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Mathematical Investigation of Flexural and Torsional Rigidity of Orthotropic Irregular Hendecagon Beam

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Abstract—**There is always a need to study the resistance of structural building components, to bending force, to angular acceleration or even to twist, in order to guide against failure. This study attempts to investigates flexural rigidity and torsional rigidity of hendecagon irregular orthotropic beam. A numerical analysis of the beam's flexural and torsional rigidity was carried out and graphs plotted with the help of computer software – MAPLE. The results show that torsion of the hendecagon beam is a function of the angle of twist and distance along the beam. The flexural rigidity of the beam is asymptotic at the end point of beam and cannot take value zero at any point along the hendecagon beam.**

***Index Terms*— Flexural Rigidity; Torsional Rigidity; Orthotropic Hendecagon Beam; Mathematical investigation.**

I. INTRODUCTION

RIGIDITY is the maximum resistance an object can offer before it deforms [1]. Torsion is the action of twisting on an object or in relation to another. Torsional rigidity is the resistance to twist. The product of the torsion constant and shear modulus gives torsional rigidity [2]. Torsional rigidity a solid beam therefore is how much the cross section of the beam resists the torsional deformation. The resistance of the the beam's cross section increases as the rigidity increases, in other words there is a positive correlation between the rigidity and the resistance of the beam. The following have effect on the torsional rigidity of the beam: the shape of the cross sectional area of the beam, the length of the beam, the shear modulus of the material the beam is made of, and the support conditions [2,3].

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On the other hand, Flexural rigidity is the resistance to bending deformation. It is depends on the elastic modulus, the second moment of area, the beam cross-section, length of the beam and the support conditions, just like in torsional rigidity. Flexural rigidity is also known as bending rigidity. [4,5,6]. Materials are considered to be orthotropic if the properties depend on the direction. An orthotropic material is unique in that material properties are dependent upon orientation. In addition to wood, as a good example of orthotropic material, other examples include composite laminates and some heavily processed metals [7].

In geometry, a hendecagon or 11-gon is an eleven-sided polygon [8]. The total effect of all the forces acting on the beam is to produce shear forces and bending moments within the beam, that in turn induce internal stresses, strains and deflections of the beam [8,9,10,11]. Beams are characterized by their manner of support, shape of cross-section, length, and their material. Structural systems usually, contain beam structures that are designed to carry lateral loads [12, 13, 14, 15, 16].

This paper investigates, mathematically, the flexural and torsional rigidity of wood beam whose cross section is an irregular polygon with eleven sides.

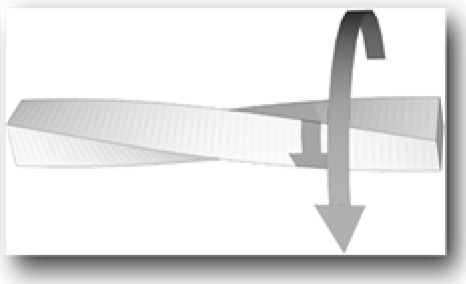

Figure 1: Torsion of an orthotropic square section beam

II. PROBLEM FORMULATION

A. *Flexural rigidity of beam problem*

The resulting curvature of the beam resulting from the applied bending moment are governed by the following equations [9]:

$$M = EI\kappa \tag{1}$$

$$M = EI\frac{d^2w}{dx^2} \tag{2}$$

Where,

w= deflection
x = distance along the beam
E = young modulus
I = second moment of area
M = bending moment

Integrating equation (2) two times gives the deflection of the beam under the applied force. In turn the flexural rigidity can be computed using the following equation;

$$K = \frac{P}{w} \tag{3}$$

Where,
K= Flexural rigidity
P = Applied force

B. *Torsional Rigidity of Beam Problem*

The torsion of a shafts of uniform cross-section is given as:

$$T = \frac{J_T}{r}\tau \tag{4}$$

$$T = \frac{J_T}{\ell}G\varphi \tag{5}$$

Where,

T is the applied torque

- τ is the maximum shear stress at the outer surface
- J_T is the torsion constant for the section.
- r is the distance between the rotational axis
- ℓ is the length of the beam.
- φ is the angle of twist
- G is the shear modulus,
- The product J_T G is called the torsional rigidity w_T.

III. ANALYSIS

For the purpose of investigation and analysis, an oak wood beam with hendecagon cross section was considered. The following parameter values were assumed: applied force (P), 10 Newton, length of beam (x), 5 meters, young modulus (E) of oak wood (along the grain), 11 GPa, bending moment.(M) 12.5, shear modulus(modulus of rigidity)(G) of wood is 13 GPa.

A. *Area moment of inertia for the hendecagon*

The area moment of inertia for the hendecagon on the XY axis – plane can be computed in general by summing contributions from each segment of the polygon. In this case the polygon has eleven vertices. When the desired reference axis is the x-axis, the area moment of inertia is given as [10].

$$I = \frac{1}{12}\sum_{i=1}^{11}\left(y_i^2 + y_i y_{i+1} + y_{i+1}^2\right)\left(x_i y_{i+1} - x_{i+1} y_i\right) \tag{6}$$

Where, x_i, y_i are the coordinates of the i-th polygon vertex, for $1 \le i \le n$. Also, x_{n+1}, y_{n+1} are assumed to be equal to the coordinates of the first vertex, i.e., $x_{n+1} = x_1$ and $y_{n+1} = y_1$.Considering the irregular hendecagon cross section with following vertices coordinates:

$$\begin{aligned}
(x_1, y_1) &= (1, 2)\\
(x_2, y_2) &= (2, 1)\\
(x_3, y_3) &= (3, 2)\\
(x_4, y_4) &= (4, 3)\\
(x_5, y_5) &= (4, 4)\\
(x_6, y_6) &= (3, 5)\\
(x_7, y_7) &= (2, 6)\\
(x_8, y_8) &= (1, 8)\\
(x_9, y_9) &= (-1, 6)\\
(x_{10}, y_{10}) &= (-2, 5)\\
(x_{11}, y_{11}) &= (-2, 4)
\end{aligned}$$

$$I = 0.083\sum\left(y_i^2 + y_i y_{i+1} + y_{i+1}^2\right)\left(x_i y_{i+1} - x_{i+1} y_i\right)$$

$$I = \frac{1}{12}\sum_{i=1}^{11}\left(y_i^2 + y_i y_{i+1} + y_{i+1}^2\right)\left(x_i y_{i+1} - x_{i+1} y_i\right)$$

I = 1, 2, 3,…, 11

$$= 0.083[(4+2+1)(1-4) + (1+2+4)(4-3) + (4+4+4)(6-2)]$$

$$I = 459.42 \tag{7}$$

From equation (2):

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$$M = EI\frac{d^2w}{dx^2} \quad (8)$$

$$\frac{M}{EI} = \frac{d^2w}{dx^2} \quad (9)$$

$$\iint \frac{M}{EI}dx^2 = \iint d^2w \quad (10)$$

$$\iint \frac{12.5}{(11)(459.42)}dx^2 = \iint d^2w \quad (11)$$

$$\iint 0.0025dx^2 = \iint d^2w \quad (12)$$

$$\int 0.0025xdx + c_1 = \int dw + k_1 \quad (13)$$

$$\frac{0.0025}{2}x^2 + c_1 + c_2 = w + k_1 + k_2 \quad (14)$$

$$0.00125x^2 + c_3 = w + k_3 \quad (15)$$

$$0.00125x^2 + C = w \quad (16)$$

When x = 5 at time t=0, w=0

$$0.00125(5)^2 + C = 0 \quad (17)$$

$$-0.03125 = C$$

Equation (16) becomes

$$0.00125x^2 - 0.03125 = w \quad (18)$$

From equation (3), the flexural rigidity of the beam under consideration is given as follows:

$$K = \frac{P}{w}$$

$$K = \frac{10}{0.00125x^2 - 0.03125} \quad (19)$$

From equation (5):

$$T = \frac{J_T}{\ell}G\varphi \quad (20)$$

$$T = 13\frac{I}{x}\varphi \quad (21)$$

$$T = 13\frac{459.42}{x}\varphi \quad (22)$$

When x = 5;

$$T = 13\frac{459.42}{5}\varphi \quad (23)$$

$$T = 1194.49\varphi \quad (24)$$

When φ = 30:

$$T = 13\frac{459.42}{x}(30) \quad (25)$$

$$T = \frac{179173.8}{x} \quad (26)$$

IV. RESULT AND DISCUSSION

Results of the investigation are represented in figure 4 through figure 13. From figure 4, it can be seen that the deflection of the hendecagon beam increases as the distance along the beam, x, increases. The deflection is however relatively small under the external force.

Torsion at different angles of twist, as shown in figure 5 through figure 8, revealed that there is a positive correlation between the torsion of the hendecagon beam and the angle of twist. Also, the torsion of the beam decreases as the distance along the beam increases. It is observed from figure 9 that the torsion of the beam at angle of twist of 90^0, represented by series 5, has the maximum amplitude, followed by torsion with angle of twist of 60^0. It continues that way till the torsion with angle of twist of 30^0. The flexural rigidity of the beam, represented in figures 10 and 11, show that its value is asymptotic to the vertical at x = 5 and asymptotic to the horizontal at value zero.

This implies that the flexural rigidity of the hendecagon beam cannot value zero at any point along the beam. In particular, the flexural rigidity cannot be determined at end point of the beam (x=5). The deflection at that is zero, confirmed in figure 4. It can therefore be deduced from the results that the higher the angle of twist, the more the hendecagon beam is subject to twisting as a result of applied load (torque). Also, the resistance of the beam against bending deformation tends to infinity at the end point of the hendecagon beam.

The flexural rigidity of the beam decreases along the lenght of the beam, but cannot be determined at the end of the beam (x = 5).

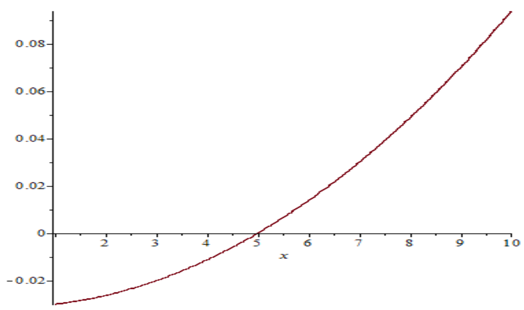

Figure 4: Deflection (w) of the beam at different lengths (x)

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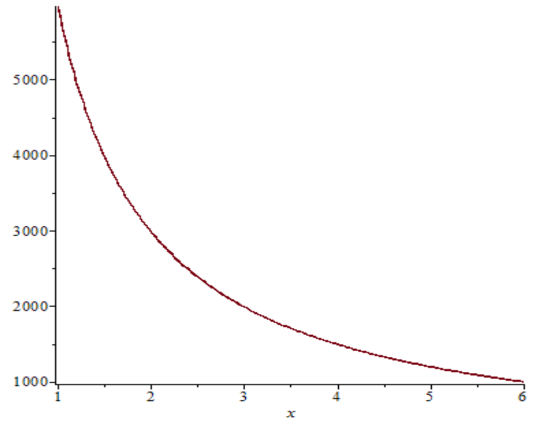

Figure 5: Torsion at angle 1^0 and different lengths (x) of the beam.

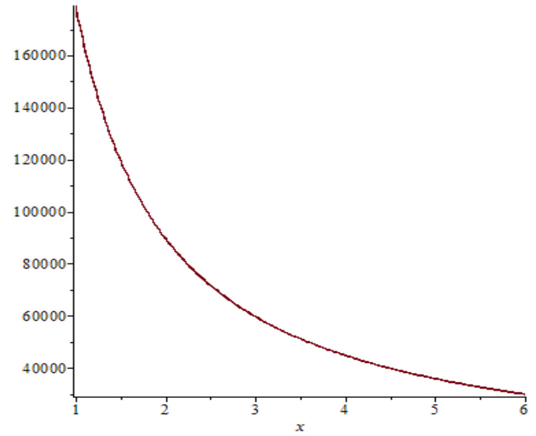

Figure 6: Torsion at angle 30^0 and different lengths (x) of the beam.

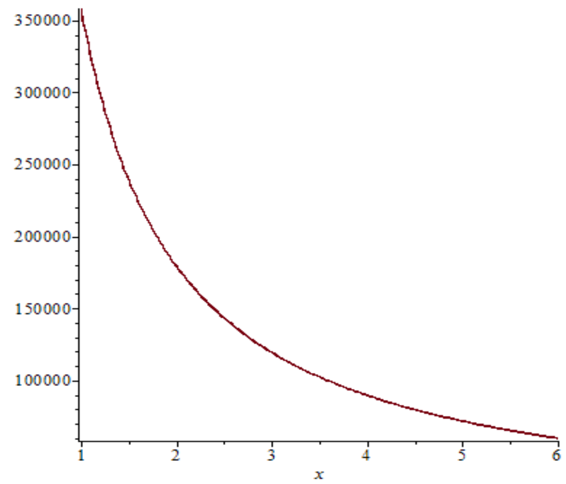

Figure 7: Torsion at angle 60^0 and different lengths (x) of the beam.

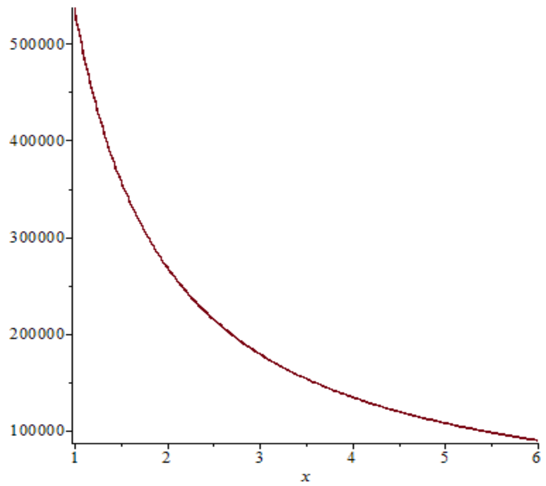

Figure 8: Torsion at angle 90^0 and different lengths (x) of the beam.

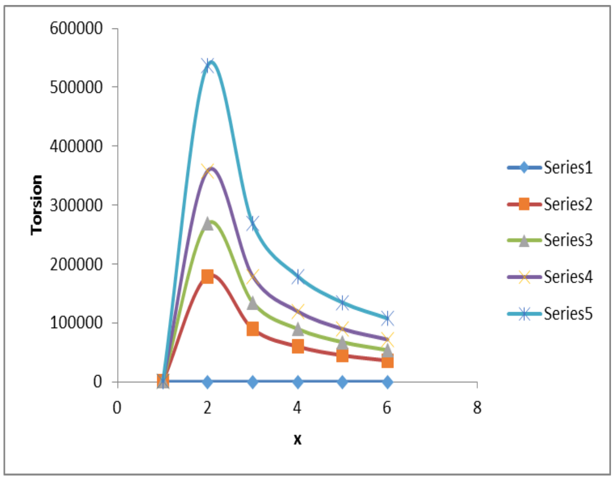

Figure 9: Torsion of the beam at different angles

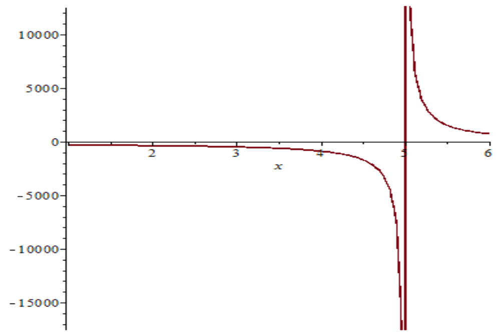

Figure 10: Flexural rigidity of the beam for distance along the beam x=1,2,…6.

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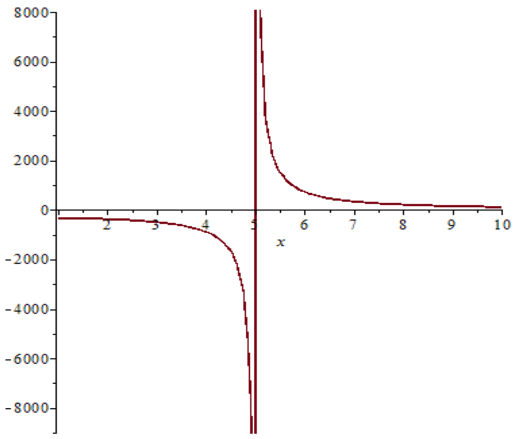

Figure 11: Flexural rigidity of the beam for distance along the beam x=1,2,…10.

V. CONCLUSION

The flexural and Torsional Rigidity of Orthotropic irregular Hendecagon beam was investigated analytically in this paper. The area moment of inertia, deflection of the beam, torsion of the beam at different angles of twist, and the flexural rigidity of the beam were computed. The results are well represented on graphs.

The torsion of the hendecagon beam is a function of the angle of twist and distance along the beam. The flexural rigidity of the beam is asymptotic at the end point of beam considered. It was observed that it cannot take the value zero at any point along the hendecagon beam. The deflection of the beam under the applied force is zero at the end of the beam (x = 5).

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