match the correct values. Therefore, it is suggested that EI can be estimated by a simple method such as finding the minimum value of the error function.

In addition, ρA was found to be multimodal. Therefore, parameter optimization using only the gradient descent method cannot reach all parameters' correct values. To solve this problem, it is necessary to improve parameter estimation methods' accuracy and efficiency by combining other optimization methods such as PSO and MCMC (Markov chain Monte Carlo) methods, as Murakami has done.

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