An Improved Relay Auto Tuning of PID Controllers for SOPTD Systems

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Abstract: Using a single symmetric relay feedback test, a method is proposed to identify all the three parameters of a stable second order plus time delay (SOPTD) model with equal time constants. The conventional analysis of relay auto-tune method gives 27% error in the calculation of k_u. In the present work, a method is proposed to explain the error in the k_u calculation by incorporating the higher order harmonics. Three simulation examples are given. The estimated model parameters are compared with that of Li et al. [4] method and that of Thyagarajan and Yu [8] method. The open loop performance of the identified model is compared with that of the actual system. The proposed method gives performances close to that of the actual system. Simulation results are also given for a nonlinear bioreactor system. The open loop performance of the model identified by the proposed method gives a performance close to that of the actual system and that of the locally linearized model.

Key words: Transfer function, identification, SOPTD model, symmetric relay, auto-tuning

1. Introduction

Åström and Hägglund [2] have suggested the use of an ideal (on-off) relay to generate a sustained oscillation of the controlled variable. The amplitude (a_0) and the period of oscillation (p_u) are noted from the sustained oscillation of the system output. The ultimate gain (k_u) and ultimate frequency (ω_u) are calculated from the principal harmonics approximation as:

$$k_u = 4h/(\pi a_0) \quad (1)$$

$$\omega_u = 2\pi/ p_u \quad (2)$$

Certain higher order model system when approximated to a FOPTD model gives a negative time constant [4]. In such cases, Li et al. [4] have suggested to identify a SOPTD model instead of a FOPTD model. SOPTD model can incorporate higher order process dynamics better than that of the FOPTD model. The controller designed based on SOPTD model gives a better-closed loop response than that is designed on a FOPTD model. It is better to have a SOPTD model with equal time constants, since only three parameters are to be identified.

$$y(s)/u(s)=k_p \exp(-Ds)/(\tau s+1)^2 \quad (3)$$

Where, 'k_p' the process gain, 'D' the time delay and 'τ' the time constant are the parameters to be estimated. Li et al. [4] have used the relay feedback method to identify SOPTD model parameters. Based on simulation studies, Li et al. [4] have reported that when the ratio of the time delay to time constant is larger, the model identified by relay autotune method gives 27% error in k_u calculations. Thyagarajan and Yu (T-Y) [8] have proposed a method of identifying a SOPTD model using symmetric relay test. If the response consists of sinusoidal oscillations with exponentially increasing magnitude that reach to steady state after some cycles, the process can be considered as second order process with equal time constants with small D/τ ratio [8]. Recently, Srinivasan and Chidambaram (S-C) [6] have proposed a method to consider higher order harmonics of the relay oscillations for a first order plus time delay (FOPTD) model. They have also proposed a method to calculate the model parameters accurately for the FOPTD system. In the present work, the method is extended to SOPTD system.

2. Consideration of higher order harmonics

Let us consider an open loop stable second order plus time delay (SOPTD) system with equal time constants (1). Consider a symmetric relay feedback system. From the Fourier series analysis, it can be easily shown [2] that a relay consists of many sinusoidal waves of odd multiples of fundamental frequency 'ω' and with the amplitude $4h/(n\pi)$ (n=1, 3, 5, ..). For a SOPTD system, the output wave is also a sinusoidal wave. Here, y(t) is the output response.

$$y(t)=[a_1 \sin(\omega_u t+\phi_1) + (1/3)\, a_3 \sin(3\omega_u t+\phi_3) + ...] \quad (4)$$

$a_1= 1/[1+(\tau\omega_u)^2]$; $a_3= 1/[1+(3\tau\omega_u)^2]$; etc.

$$\phi_1=-D\omega_u - 2\tan^{-1}(\tau\omega_u) = -\pi;\ ;\phi_3 =-3D\omega_u - 2\tan^{-1}(3\tau\omega_u);\ \phi_j = -j\, D\omega_u - 2\tan^{-1}(j\tau\omega_u) \quad (5)$$

Equation (4) can be written as:

$$y(t) = a_1\, [\sin(\omega_u t+\phi_1) + \ldots] \quad (6)$$

If $\tau\omega_u$ is assumed very large, then $a_3 \sin(3\tau\omega_u +\phi_3)$, $a_5 \sin(5\tau\omega_u +\phi_5)$ etc. will be neglected. In what follows, we will consider the higher order dynamics for the calculation of k_u and in the identification of the model parameters. Let us derive approximate evaluation of y(t) for the limiting cases of smaller $\tau\omega_u$ and separately for larger $\tau\omega_u$. Wherever the relay oscillations are close to a rectangular waveform, the results for smaller $\tau\omega_u$ is to be used. If the relay oscillations are close to a sinusoidal waveform, then the standard equation considering fundamental frequency of oscillation is used.

$$\phi_1 = -D\omega_u - 2\tan^{-1}(\tau\omega_u) =- \pi$$

$$\tan^{-1}(j\tau\omega_u)=j\tan^{-1}(\tau\omega_u) \quad (7)$$

$$\phi_3 = -3D\omega_u - 2\tan^{-1}(3\tau\omega_u) =- 3\pi \quad (8)$$

Similarly, $\phi_5 = -5\pi$;…; $\phi_N = -N\pi$ (9)

Hence, (6) becomes as:

$$y(t) = a_1[-\sin(\omega_u t) -(1/3)\, b_3 \sin(3\omega_u t)-.] \quad (10)$$

where $b_3 = [1+(\tau\omega_u)^2]/[1+(3\tau\omega_u)^2]$; (11)

Value of $\tau\omega_u$ can be neglected when compared to 1 and hence the values of b_1, b_3, .can each be approximated to 1. Hence, (10) becomes:

$$y(t) = a_1[-\sin(\omega_u t) -(1/3) \sin(3\omega_u t) - \cdots] \quad (12)$$

Here the value of 'a_1' is to be calculated. The value of 'a_1' is not the amplitude what we observe from the output oscillation. Let us estimate the error involved in k_u by using only the principle harmonics in the analysis of relay testing. From the output oscillations, it is possible to calculate y(t*) at any time 't*'. Let, ω_u is the frequency of observed oscillations. From (12), we get

$$a_1 = y(t^*) / \Sigma\, [\sin(i\omega_u t)/i] \quad (13)$$

Let us consider the time (t*) at which

$\omega_u\, t^* = 0.5\pi.$ (14)

Then (12) becomes:

$y(t^*)= a\, [1-(1/3)+(1/5) -(1/7)+(1/9)+\cdots]$ (15)

Where, 'a' is the modulus of a_1. In the above equation, let 'N' be the number of terms considered in (15). Using the limiting value for the summation term (0.25π):

$a = 1.273\, y(t^*)$ (16)

From the relay oscillation test, the value of ω_u is noted and the t* is calculated using (14). The value of the process output y(t*) is noted at t*. Then by selecting proper number of terms in (15), the calculated amplitude is obtained. The consideration of all higher order harmonics gives $a= 1.273\, y(t^*)$. This limiting value shows that a maximum error 27% in k_u is obtained by using the conventional analysis (principle harmonics method). The observed amplitude (a_0) is always less than the corrected amplitude (a). Later it will be shown that the actual maximum error in k_u will be of 27% [percentage error in ultimate gain is given by $(k_{u(\text{principle harmonics})}-k_{u(\text{exact})})/\ k_{u(\text{exact})}$]. The calculated value of k_u by the conventional method is always greater than the exact values. The method is verified for SOPTD system with various D/τ ratios (refer Table I). The present study shows that the value of N=5 gives better results on calculated k_u. It is also observed that, depending upon the D/τ ratio the system will filter out some of the higher order harmonics (not all). Hence, by incorporation terms for all N may lead to an error in the estimation of the ultimate gain (refer Table I). The actual number of the higher order harmonics to be considered, is also depends on the dynamics of the process. When there is no initial dynamics, a value of N=5 is suggested. If the system response shows any initial dynamics, then N=7 or N=9 is recommended.

3. Proposed method

From the definition of the Laplace transform [3]

$$y(s) = \int_0^\infty y(t)\exp(-st)\,dt \tag{17}$$

$$u(s) = \int_0^\infty u(t)\exp(-st)\,dt \tag{18}$$

The above integral can be evaluated for a particular value of s (say s_1). It is suggested to use the value $s_1= 8/t_s$ where 't_s' is the time at which three repeated cycles of oscillations appear in the output. The reason for taking $s_1=8/t_s$ is that, for $t > t_s$, because of very small value of the term $\exp(-s_1 t)$, the contributions by subsequent terms is negligible while evaluating the integral. Let the above resulting integral value be denoted as $y(s_1)$ and u(s1).

$y(s_1)/u(s_1) = k_p \exp(-D\, s_1)/(\tau s_1+1)^2$ (19)

$[y(s_1)/u(s_1)]\, (\tau s_1 + 1)^2 - k_p \exp[-D\, s_1] = 0$ (20)

From the amplitude criterion:

$k_p\, k_u/(\tau\, \omega_u\,)^2+1 = 1$ (21)

k_u used in the above equation is the corrected k_u giving

$v = \tau\omega_u = [(k_p\, k_u)-1]^{0.5}$ (22)

From the phase angle criterion:

$D = [\pi-2 \tan^{-1}(v)]/\omega_u$ (23)

$[y(s_1)/u(s_1)]\, [(v/\omega_u)s_1 + 1]^2 - k_p \exp\{-(s_1/\omega_u)\, [\pi-2 \tan^{-1}(v)]\} = 0$ (24)

using (22), (23) and (24), we get k_p τ and D.

4. Simulation Results

Three simulation examples are considered using symmetric relay with relay height ±1.

Case study 1: Consider SOPTD system with equal time constants as:

$G(s)=\exp(-0.5s)/(20s+1)^2$ (25)

The relay response consists of sinusoidal oscillations with exponentially increasing magnitude that reach steady state oscillations after fewer cycles. This is a typical response of a SOPTD model with equal time constants for low D/τ ratios [8]. Table II gives model identified using proposed method, S-C method [6], Li method and T-Y method [8]. Fig. 1 shows the open loop response of the actual and identified models for a unit step change in the set point. The proposed method gives response close to that of the actual system. Now let us design a controller based on the identified model parameters. Skogestad [7] suggested simple IMC ideal PID settings to design a controller for good robustness and the fast response with τ_c =D. Now, the controller is designed based on model identified (refer to Table II) and the performance is evaluated on the actual system. Fig. 2 shows that the response of the proposed method is close to the actual system. Now, to verify the performance of the controller in case of parameter uncertainty, the closed loop performance is evaluated on the perturbed system (keeping τ=30 in (25)). The proposed method gives response close to the actual system.

Case study 2: Consider a TOPTD with equal time constant system considered by Luyben [4]

$G(s) = 0.125 \exp(-s)/(s+1)^3$ (26)

Table II gives model identified by the proposed, S-C, Li and T-Y method. Fig. 4 shows the open loop system response of the actual and identified models. The proposed method gives a response close to that of the actual system. The controller is designed based on model identified using [7] (refer to Table II) and the performance is evaluated on the actual system (refer Fig. 5). The ISE value for proposed method is 3.85 and ISE for Li et al. method is 4.03 and 4.53 for the for S-C method. Hence, proposed method is preferred.

Case study 3 Consider TOPTD model [4] as:

$G(s)=\exp(-2s)/[(s+1) (10s+1) (20s+1)]$ (27)

Table II gives the identified model and the actual model. Fig. 6 shows the open loop performance for the unit step change in the set point. The proposed method gives response close to the actual system. For the closed loop performance, the controller is designed on the identified models and the performance is evaluated on the actual system for unit step change in the set point. Since error persists for long time ITAE is used to compare the closed loop performance. The ITAE for proposed method is 339.45, 1007.83 for the Srinivasan and Chidambaram method and 448.65 for Li et al. method. The ITAE value shows the proposed method is referred.

4.1 Effect of measurement noise

The effect of measurement noise on the accuracy of the estimation of the parameters is considered. The example considered for simulation, is [8]:

$G(s) = \exp(-10s)/(s+1)^2$ (28)

The measurement noises with a zero mean Gaussian distribution and a standard deviation of 0.5 %, 1% and separately of 1.5 % is added to the output of the system. In the identification test, the corrupted signal is used in the feed back control and for the system output. Once the initial dynamics are died out, the amplitude and the period of oscillation are calculated. Table III gives the identified parameters using the

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proposed method. As the noise level increases, the parameter estimation deteriorates. It is desirable to use a first order filter to remove the noise.

4.2 Effect of load

The effect of the load on parameter identification is also considered. The load of 0.02 and separately of 0.04 is introduced. The transfer function for the load is assumed that of the process transfer function. The load affected output signal is used for the feedback relay and as the system output for model identification purposes. The effect of load on model identification is reported in Table III. It is observed that, there is no significant effect of the load on the model identification. Hence, the identification method is robust.

4.3. A nonlinear bioreactor system

Consider a nonlinear continuous bioreactor. The dimensionless model equations are given by [1]:

$$dX_1/dt = (\mu - D)X \qquad (29)$$

$$dX_2/dt = (X_{2f} - X_2)D - (\mu X_1/\gamma) \qquad (30)$$

where,

$$\mu = (\mu_m X_2)/(K_m + X_2) \qquad (31)$$

Here X_1, X_2, X_{2f} are the dimensionless concentration of biomass cell, substrate and substrate feed respectively. 'D' is the dilution rate and 'μ' is the specific generation rate. The model parameter [1] as:

$$\gamma = 0.4\ \mathrm{g/g}; \quad X_{2f} = 1.0\ \mathrm{g/l}; \quad \mu_m = 0.4\,\mathrm{h}^{-1}$$

$$k_m = 0.05\ \mathrm{g/l}; \quad k_f = 0.4545\ \mathrm{g/g}, \quad D = 0.36\ \mathrm{1/h}^{-1}$$

The solution of (29) to (31) gives the following stable steady state

$[X_1, X_2] = [0.22, 0.45]$ (32)

Initially, the system is assumed to be at the stable steady state condition. At time t = 0, X_1=0.22 and X_2=0.45. At this condition the dilution rate (D) is 0.36 l/hr The substrate feed concentration is considered as the manipulated variable in order to control the cell mass concentration (X_1) at the stable steady state at X_1=0.22. A delay of four hour is considered in the measurement of X_1. For the given condition of the stable operating point (32), the local linearized model is obtained as:

$G(s) = 0.4 \exp(-4s)/(63.29\, s^2 + 25.59\, s + 1)$ (33)

A symmetrical relay with relay height (h) =0.02 is conducted. Relay output response (deviation value from the steady state point of X=0.22) is shown in Fig. 8. An open loop response of the nonlinear model is evaluated for a step change in X_1 from 0.22 to 0.242. Fig. 9 shows, the response of the proposed method close to that of the actual system. The response based on the identified model of T-Y [8] completely mismatch with that of the actual bioreactor system response.

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Table I Effect of higher order harmonics on k_u

D/τ	k_u for N					k_u	
	1	3	5	7	∞	Exact	Error
4	1.33	1.25	1.21	1.19	1.1	1.2	4
6	1.28	1.16	1.12	1.10	1.0	1.1	11
8	1.27	1.12	1.08	1.06	1.0	1.0	17
10	1.27	1.11	1.07	1.05	1..0	1.0	19
20	1.27	1.10	1.06	1.04	1.0	1.0	25
40	1.27	1.10	1.06	1.04	1.0	1.0	27

Table II Identified model parameters and controller settings

Case		Act.	Propo.	S-C	Li	T-Y
1	k_p	1	1.0	0.37	1.19	3.03
	D	0.5	0.54	3.86	0.52	0.32
	τ	20	20.81	64.79	20.09	35.4
	k_c	120	102.9	22.35	94.62	291
	τ_I	24	25.21	30.89	24.26	38.03
	τ_D	3.33	3.63	-	3.45	2.43
2	k_p	0.125	0.12	0.13	0.24	103
	D	1	1.49	2.14	1.05	0.04
	τ	1	1.32	2.55	2.07	48.6
	k_c		6.94	4.51	8.24	4.84
	τ_I		2.64	2.55	4.15	0.35
	τ_D		0.66	--	1.03	--
3	k_p	1	0.96	1.84	2.15	185
	D	2	3.0	7.63	2.0	0.20
	τ_1	1	14.2	90.36	21.79	207
	τ_2	10				
	τ_3	20				
	k_c		4.87	3.21	5.98	
	τ_I		28.41	61.0	37.79	
	τ_D		7.10		9.22	

Table III Effect of measurement noise and load

Effect considered for	σ (%)	Identified parameters		
		k_p	τ	D
Measurement noise with a zero mean Gaussian distribu.	0.5	0.97	0.81	9.99
	1.0	0.99	0.99	9.69
	1.5	1.0	0.99	9.74
Load	0.02	0.99	0.81	10.0
	0.04	1.02	0.91	9.88

Model used for simulation: $G(s)= \exp(-10 s)/(s+1)^2$

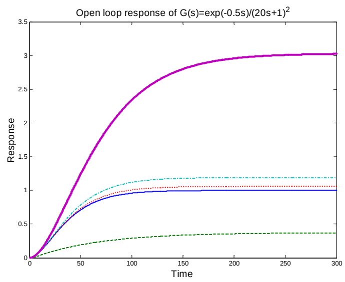

Fig. 1

Open loop response comparison for case study-1

Solid: Actual system
Dash: S-C method
Dotted: Proposed method
Dash-dot: Li et al. method
Solid (Thick): T-Y method

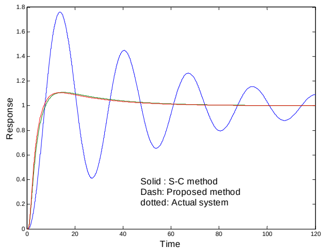

Fig. 2 Closed loop response comparison for case study-1

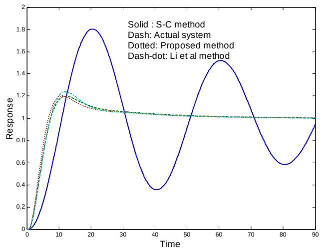

Fig. 3 Closed loop response for unit step change in set point for a perturbed system (with uncertainty in time constant)

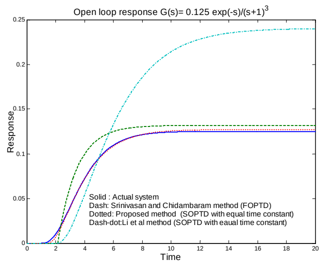

Fig. 4 Open loop response of case study-2

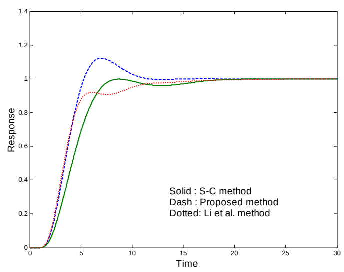

Fig. 5: Closed loop response for unit change in set point for case study-2

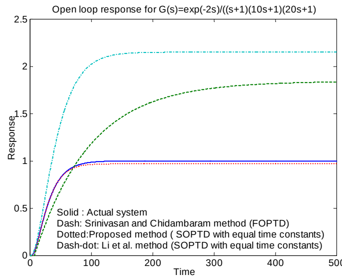

Fig. 6 Open loop response of case study-3

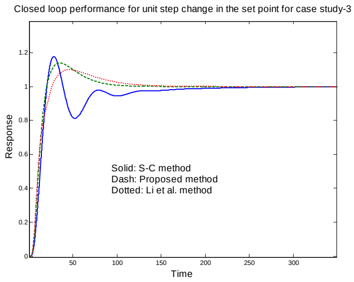

Fig. 7: Closed loop response for unit step change in set point for case study-3

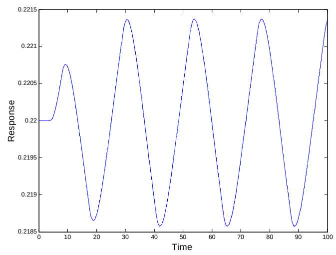

Fig. 8 Relay feedback response, bioreactor system

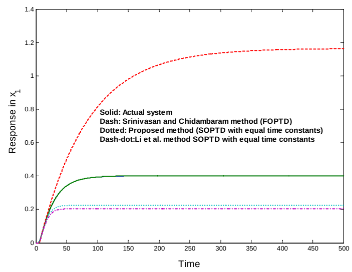

Fig. 9: Open loop response of bioreactor system