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Basic Research on the Bending Problem of Kirchhoff-Love Plates Using Hermite Elements

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***Abstract*—A new Hermite basis function was proposed for the pure bending problem of Kirchhoff-Love bent plate elements. Finite element analysis was performed on a square flat plate model. The results were not as good as the existing known basis functions.**

***Index Terms*— FEM, Kirchhoff-Love plates, Hermite elements, The basis function**

I. Introduction

FEM (Finite Element Method) is the most popular method of numerical simulation for solving partial differential equations. Solutions of partial differential equations are always continuous functions, while numerical schemes treat them as discrete. The discretization process in FEM is realized by interpolation using a Lagrangian or Hermite basis. The former is a preferred choice in many FEM codes but has many problems in terms of computational accuracy and stability. On the other hand, despite its complications, the latter is an effective way to avoid these problems and to improve results. However, the study of Hermitian bases in high-dimensional spaces has not been very active. Thus, this study aims to find new Hermitian bases and examines their performances.

In this study, Kirchhoff-Love Plate Problem is solved by using a traditional base [1] and new Hermitian bases, respectively, and the obtained solutions are compared with the analytical solution. The performance of each basis is evaluated by errors from the analytical solution.

II. Theory

1. Kirchhoff-Love Plate Theory

Kirchhoff-Love Plate is a plate satisfying the following three conditions:

1) A line segment that was perpendicular to the neutral plane before deformation is kept perpendicular after deformation.
2) The length of the line segment does not change after deformation.
3) The normal stress acting on the surface parallel to the neutral plane can be ignored.

From these assumptions, parallel displacements can be expressed by the differential of vertical displacement. Let the displacements of x-, y- and z-directions be $U(x,y,z)$, $V(x,y,z)$ and $W(x,y,z)$, respectively.

$$U(x,y,z) = -z\frac{\partial w}{\partial x} \tag{1}$$

$$V(x,y,z) = -z\frac{\partial w}{\partial y} \tag{2}$$

$$W(x,y,z) = w(x,y) \tag{3}$$

where w indicates the vertical displacement of the neutral plane. The neutral plane is at $z = 0$.

The components of strain tensor are as follows:

$$\varepsilon_x = \frac{\partial U}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} \tag{4}$$

$$\varepsilon_y = \frac{\partial V}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} \tag{5}$$

$$\gamma_{xy} = \frac{1}{2}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right) = -\frac{z}{2}\frac{\partial^2 w}{\partial x \partial y} \tag{6}$$

In the Kirchhoff-Love theorem, $\varepsilon_z = \gamma_{yz} = \gamma_{zx} = 0$. Let Young's modulus and Poisson's ratio be E and ν, respectively. The components of stress tensor can be related to the components of strain tensor as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & \\ \nu & 1 & \\ & & 1-\nu \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{7}$$

The bending moments tensor are as follows:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2}\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z\,\mathrm{d}z \tag{8}$$

Let shear forces of x-, y-directions and load be Q_x, Q_y, and q, respectively. Then, the equations of equilibrium are shown as follows:

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0 \tag{9}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \tag{10}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \tag{11}$$

In summary, the following equation is obtained.

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q \tag{12}$$

Let the flexural rigidity of the plate be $D(= Eh^3/12(1+\nu)(1-\nu))$. Putting all the equations together, the equation becomes

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$$D\left\{\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}\right)+\nu\ \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2}\right)\right. \\ +2(1-\nu)\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial^2 w}{\partial x\partial y}\right) \\ +\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}\right) \\ \left.+\nu\ \frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2}\right)\right\}=q \quad (13)$$

When ω, S and R denote the weight, surface area, and residual, respectively, the Galerkin Method can be written as follows:

$$\int_S \omega R \mathrm{d}S = 0 \quad (14)$$

When the deflection w is approximated as $w=\boldsymbol{w}\cdot\boldsymbol{N}$, by substituting the basis function $\boldsymbol{N}$ and Eq. (13) into R, Eq. (14) can be rewritten as

$$\left[\int_{\boldsymbol{S}} D(\mathbf{K}_1+\mathbf{K}_2+\mathbf{K}_3+\mathbf{K}_4+\mathbf{K}_5)\,\mathrm{d}\boldsymbol{A}\right]\ \boldsymbol{w} \\ =\int_{\boldsymbol{S}} \boldsymbol{N}\boldsymbol{q}\ \mathrm{d}\boldsymbol{A} \quad (15)$$

where

$$\mathbf{K}_1=\frac{\partial^2\boldsymbol{N}}{\partial x^2}\frac{\partial^2\boldsymbol{N}^T}{\partial x^2} \\ \mathbf{K}_2=\nu\frac{\partial^2\boldsymbol{N}}{\partial x^2}\frac{\partial^2\boldsymbol{N}^T}{\partial y^2} \\ \mathbf{K}_3=2(1-\nu)\frac{\partial^2\boldsymbol{N}}{\partial x\partial y}\frac{\partial^2\boldsymbol{N}^T}{\partial x\partial y} \\ \mathbf{K}_4=\frac{\partial^2\boldsymbol{N}}{\partial y^2}\frac{\partial^2\boldsymbol{N}^T}{\partial y^2} \\ \mathbf{K}_5=\nu\frac{\partial^2\boldsymbol{N}}{\partial y^2}\frac{\partial^2\boldsymbol{N}^T}{\partial x^2} \quad (16)$$

Summarizing with the stiffness matrix $\mathbf{K}$, vector $\boldsymbol{w}$, and external force vector $\boldsymbol{f}$, the simultaneous equation $\mathbf{K}\boldsymbol{w}=\boldsymbol{f}$ is obtained.

2. About Hermite Elements

The normalized coordinate system of elements is ξ (-1≤ ξ ≤1). The condition of the basis functions H is expressed as follows:

$$H_{2i-1}(\xi_j)=\delta_{ij},\quad \frac{\partial H_{2i}}{\partial\xi}(\xi_j)=\delta_{ij} \quad (17)$$

where δ_{ij} is Kronecker's delta, and i and j both take the values 1 and 2. The nodal positions are $\xi_1=-1$ and $\xi_2=1$. The Hermite basis function satisfying the condition can be obtained as follows:

$$H_1=\frac{1}{4}(\xi-1)^2(\xi+2) \\ H_2=\frac{1}{4}(\xi-1)^2(\xi+1) \\ H_3=\frac{1}{4}(\xi+1)^2(\xi-2) \\ H_4=\frac{1}{4}(\xi+1)^2(\xi-1) \quad (18)$$

We will assume that there are two nodes x_1 and x_2 in the global coordinate system x. The displacements at the nodes are u_1 and u_2, and the rotation angles are θ_1 and θ_2. The displacement and rotation angle are given as $u(x)$ and $\theta(x)=\partial u/\partial x$, respectively. $u(x)$ and $\theta(x)$ can be approximated by the Hermite basis function as follows.

$$u(x)=\boldsymbol{N}(x)\cdot\boldsymbol{u}=\mathbf{A}\boldsymbol{H}(\xi)\cdot\boldsymbol{u} \quad (19)$$

$$\theta(x)=\frac{\partial u}{\partial x}=\frac{\partial\boldsymbol{N}}{\partial x}\cdot\boldsymbol{u}=\left(\frac{\partial x}{\partial\xi}\right)^{-1}\mathbf{A}\frac{\partial\boldsymbol{H}}{\partial\xi}\cdot\boldsymbol{u} \quad (20)$$

where $\boldsymbol{H}^T=\{H_1\ H_2\ H_3\ H_4\}$, $\boldsymbol{u}^{\mathrm{T}}=[u_1\quad\theta_1\quad u_2\quad\theta_2]$, and $\mathbf{A}$ is the correction matrix for dimensionality matching. In summary, the correction matrix $\mathbf{A}$ can be written as

$$\mathbf{A}=\begin{bmatrix} \begin{matrix}1 & 0\\ 0 & \frac{(x_2-x_1)}{2}\end{matrix} & \boldsymbol{I} \\ \boldsymbol{I} & \begin{matrix}1 & 0\\ 0 & \frac{(x_2-x_1)}{2}\end{matrix}\end{bmatrix} \quad (21)$$

The two-dimensional function $\boldsymbol{H}$ is obtained as follows:

$$\boldsymbol{H}^T=\{\boldsymbol{H}_{(1)}^T\ \boldsymbol{H}_{(2)}^T\ \boldsymbol{H}_{(3)}^T\ \boldsymbol{H}_{(4)}^T\} \quad (22)$$

where $\boldsymbol{H}_{(k)}^T=\{H_{3k-2}\ H_{3k-1}\ H_{3k}\ \}$. By multiplying Hermite basis functions shown in Eq. (18) in each dimension, the content of $\boldsymbol{H}_{(k)}^T$ is as follows:

$$H_{3k-2}(\xi,\eta)=H_{2i-1}(\xi)\cdot H_{2j-1}(\eta) \\ H_{3k-1}(\xi,\eta)=H_{2i}(\xi)\cdot H_{2j-1}(\eta) \\ H_{3k}(\xi,\eta)=H_{2i-1}(\xi)\cdot H_{2j}(\eta) \quad (23)$$

where η is -1≤ η ≤1 and the normalized coordinate system of elements, and the combinations of k, i, and j are as follows:

$$(k,i,j)=(1,1,1),(2,2,1),(3,2,2),(4,1,2) \quad (24)$$

The displacement of the z-direction is $w(x,y)$, and the rotation angles in the x- and y-directions are $\theta^x(x,y)$ and $\theta^y(x,y)$. The $w(x,y)$, $\theta^x(x,y)$, and $\theta^y(x,y)$ can also be approximated by the Hermite basis function as follows.

$$w(x,y)=\boldsymbol{N}\cdot\boldsymbol{w}=\mathbf{A}\boldsymbol{H}(\xi,\eta)\cdot\boldsymbol{w} \\ \theta^x(x,y)=\frac{\partial}{\partial x}w(x,y)=\mathbf{A}\frac{\partial\boldsymbol{H}}{\partial x}\cdot\boldsymbol{w} \\ \theta^y(x,y)=\frac{\partial}{\partial y}w(x,y)=\mathbf{A}\frac{\partial\boldsymbol{H}}{\partial y}\cdot\boldsymbol{w} \quad (25)$$

where $\boldsymbol{w}^T=[w_1\ \theta^{x1}\ \theta^{y1}\ w_2\ \theta^{x2}\ \theta^{y2}\ w_3\ \theta^{x3}\ \theta^{y3}\ w_4\ \theta^{x4}\ \theta^{y4}]$.
In summary, the equations look like follows:

$$\begin{Bmatrix} w(x,y)\\ \theta^x(x,y)\\ \theta^y(x,y)\end{Bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & \frac{\partial\xi}{\partial x} & \frac{\partial\eta}{\partial x}\\ 0 & \frac{\partial\xi}{\partial y} & \frac{\partial\eta}{\partial y}\end{bmatrix}\begin{Bmatrix}\boldsymbol{H}\cdot\boldsymbol{w}\\ \mathbf{A}\frac{\partial\boldsymbol{H}}{\partial\xi}\cdot\boldsymbol{w}\\ \mathbf{A}\frac{\partial\boldsymbol{H}}{\partial\eta}\cdot\boldsymbol{w}\end{Bmatrix} \quad (26)$$

By using the x-coordinates x_1,x_2,x_3,x_4, and y-coordinates y_1,y_2,y_3,y_4 of the nodes of a quadrangle element, the correction matrix $\mathbf{A}$ is as follows:

$$\mathbf{A}=\begin{Bmatrix}\mathbf{A}_1 & 0 & 0 & 0\\ 0 & \mathbf{A}_2 & 0 & 0\\ 0 & 0 & \mathbf{A}_3 & 0\\ 0 & 0 & 0 & \mathbf{A}_4\end{Bmatrix} \quad (27)$$

where $\mathbf{A}_{\mathrm{k}}$ (k = 1,2,3,4) is as follows:

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$\mathbf{A}_1$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{(x_2 - x_1)}{2} & \frac{(y_2 - y_1)}{2} \\ 0 & \frac{(x_4 - x_1)}{2} & \frac{(y_4 - y_1)}{2} \end{bmatrix}$$

$\mathbf{A}_2$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{(x_2 - x_1)}{2} & \frac{(y_2 - y_1)}{2} \\ 0 & \frac{(x_3 - x_2)}{2} & \frac{(y_3 - y_2)}{2} \end{bmatrix}$$

$\mathbf{A}_3$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{(x_3 - x_4)}{2} & \frac{(y_3 - y_4)}{2} \\ 0 & \frac{(x_3 - x_2)}{2} & \frac{(y_3 - y_2)}{2} \end{bmatrix}$$

$\mathbf{A}_4$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{(x_3 - x_4)}{2} & \frac{(y_3 - y_4)}{2} \\ 0 & \frac{(x_4 - x_1)}{2} & \frac{(y_4 - y_1)}{2} \end{bmatrix} \quad (28)$$

3. Proposal of a New Basis Function

The two new basis functions are proposed. Using 12 unknown coefficients α_{kl}, the basic function is given as follows:

$$H_i^{(t)}(\xi, \eta) = \sum_{k=0}^{3} \sum_{l=0}^{3} \alpha_{kl}^{(t)} \xi^k \eta^l \quad (29)$$

where i is from 1 to 12, and k and l are the order of the terms, and t indicates the function number used in this paper. When creating the basis function $\boldsymbol{H}$, three conditions are given as follows at the four nodes of the flat plate element.

If $m = 3k - 2$

$$H_m(\xi_i, \eta_j) = 1 \text{ and the others} = 0$$

If $m = 3k - 1$

$$\frac{\partial H_m(\xi_i, \eta_j)}{\partial \xi} = 1 \text{ and the others} = 0 \quad (30)$$

If $m = 3k$

$$\frac{\partial H_m(\xi_i, \eta_j)}{\partial \eta} = 1 \text{ and the others} = 0$$

where m is from 1 to 12, and k is from 1 to 4, and i and j both take the values 1 and 2. $\xi_1 = -1$ and $\xi_2 = 1$, and $\eta_1 = -1$ and $\eta_2 = 1$.

III. Numerical calculation conditions

There are three basic functions. Each of them will be used to verify the flat plate under a uniformly distributed load. Numerical calculations were performed for two boundary conditions. The material parameters are shown in TABLE I and the boundary conditions are shown in Eq (31) and Eq (32). In the finite element method, the Gaussian quadrature is used for integration. There are 25 (5x5) integration points in the Gaussian quadrature.

TABLE I
PARAMETER OF MODEL

Symbol	Quantity	Unit	Value
E	Young's modulus	MPa	200×10^3
v	Poisson's ratio		0.3
L	A side of square	mm	400
t	Thickness of the plate	mm	10
D	Flexural rigidity	$N \cdot mm$	1.83×10^7

1, 4-sided simply supported (B.C.1)

At $x = 0, L$

$$w = 0, \frac{\partial w}{\partial y} = 0$$

At $y = 0, L$ (31)

$$w = 0, \frac{\partial w}{\partial x} = 0$$

A uniformly distributed load $q(x, y) = q_0$ (= -0.2) (MPa) is applied to the entire plate. The theoretical solution [2] of the maximum displacement in the z-direction was obtained from $w = 0.00406 q_0 L^4$/D=1.1349(mm).

2, 4-sided fixed (B.C.2)

At x,$y = 0, L$

$$w = 0, \frac{\partial w}{\partial x} = 0, \frac{\partial w}{\partial y} = 0 \quad (32)$$

The load is q_0= -0.1 (MPa). The theoretical solution [2] $w = 0.00126 q_0 L^4$/D=1.7611(mm).

IV. Results and Discussions

A. Simple Hermite ($t = 1$)

This function is not a new discovery and is originally known. From Eq. (22), the function can be found. TABLE II shows the results of the two boundary conditions. Each result is a different number of elements. TABLE III shows the coefficients of that function. The term $\xi^2\eta^2$ is not used. Fig.1 and Fig.2 is the figure of the deformed plate under B.C.1 and B.C.2. The number of elements in those diagrams is 400(20x20). Also, the number of elements in the diagrams that will be displayed in this paper is the same and those figures show a magnified deformation in the z-direction.

TABLE II
RESULTS OF SIMPLE HERMITE ($t = 1$)

B.C.	1	2
NUM.ELEM	MAXIMUM DIPLACEMENT	
4x4	0.9657	1.6932
6x6	1.0294	1.7054
8x8	1.0523	1.7144
10x10	1.0630	1.7194
20x20	1.0774	1.7268
40x40	1.0810	1.7287

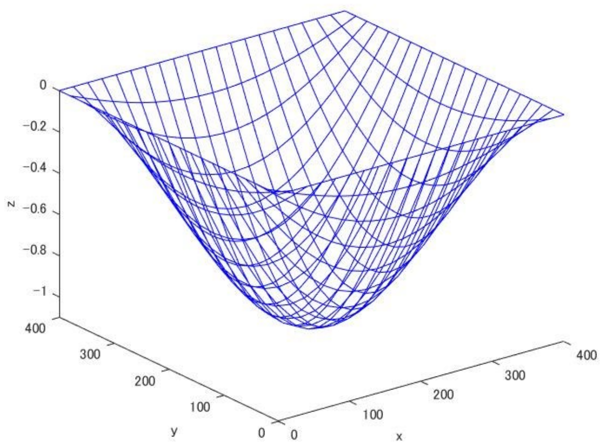

Fig .1 Deformed plate under B.C.1 ($t = 1$)

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TABLE III
COEFFICIENTS OF BASIS FUNCTIONS $\alpha_{kl}^{(1)}$: Simple Hermite

$(t=1)$		i											
k	l	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1/4	1/8	1/8	1/4	-1/8	1/8	1/4	-1/8	-1/8	1/4	1/8	-1/8
	1	-3/8	-3/16	-1/8	-3/8	3/16	-1/8	3/8	-3/16	-1/8	3/8	3/16	-1/8
	2	0	0	-1/8	0	0	-1/8	0	0	1/8	0	0	1/8
	3	1/8	1/16	1/8	1/8	-1/16	1/8	-1/8	1/16	1/8	-1/8	-1/16	1/8
1	0	-3/8	-1/8	-3/16	3/8	-1/8	3/16	3/8	-1/8	-3/16	-3/8	-1/8	3/16
	1	9/16	3/16	3/16	-9/16	3/16	-3/16	9/16	-3/16	-3/16	-9/16	-3/16	3/16
	2	0	0	3/16	0	0	-3/16	0	0	3/16	0	0	-3/16
	3	-3/16	-1/16	-3/16	3/16	-1/16	3/16	-3/16	1/16	3/16	3/16	1/16	--3/16
2	0	0	-1/8	0	0	1/8	0	0	1/8	0	0	-1/8	0
	1	0	3/16	0	0	-3/16	0	0	3/16	0	0	-3/16	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	-1/16	0	0	1/16	0	0	-1/16	0	0	1/16	0
3	0	1/8	1/8	1/16	-1/8	1/8	-1/16	-1/8	1/8	1/16	1/8	1/8	-1/16
	1	-3/16	-3/16	-1/16	3/16	-3/16	1/16	-3/16	3/16	1/16	3/16	3/16	-1/16
	2	0	0	-1/16	0	0	1/16	0	0	-1/16	0	0	1/16
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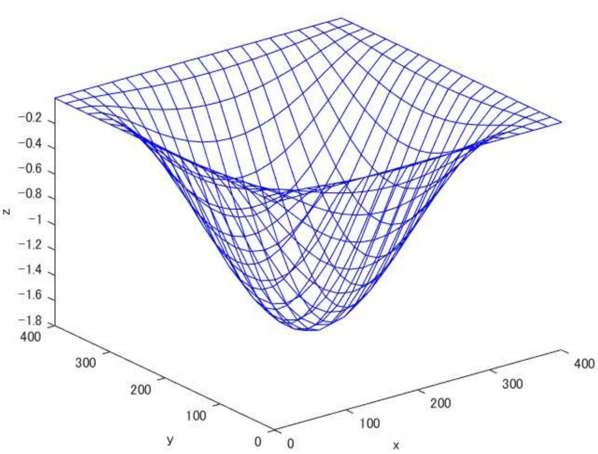

Fig .2 Deformed plate under B.C.2 $(t=1)$

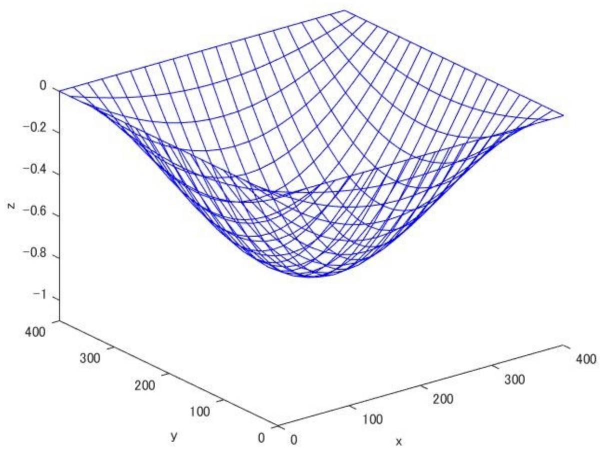

Fig .3 Deformed plate under B.C.1 $(t=2)$

B. New Hermite 1 $(t=2)$

This function is the first of the two newly proposed functions. TABLE IV shows the results of the two boundary conditions. The results were a different number of elements. TABLE V shows the coefficients of this function. All coefficients of the four terms (η^2, $\xi\eta$, $\xi\eta^2$, $\xi^2\eta$) are zero. Because there are three more of those terms than $(t=0)$, the accuracy of $(t=1)$ is reduced. Fig.3 and Fig.4 is the figure after deformation of the plate under B.C.1 and B.C.2.

TABLE IV
RESULTS OF NEW HERMITE 1 $(t=2)$

B.C.	**1**	**2**
NUM.ELEM	**MAXIMUM DIPLACEMENT**	
4x4	0.7608	1.5208
6x6	0.7878	1.4857
8x8	0.7968	1.4569
10x10	0.8009	1.4334
20x20	0.8065	1.4257
40x40	0.8079	1.4214

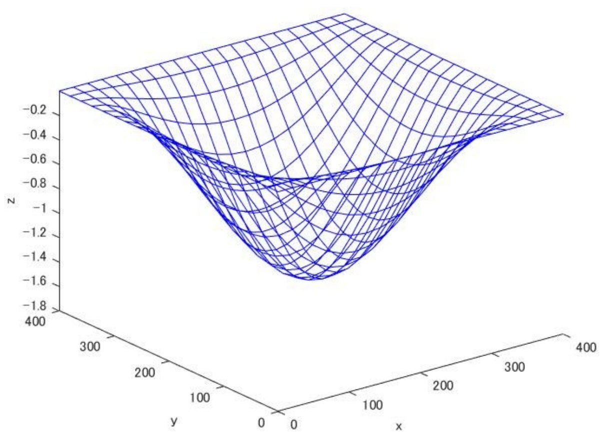

Fig .4 Deformed plate under B.C.2 $(t=2)$

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TABLE V
COEFFICIENTS OF BASIS FUNCTIONS $\alpha_{kl}^{(2)}$: New Hermite 1

$(t=2)$		i											
k	l	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1/4	1/8	0	1/4	-1/8	0	1/4	-1/8	0	1/4	1/8	0
	1	-3/8	0	-1/8	-3/8	0	-1/8	3/8	0	-1/8	3/8	0	-1/8
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	1/8	-1/8	1/8	1/8	1/8	1/8	-1/8	-1/8	1/8	-1/8	1/8	1/8
1	0	-3/8	-1/8	0	3/8	-1/8	0	3/8	-1/8	0	-3/8	-1/8	0
	1	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	3/8	1/8	0	-3/8	1/8	0	3/8	-1/8	0	-3/8	-1/8	0
2	0	0	-1/8	1/8	0	1/8	1/8	0	1/8	-1/8	0	-1/8	-1/8
	1	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	-1/8	0	0	-1/8	0	0	1/8	0	0	1/8
	3	0	1/8	0	0	-1/8	0	0	1/8	0	0	-1/8	0
3	0	1/8	1/8	-1/8	-1/8	1/8	1/8	-1/8	1/8	-1/8	1/8	1/8	1/8
	1	3/8	0	1/8	-3/8	0	-1/8	3/8	0	-1/8	-3/8	0	1/8
	2	0	0	1/8	0	0	-1/8	0	0	1/8	0	0	-1/8
	3	-1/2	-1/8	-1/8	1/2	-1/8	1/8	-1/2	1/8	1/8	1/2	1/8	-1/8

C. New Hermite 2 $(t=3)$

This function is the second of the two newly proposed functions. TABLE VI shows the results of the two boundary conditions, with different number of elements. TABLE VII shows the coefficients of this function. All coefficients of the term $\xi^3\eta^3$ is zero. Compared to $(t=1)$, zero coefficients in the higher order terms make the results inaccurate. Fig.5 and Fig.6 is the figure after deformation of plate under B.C.1 and B.C.2.

TABLE VI
RESULTS OF NEW HERMITE 2 $(t=3)$

B.C.	1	2
NUM.ELEM	**MAXIMUM DIPLACEMENT**	
4x4	0.8584	1.5050
6x6	0.9150	1.5159
8x8	0.9354	1.5239
10x10	0.9449	1.5284
20x20	0.9577	1.5349
40x40	0.9609	1.5366

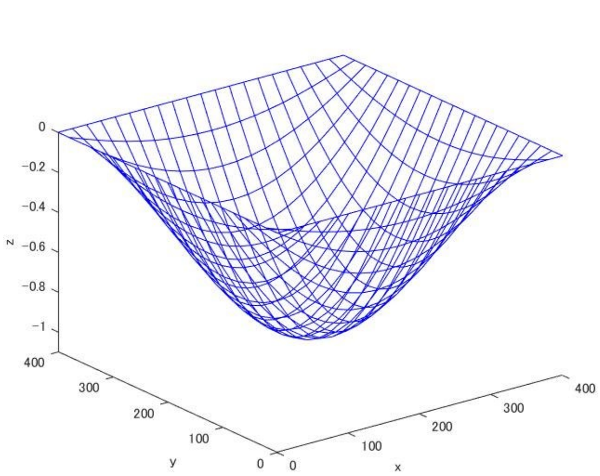

Fig .5 Deformed plate under B.C.1 $(t=3)$

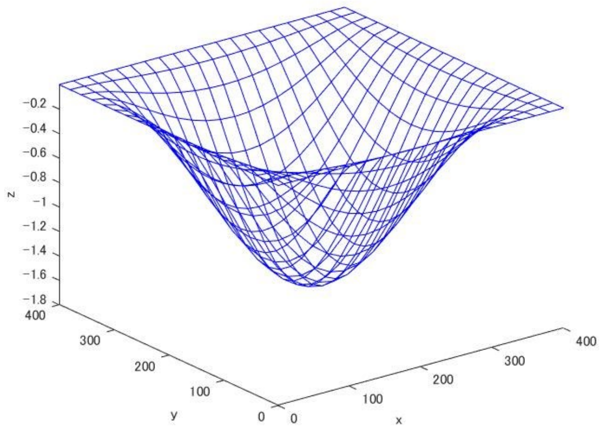

Fig .6 Deformed plate under B.C.2 $(t=3)$

Fig.7 and Fig.8 are the graphs showing the errors between the numerical results and the theoretical solution for each of the three functions. The x-axis of the graphs is the number of elements in the x-direction, and the y-axis is the errors (%). The logarithmic scale is used for x-axis. Under the B.C.1, with 20x20 elements, the errors of Simple $(t=1)$ is 5.07% while New Hermite 1 $(t=2)$ and New Hermite 2 $(t=3)$ are 28.93 and 15.61%. These results are not good. Under the B.C.2, with 20x20 elements, the errors of Simple $(t=1)$ is 1.95% while New Hermite 1 $(t=2)$ and New Hermite 2 $(t=3)$ are 19.29 and 12.75%. They are also bad results. Under B.C.1, all cases showing poor accuracy are caused by the inability to correctly represent the boundary conditions. The New Hermite 1 had the worst results in both conditions and the New Hermite 2 was not better than the Simple precision.

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TABLE VII
COEFFICIENTS OF BASIS FUNCTIONS $\alpha_{kl}^{(3)}$: New Hermite 2

$(t=3)$		i											
k	l	1	2	3	4	5	6	7	8	9	10	11	12
0	0	3/16	0	0	3/16	0	0	3/16	0	0	3/16	0	0
	1	-5/16	-1/16	-1/16	-5/16	1/16	-1/16	5/16	-1/16	-1/16	5/16	1/16	-1/16
	2	1/16	1/8	0	1/16	-1/8	0	1/16	-1/8	0	1/16	1/8	0
	3	1/16	-1/16	1/16	1/16	1/16	1/16	-1/16	-1/16	1/16	-1/16	1/16	1/16
1	0	-5/16	-1/16	-1/16	5/16	-1/16	1/16	5/16	-1/16	-1/16	-5/16	-1/16	1/16
	1	1/2	1/8	1/8	-1/2	1/8	-1/8	1/2	-1/8	-1/8	-1/2	-1/8	1/8
	2	-1/16	1/16	1/16	1/16	-1/16	-1/16	1/16	-1/16	1/16	-1/16	-1/16	-1/16
	3	-1/8	0	-1/8	1/8	0	1/8	-1/8	0	1/8	1/8	0	-1/8
2	0	1/16	0	1/8	1/16	0	1/8	1/16	0	-1/8	1/16	0	-1/8
	1	-1/16	1/16	-1/16	-1/16	-1/16	-1/16	1/16	1/16	-1/16	1/16	-1/16	-1/16
	2	-1/16	-1/8	-1/8	-1/16	1/8	-1/8	-1/16	1/8	1/8	-1/16	-1/8	1/8
	3	1/16	1/16	1/16	1/16	-1/16	1/16	-1/16	1/16	1/16	-1/16	-1/16	1/16
3	0	1/16	1/16	-1/16	-1/16	1/16	1/16	-1/16	1/16	-1/16	1/16	1/16	1/16
	1	-1/8	-1/8	0	1/8	-1/8	0	-1/8	1/8	0	1/8	1/8	0
	2	1/16	1/16	1/16	-1/16	1/16	-1/16	-1/16	1/16	1/16	1/16	1/16	-1/16
	3	0	0	0	0	0	0	0	0	0	0	0	0

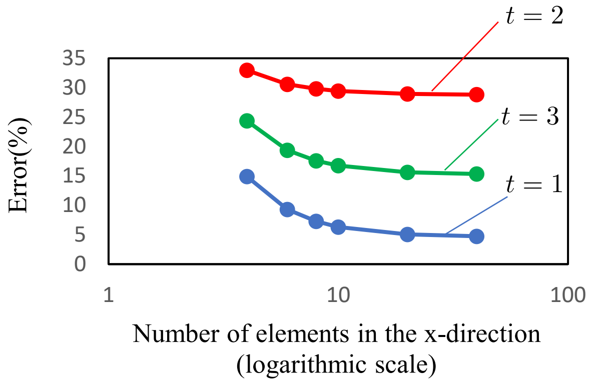

Fig .7 The errors of numeric calculation under B.C.1

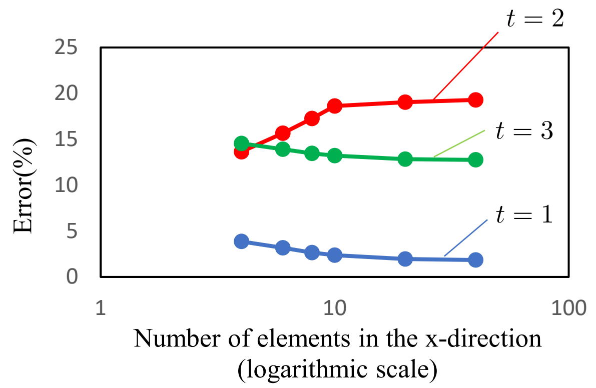

Fig .8 The errors of numeric calculation under B.C.2

V. Conclusion

In this study, the two basis functions were proposed. But they are inferior in accuracy to the existing basis functions. This time, it was found once again that the existing basis functions are excellent.

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