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Identification of Time-Varying Compliance and its Role in Cardiac Energetics using Computer Simulation

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***Abstract*— Using an Adaptive Identification Algorithm, the variations of Arterial Compliance were monitored during the cardiac cycle. A computer-simulated experiment demonstrated the contribution of a time-varying Compliance in reducing the Energetic Load on the Heart.**

***Index Terms*— Arterial Compliance, Cardiac Load, Computer Simulation, Energy, Model.**

I. Introduction

Several electrical models of the Arterial System exist [1], [2], [3]. They are grouped into two categories, the lumped-parameter and the distributed-parameter ones. We chose a well-established lumped-parameter model: the 3-element Windkessel, first proposed by Westerhof [4]. A circuit representation of the model is shown in Fig. 1.

The decision to use this particular model was influenced by the study of Burkhoff et al. [5] who examined the validity of this model, and found it to be a reasonable representation of cardiac load for purposes of predicting stroke volume, stroke work, oxygen consumption, and systolic and diastolic aortic pressures. In addition, the model is fairly simple, with only three passive elements representing the arterial system as the cardiac load.

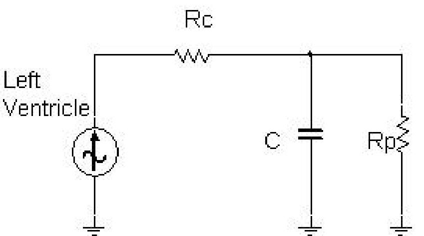

Fig. 1. The 3-element Windkessel model

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In this model the Compliance C represents the compliance of the Aorta and the large arteries. Rp is represents the resistance of the deep circulation, and responds to the metabolic needs of the body and to the Mean Arterial Pressure. Rc is the characteristic Impedance, i.e. the impedance theoretically measured when the animal is totally vasodilated. It is unlikely that either one of the two resistive elements change within one beat, during steady-state function.

However there is evidence the Compliance *C* changes during a cardiac cycle [6], [7], [8], [9], [10] even in steady-state operation of the heart. In this study we attempt to monitor the instantaneous changes in Compliance during a cycle, and the contribution of these changes in the reduction of the energetic load on the Heart.

II. Materials And Methods

A. Physiological Data Acquisition

Nine female Yorkshire pigs, 40-47 Kg, were preanesthetized with ketamine HCl(20 mg/kg) and xylazine (0.05 mg/kg). The pigs were then anesthtized with isoflurane. A midline tracheostomy was performed and the trachea was intubated. End-tidal CO2 was monitored, and arterial blood gases were kept within normal limits. A carotid artery was cannulated just long enough to reach the aortic root. Aortic Pressure was measured using a *Spectramed* model P23-XL pressure transducer. A midline sternotomy was performed and, and the aortic root was dissected free. An ultrasonic flowmeter transducer (Transonic Systems) was placed around the aortic root. It was connected to a Transonics Systems Model T201 flowmeter. Pressure and Flow were recorded on analog tape and then digitized.

B. Identification Technique

Resistance Rc was calculated as the ratio of the maximum derivative of Aortic Pressure P_A over the maximum derivative of Aortic Flow Q, according to a technique verified by Lucas [11]:

$$Rc = \frac{(dP_A / dt)_{MAX}}{(dQ / dt)_{MAX}} \quad (1)$$

Resistance Rp was calculated as the difference between the ratio of Mean Aortic Pressure and Mean Aortic Flow, and Rc:

$$Rp = \frac{P_{A\,MEAN}}{Q_{MEAN}} - Rc \quad (2)$$

The input into the system of Figure 1 can be either the Aortic Pressure or the Aortic Flow. If the Aortic Flow is chosen, then the Transfer Function is the Peripheral Impedance:

$$Z(s) = \frac{s + \frac{Rc + Rp}{CRcRp}}{s + \frac{1}{CRp}} \quad (3)$$

or if Rc<<Rp we can simplify the expression:

$$Z(s) = \frac{s + \frac{1}{CRc}}{s + \frac{1}{CRp}} \quad (4)$$

This Transfer Function is the most commonly used, but its Impulse Response will decay with a time constant τ=C*Rp. If we choose the Aortic Pressure as the input to the system, the new Transfer Function will be the inverse of Z(s), or the Peripheral Admittance Y(s):

$$Y(s) = \frac{s + \frac{1}{CRp}}{s + \frac{1}{CRc}} \quad (5)$$

The Impulse Response of (IR) of Y(s) will decay with a time constant τ=C*Rc, and since Rc<<Rp this time constant is much shorter, or the IR will decay faster with time. The method used in this study relies on our ability to estimate the IR at every instant of time during a cardiac cycle. For that purpose we use an Adaptive Algorithm that has to converge as soon as possible. Thus, it is more practical to attempt estimating a shorter IR, so we chose the Admittance system.

Most techniques used for the evaluation of the Windkessel models assume a time-invariant system, and rely on frequency spectra of pressure and flow. These techniques are thus inadequate for our study. The identification algorithm we used is based on the Least Mean Square (LMS) Algorithm [12]. This Algorithm has been shown to be capable of very fast convergence, and efficient in the identification of time-varying systems. It provides an Estimate of the Impulse Response (EIR) of the system at every point in time. Thus, we end up with a series of EIRs, one for every point. All these Impulse Responses are time-decaying exponentials, and we derive the Compliance C by matching their time constant(s) to C*Rc, where C is unknown and varying, but Rc is fixed and known from Equation (1). Thus from a series of EIRs we end up with a series of Compliance estimates.

The output of the Admittance System, Aortic Flow Q, is the convolution of Aortic Pressure and the Impulse Response Y of the system.

$$Q(n) = \sum_{k=0}^{L} P(n-k)Y(k) \quad (6)$$

Theoretically the convolution should extend over infinity, but only a limited number of data points L are available. The output of the Estimated System is Q_E, and it is equal to the convolution of the same Pressure and the Estimate Impulse Response (EIR).

$$Q_E(n) = \sum_{k=0}^{L} P(n-k)Y_E(k) \quad (6a)$$

To obtain the EIRs of the Admittance system, we set up a scheme as shown in Fig. 2. At every data point *n*, We compare the output of the Real System, which is the Aortic Flow Q(n), with the output of the Estimate System whose output is the Estimate Aortic Flow $Q_E(n)$. The two outputs are subtracted at every point to generate the Error *e(n)*. This Error is used to update our estimate of the system's Impulse Response, according to the LMS algorithm:

$$Y_E(k)_{n+1} = Y_E(k)_n + 2\mu * e(k) * P(n-k),$$
k=0,1,2,…L (7)

Where μ is a convergence factor. As the algorithm converges this EIR becomes a better approximation of the actual IR. As the EIR is updated at every sample point in time, it can presumably track the changes in the actual IR of the system. This way we obtain a series of EIR, one for every instant of time we sampled the data at.

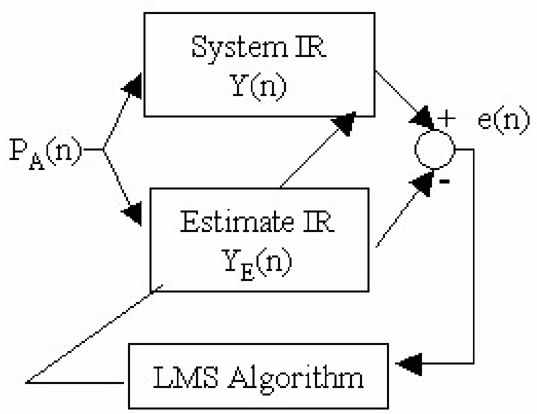

Fig. 2. Identification of Impulse Response of Admittance System Using LMS Algorithm.

Speed of convergence *also* depends on the number of weights needed to represent the IR of the system. This was the reason we chose to estimate the Admittance's IR which decays faster than the Impedance's. The convergence factor μ is also critical. A large μ makes the convergence faster, but the estimates tend to oscillate around the true solution resulting in poorer representation of the system. If μ is chosen too small, the algorithm becomes too slow to follow a fast-changing system. There are theoretical considerations on the choice of μ, but trial and error was our method of choice.

Fig. 3 demonstrates a typical Compliance during a cardiac cycle. It starts with a low value and increases at the beginning of systole, reaching a peak just before the peak of Aortic Pressure. It then decays back to the original value. These results are qualitatively consistent with other studies, especially [8] and [9]. Note however that in [8] the variation of the Pulmonary Arterial Compliance was monitored.

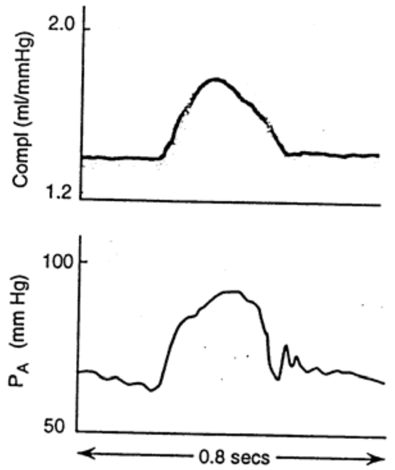

Fig. 3. Compliance as it varies during a cardiac cycle and Aortic Pressure for reference.

Table I. Data from Pigs. Flow in ml/s, C in ml/mmHg. Clow=pre-systolic compliance, Cpeak=peak Compliance during systole.

Pig	Q	Clow	Cpeak
1	39	1.57	2.29
2	34	1.2	1.81
3	50	1.5	2.11
4	35	1.3	1.93
5	34	0.58	1.05
6	35.3	0.9	1.52
7	28	2.1	2.89
8	55	0.8	1.44
9	31	1.12	1.98

C. *The Computer Simulation Experiment*

We suspected an important physiological contribution of the time-variation of Arterial Compliance. We investigated its contribution to facilitating the work of the Left Ventricle of the Heart during ejection. We decided to compare the Energy expended by the Ventricle in the case of a non-varying Compliance to the case of a varying one. Effectively, this comparison could only be done in simulation. Triangular shaped waveforms simulating Aortic Flow were fed to computer-simulated systems like the one shown in Figure 1. Every simulated system represented component values calculated for an animal (pig).

In the case of every animal a system with fixed components was used first, with Compliance set to the pre-systolic value. Next the same waveforms were fed to a system whose Compliance was time-varying in the way it was found to. Aortic Pressure waveforms were generated from the simulated systems. Fig. 4 demonstrates a set of these waveforms from a fixed-compliance case. In all cases a *proxy* for the Energy was calculated as the integral of the product of Pressure and Flow integrated over time, after the system had reached steady state. So the initial transients were ignored.

$$Energy = \int P_A Q dt \quad (8)$$

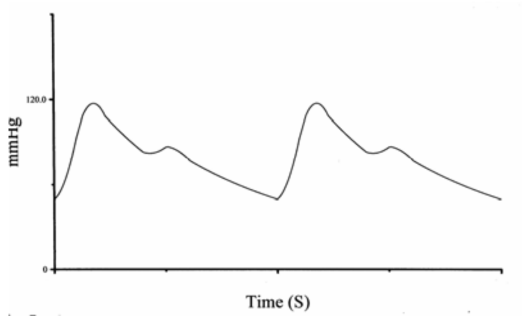

Fig. 4. Simulated Aortic Pressure waveforms from simulation program for fixed Compliance.

Table II summarizes the results of the simulation experiment. It was found that for every animal's simulated Arterial System, the time-variation of Compliance results in less Energy required by the Heart.

Table II. P1: Energy units in case of varying Compliance. P2: Energy units in case of Compliance fixed to its low value

Pig	E1(mmHg8ml)	E2(mmHg*ml)	P2/P1
1	2925	3612	1.23
2	2380	2892	1.21
3	4100	5214	1.27
4	2625	3112	1.18
5	2924	3201	1.09
6	2682	3152	1.17
7	2212	2766	1.25
8	2232	2733	1.22
9	3925	4688	1.19

III. Conclusion

Increased Compliance during systole facilitates the emptying of the Left Ventricle into the Proximal Aorta. Most of the Compliance represented in the model of Figure 1 exists in the Aorta and the large arteries immediately adjacent to it. This is the area the Ventricle empties the Stroke Volume into. When Compliance was not allowed to increase during the contraction phase, the Energetic Requirements increased. In [10] it was demonstrated that a higher compliance was strongly associated with a decrease in the energetic cost of contraction. After the contraction of the Ventricle, the Proximal Aorta stiffens, and thus its Compliance is reduced. We think this is important, for a stiffer Aorta would propel the blood just injected into the deep circulation faster. The proximal Aorta empties its contents into the deep circulation with a time constant approximately equal to C*Rp, so a smaller C would reduce the required time.

REFERENCES

[1] H. Akaike, "A new look at the statistical model identification", *IEEE Trans. Automat. Contr.* AC-19, 1974, pp. 716-723.

[2] J. T. Apter, "An Analysis of the Aortic Pressure Curve During Diastole", *Bull. Math. Biophys. 25* ,1963, pp. 325-339.

[3] M. Karamanoglu, "A System for Analysis of Arterial Blood Pressure Waveforms in Humans", *Computers and Biomedical Research 30* ,1997, pp. 244-255.

[4] N. Westerhof, G. Elzinga, and P. Sipkema, "An artificial arterial system for pumping hearts", *Journal of Applied Physiology 31* ,1971, pp. 776-781.

[5] D. Burkhoff, J. Alexander Jr and J. Schipke**, "**Assessment of Windkessel as a model of aortic input impedance", *Am J Physiol Heart Circ Physiol 255*, 1988, pp. H742-753.

[6] P. Segers, N. Stergiopulos and N. Westerhof, "Quantification of the Contribution of Cardiac and Arterial Remodeling to Hypertension ", *Hypertension* 2000;36. pp. 760-765 (models elastance as time-varying, basic the same we find)

[7] T. Wronski, P. B. Persson, E. Seeliger, A. Harnath, and B. Flemming, **"**Coupling of left ventricular and aortic volume elasticity in the rabbit", *Am J Physiol Regul Integr Comp Physiol* vol.279(2), Aug 2000, pp. R539-R547.

[8] B.J. Grant, J.M. Fitzpatrick and B.B. Lieber, "Time-varying pulmonary arterial compliance". *J Appl Physiol* 70, 1991, pp. 575-583.

[9] P. Segers, N. Stergiopulos, N. Westerhof, " Quantification of the Contribution of Cardiac and Arterial Remodeling to Hypertension", 2000;36;760-765 *Hypertension*; 36, 2000, pp. 760-765.

[10] P. Kolh, V. D'Orio, B. Lambermont, P. Gerard, C. Gommes, R. Limet, "Increased aortic compliance maintains left ventricular performance at lower energetic cost". *Eur J Cardiothorac Surg* 2000;17 pp. 272-278.

[11] L. C. Lucas, W.R. Benson, B. Ha and G.W. Henry, "Comparison of the time domain algorithms for estimating aortic characteristic impedance in humans". *IEEE Trans. Biomed. Eng*. 35, 1988, pp. 62-67.

[12] S. Haykin, "Introduction to adaptive filters", New York, NY: *MacMillan Publishing Company*, 1984, pp. 90-126.

[13] A.M. Vrettos, "Role of Arterial Compliance in the energetic efficiency of the ventricular/vascular system", *Abstract of Dissertation, Univ. of Kentucky*, 1993.