Proceedings of the World Congress on Engineering and Computer Science 2007
WCECS 2007, October 24-26, 2007, San Francisco, USA

Inverse Design Methodology for the Stability Design of Elliptical Bearings Operating with Non-Newtonian Lubricants

Raghunandana. K.

Abstract— **The elliptical bearing is the most widely used in turbines, and is probably the easiest noncircular bearing to make. To design the elliptical bearing, a few important characteristics, like load carrying capacity, flow requirement and power loss due to viscous friction are to be predicted correctly. These parameters can be determined if the pressure within the bearing is known. To have a complete knowledge of these, the generalized Reynolds equation containing viscosity, density, film thickness, surface motion and time as parameters is to be solved. The solution procedure utilized here is an inverse one. A data bank is generated for non-dimensional load carrying capacity in terms of Sommerfeld number, friction force and end flow at different eccentricity ratios. Since the data bank obtained is discreet, curve fitting is done to make them continuous. Using the perturbed pressures it has been possible to compute the four components of stiffness and damping coefficients of the bearings. Apart from the stiffness and damping coefficients another dynamic quantity of interest, namely the threshold of oil whirl instability has been calculated.**

***Index Terms*— Elliptical Bearing, Mass Parameter, Non-Newtonian, Stability.**

I. Introduction

In conventional bearing design the bearing characteristics are determined by assuming the lubricant as Newtonian and that the bearing shell as rigid. Several investigators [1, 2, and 3] have studied the performance characteristics of journal bearings under these assumptions. Since the early experiments of Newkirk [4, 5] who reported instability in circular bearings and came to the conclusion that this bearing type is often inadequate for high speed applications, much effort has been spent on the development of other bearing shapes in order to improve the bearing stabilizing capacity. Static and dynamic properties of a large number of fixed geometry bearings have been studied by several authors. The majority are concerned with the determination of the effect of parameters believed or known to affect stability on dynamic coefficient and stability limits. Some effort has been made also on the nonlinear analysis in order to verify the validity of the linearized stability limit or to predict the journal motion above the threshold speed.

Manuscript received June 9, 2007. This work was supported in part by the Manipal Institute of Technology, Manipal University.

Raghunandana. K. is with Manipal Institute of Technology, Manipal – 576 104, India. (Phone: 91-9448803941; fax: 91-0820-71071; e-mail: rkurkal@yahoo.co.in)

To enhance certain characteristics of the lubricants, various additives i.e., solids or liquids in the form of small particles, are added to the lubricant and the relation between shear stresses and shear strain rates for these lubricants are nonlinear. When the viscosity for these lubricants depends on shear strain rates, the lubricant is called non-Newtonian. Albert and Thamer [6] showed that the load capacity and the frictional force of the slider bearing increase with increase in concentration of additives and contaminants. Prakash and Sinha [7] determined the static characteristics of a journal bearing with micropolar fluids. Prabhakaran Nair et al. [8] also determined static and dynamic characteristics of circular rigid bearings with micropolar fluids. Dien and Elrod [9] have developed a regular perturbation expansion for velocity and pressure fields for non-Newtonian lubricants. Their analysis results in a slightly modified form of the Reynolds' equation.

To determine the sub synchronous whirl stability limit of a rigid rotor two methods are available, the linearized perturbation and the nonlinear transient analysis techniques. In the linear analysis the linearized oil film displacement and velocity are used to model the influence of hydrodynamic forces. Of these two methods the linearized perturbation method has been extremely popular because of its simplicity and limited computational requirements. A comprehensive range of mathematical models are available to represent the performance of bearing under engine operating conditions. Booker [10] introduced a direct technique, known as the mobility method, to evaluate the journal orbit under given dynamic load conditions. Moes and Bosma [11] developed finite length mobility, summing the short and long bearing mobility.

The current trend in the design of high speed rotating machinery is to have smaller weight and size with maximum efficiency and stability. In such applications multi-lobe journal bearings are often used to obtain better dynamic stability than a system with plain cylindrical journal bearing. The existing literature shows that the computer aided inverse design study on static and dynamic performance characteristics of a two-lobe journal-bearing operating under power law fluid are scarce. Hence, it is felt that there is a need to recompute the design of a two-lobe journal bearing considering the proposed inverse technique method. The present paper reports an inverse design methodology for the design of elliptical bearings in which a data bank is generated for a given eccentricity ratio (not for the applied load). Suitable sort of relationship between Sommerfeld number and eccentricity ratio is obtained. Similarly, the same for flow and friction force. This is more comprehensive for selection of the bearing dimensions and hence the stability.

NOMENCLATURE:

d Distance between center of one lobe to geometric center of bearing (m)
D Diameter of journal (m)
e_1, e_2 Eccentricity with respect to geometric centre of the bearing (m)
F_r, F_ϕ Hydrodynamic forces (N)
h Film thickness, $c(1+\varepsilon_{1,2}\cos\theta)\,(m)$ $\bar{h} = h/c$
I Second invariant of strain rate tensor
m Pseudoplastic viscosity constant $\left(Ns^n/m^2\right)$
$\overline{M}$ Mass parameter $\left(M_r c\omega^2/W_0\right)$ (dimensionless)
n Power law index (dimensionless)
p Film pressure (Pa) $\bar{p} = pc^2/\eta'\omega R^2$ (dimensionless)
Q Side flow rate (m^3/sec)
R Radius of either lobe (m)
s specific heat of lubricant, (J/kg/^{0}K)
t Time (s)
U Surface velocity in X-direction (m/s)
W_0 Steady load, $\overline{W}_0 = W_0 c^2/\eta'\omega R^3 L$ (dimensionless)
x,z Co-ordinates
$\theta, \bar{z}$ Dimensionless co-ordinates x/R , $\bar{z}/(L/2)$
δ Ellipticity ratio (d/c)
$\varepsilon_1, \varepsilon_2$ Lower and upper lobe eccentricity ratios
τ Dimensionless time, $\omega_p\, t$
ρ Density of lubricant, (kg/m^3)
ω, ω_p Angular velocity of journal, whirl frequency (rad/sec)
η Absolute viscosity of lubricant (Ns/m^2)
ϕ_1, ϕ_2 Attitude angles with respect to lobe centers (rad)
Ω Whirl ratio ω_p/ω

II. METHODOLOGY

Steady-State Characteristics:

With the basic assumptions in the theory of hydrodynamic lubrication, the modified Reynolds' equation derived by Dien and Elrod [9] can be written under dynamic condition as:

$$\frac{\partial}{\partial x}\left(\frac{h^{2+n}}{n}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^{2+n}\frac{\partial p}{\partial z}\right)=6m\,U^n\frac{\partial h}{\partial x}+12m\,U^{n-1}\frac{\partial h}{\partial t} \quad (1)$$

Here the viscosity, η is dependent on the second invariant of the strain rate tensor and is given as $\eta = mI^{\frac{n-1}{2}}$. The modified viscosity is $\mu' = m\left(\frac{U}{C}\right)^{n-1}$. When n = 1, $\eta^1 = m = \eta$ and the equation **(1)** reduces to Reynolds' equation for the Newtonian lubricant which when non-dimensionalised using rotating co-ordinates, is

$$\frac{\partial}{\partial\theta}\left(\frac{\bar{h}^{2+n}}{n}\frac{\partial\bar{p}}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^{2+n}\frac{\partial\bar{p}}{\partial\bar{z}}\right)=6\frac{\partial\bar{h}}{\partial\theta}-12\Omega\frac{\partial\phi}{\partial\tau}\frac{\partial\bar{h}}{\partial\theta}+12\Omega\frac{\partial\bar{h}}{\partial\tau} \quad (2)$$

Assuming that the journal whirls about its mean steady state position given by ε_0 and ϕ_0, for the first order perturbation, the pressure and film thickness can be expressed as [12]:

$$\bar{p} = \bar{p}_0 + \varepsilon_1 e^{i\Omega\tau}\bar{p}_1 + \varepsilon_0\phi_1 e^{i\Omega\tau}\bar{p}_2 \quad \textbf{(3)}$$

$$\bar{h} = \bar{h}_0 + \varepsilon_1 e^{i\Omega\tau}\cos\theta + \varepsilon_0\phi_1 e^{i\Omega\tau}\sin\theta \quad \textbf{(4)}$$

Where $\varepsilon = \varepsilon_0 + \varepsilon_1 e^{i\Omega\tau}$ and $\phi = \phi_0 + \phi_1 e^{i\Omega\tau}$ **(5)**

Substituting equations **(3-5)** in equation **(2)** and collecting the zeroth and first order terms for ε_1 and $\varepsilon_0\ \phi_1$, three set of equations are obtained. The solution of these equations gives the steady state and dynamic pressures. The design procedure of bearing can be simplified by the computer aided design. A designer is usually provided with three input data, such as diameter of the journal, radial load and speed. Knowing the specific application of the bearing the length of the bearing is assumed. The recommended radial clearance ratios for different alloys vary in the range of 0.0005 to 0.001. Bearing diameter is determined using the recommended value of C/R ratio. The non dimensionalised Reynolds equation is solved for the steady state pressure distribution with the following boundary conditions using the finite difference method.

$$\bar{p}_o = 0 \ at\ \bar{z} = \pm 1.0$$
$$\bar{p}_o = \partial\bar{p}_0/\partial\theta = 0 \ for\ \theta_2 \le \theta \le \theta_1 \quad \textbf{(6)}$$

where θ_1 *and* θ_2 are the angular co-ordinates where film starts and cavitates. The two lobes of the bearing (Fig.1) are considered as partial bearings initially and then the pressure fields are evaluated.

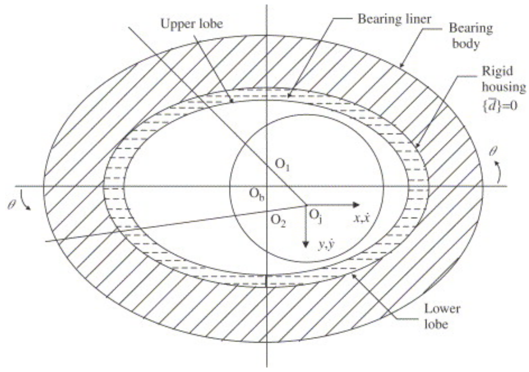

Figure 1: Elliptical Bearing

Eccentricity ratios of the lower lobe and upper lobe are given respectively as

$$\varepsilon_1^2 = \varepsilon^2 + \delta^2 + 2\varepsilon\,\delta\cos\phi \ \text{ and } \ \varepsilon_2^2 = \varepsilon^2 + \delta^2 - 2\varepsilon\,\delta\cos\phi \quad \textbf{(7)}$$

The film thickness is evaluated from

ISBN:978-988-98671-6-4

$\bar{h}=1+\varepsilon_1\cos\theta \quad for \ \pi-\varphi-\phi_1 \le \pi+\varphi-\phi_1$ and

$\bar{h}=1+\varepsilon_2\cos\theta \quad for \ \pi-\varphi+\phi_2 \le \pi+\varphi-\phi_2$. **(8)**

Forces are found out by integrating the pressure in the film region:

$$\bar{F}_{\varepsilon 1,2} = \iint \bar{p}\cos\theta \, d\theta \, d\bar{z} \text{ and } \bar{F}_{\phi 1,2} = \iint \bar{p}\sin\theta \, d\theta \, d\bar{z} \quad \textbf{(9)}$$

The resultant fluid film forces are then found from:

$$\bar{F}_{\varepsilon} = \bar{F}_{\varepsilon 1}\cos(\phi-\phi_1) - \bar{F}_{\varepsilon 2}\cos(\phi+\phi_2) + \bar{F}_{\phi 1}\sin(\phi-\phi_1) - \bar{F}_{\phi 2}\sin(\phi+\phi_2)$$

$$\bar{F}_{\phi} = \bar{F}_{\varepsilon 1}\sin(\phi-\phi_1) + \bar{F}_{\varepsilon 2}\sin(\phi+\phi_2) + \bar{F}_{\phi 1}\cos(\phi-\phi_1) + \bar{F}_{\phi 2}\cos(\phi+\phi_2) \quad \textbf{(10)}$$

For a given value of ε and ϕ if the summation of horizontal components of the forces do not vanish, then a new value of ϕ is provided for the same value of ε. Therefore an iteration process is adopted to evaluate the exact value of ϕ.

Steady-state load carrying capacity is given by,

$$\bar{W} = \sqrt{\bar{F}_{\varepsilon 0}^{\,2} + \bar{F}_{\phi 0}^{\,2}} \quad \textbf{(11)}$$

Where $\bar{F}_{\varepsilon 0}\, and\, \bar{F}_{\phi 0}$ are the steady-state fluid film forces.

Attitude angle is defined as, $\phi = \tan^{-1}\left(\dfrac{\bar{F}_{\phi 0}}{\bar{F}_{\varepsilon 0}}\right)$ **(12)**

The load is then expressed in terms of Sommerfeld number.

$$S = 1/\pi\bar{W} \quad \textbf{(13)}$$

The end flow is evaluated from the expression:

$$\bar{Q} = -\int_{\theta_1}^{\theta_2} \frac{\bar{h}_o^{\,3}}{6}\left.\frac{\partial \bar{p}_o}{\partial \bar{z}}\right|_{\bar{z}=1} d\theta \quad \textbf{(14)}$$

Friction force is found from:

$$\bar{F} = \int_0^1\int_{\theta_1}^{\theta_2} \frac{\bar{h}_o}{2}\frac{\partial \bar{p}_o}{\partial \theta} d\theta\, d\bar{z} + \int_0^1\int_0^{2\pi} \frac{1}{\bar{h}_o} d\theta\, d\bar{z} \quad \textbf{(15)}$$

A data bank is generated for non-dimensional load carrying capacity in terms of Sommerfeld number, friction force and end flow at eccentricity ratios 0.1 to 0.9 for the selected length and diameter of the bearing. The solution procedure outlined here is an inverse one. However, in practice, as mentioned earlier, one has to design a bearing for a given load, in addition to journal diameter and operating speed. As the data bank generated is for a given eccentricity ratio (not for the applied load), it becomes necessary to have some sort of relationship between Sommerfeld number (hence load) and eccentricity ratio. Similarly, the same for flow and friction force. There are large number of lubricants (non-Newtonian) available. Consulting oil manufacturers' data for these oils, a suitable lubricant is chosen. The lubricant properties, such as specific heat, mass density, power law index (which has to be experimentally found by a rheometer) and viscosities are to be fed as the input data.

Data bank obtained for various eccentricity ratios, for the specified length and diameter of the bearing is discreet. To make them continuous, we go for curve fitting. The best possible curve fitting equations used are:

Temperature Vs Viscosity for non-Newtonian lubricant

$$V = A_1\left(e^{\lambda_1 t}\right) + A_2\left(e^{\lambda_2 t}\right) \quad \textbf{(16)}$$

Sommerfeld number Vs Eccentricity Ratio:

$$Log\ S = m_1(\varepsilon) + c_1 \quad \textbf{(17)}$$

End flow Vs. Eccentricity Ratio:

$$\bar{Q} = m_2(\varepsilon) + c_2 \quad \textbf{(18)}$$

Friction force Vs. Eccentricity Ratio:

$$\bar{F} = B_1\left(e^{\lambda_3 \varepsilon}\right) + B_2\left(e^{\lambda_4 \varepsilon}\right) \quad \textbf{(19)}$$

where $A_1\ A_2\ B_1\ B_2\ m_1\ m_2\ c_1\ c_2\ \lambda_1\ \lambda_2\ \lambda_3\ \lambda_4$ are constants which are determined by using the data stored. For the specified load to be supported by the bearing at the given speed, and for the properties (power law index, viscosity, specific heat, density) of the lubricant, the Sommerfeld number is calculated. The end flow and friction factor for the operating eccentricity ratio is determined from equations **(14-15).**

As the shearing takes place in the oil, the temperature is likely to increase and hence the viscosity decreases. The thermal equilibrium has to be attained so that there is no drastic fall in viscosity of oil. This is accomplished by using the so-called *"two-thirds temperature rise"* rule originally given by Cameron. The heat generated is calculated from

$$H_g = FR\omega \quad \textbf{(20)}$$

Heat dissipated is calculated from $H_d = \rho Q s \Delta T$ **(21)**

where s and Δt are the specific heat of the lubricant and the temperature rise respectively.

In practice, heat is conducted through the bearing housing and the shaft. For simplicity it is assumed that heat generated goes into the oil. $H_g = H_d$. Thus $F R\omega = \rho\ Q\ s\ \Delta T$. Hence

$$\frac{\Delta T}{\eta'} = \frac{\pi N' L^2 \bar{F}}{C^2 \rho s \bar{Q}} \quad \textbf{(22)}$$ where N^1 is in rps.

The right hand of the above equation is computed from a given bearing, operating under certain condition and the effective temperature is evaluated. At this effective temperature the steady-state pressure distribution is found and the load carrying capacity is evaluated. The load carrying capacity thus evaluated is checked with the given load. It becomes necessary to adjust the various input quantities like length, clearance and lubricant characteristics so that the lubricated load is slightly higher than that of the applied load. When this condition is fulfilled, the preliminary design dimensions are noted. Now the designed bearing is checked for stability, as these bearings are prone to instability at higher speeds.

Stiffness and Damping Coefficients:

After having obtained the equilibrium temperature and steady-state solution, the perturbed pressures are solved by the similar way with the use of modified boundary conditions. The non-dimensional stiffness and damping coefficients are found from the following equations:

Comparison of steady state results with [14] (Table IV)		
Parameter	Present Result	Result from **[14]**
L/D Ratio	1	1
Eccentricity ratio	0.490	0.5
Non Dimensional Load	2.562	2.61
Non dimensional Side leakage	0.577	0.612

$$\overline{K}_{\varepsilon\varepsilon} = -\mathrm{Re}\left[\int_0^1 \int_{\theta_1}^{\theta_2} \overline{p}_1 \cos\theta\, d\theta\, d\overline{z}\right] \qquad \overline{K}_{\varepsilon\phi} = -\mathrm{Re}\left[\int_0^1 \int_{\theta_1}^{\theta_2} \overline{p}_2 \cos\theta\, d\theta\, d\overline{z}\right]$$

$$\overline{B}_{\varepsilon\varepsilon} = -\mathrm{Im}\left[\int_0^1 \int_{\theta_1}^{\theta_2} \overline{p}_1 \cos\theta\, d\theta\, d\overline{z}\right] \Big/ \Omega \qquad \overline{B}_{\varepsilon\phi} = -\mathrm{Im}\left[\int_0^1 \int_{\theta_1}^{\theta_2} \overline{p}_2 \cos\theta\, d\theta\, d\overline{z}\right] \Big/ \Omega$$

(23)

Where $\overline{K}_{ij} = \dfrac{K_{ij} c^3}{\eta^1 \omega R^3 L}$ and $\overline{B}_{ij} = \dfrac{B_{ij} c^3}{\eta^1 R^3 L}$. The other coefficients $\overline{K}_{\phi\varepsilon}, \overline{K}_{\phi\phi}, \overline{B}_{\phi\varepsilon}\ and\ \overline{B}_{\phi\phi}$ can be easily written by analogy.

Stability Analysis:

The stiffness and damping coefficients thus obtained are used in the study of stability of a rigid rotor. It is possible for a realistic flexible rotor-bearing system to carry out a complete analysis accounting for all factors external to the bearings. Each of the two bearing is assigned a journal mass equal to one half the rotor mass [13]. It then becomes possible to write the journal equation of motion along the line of centers as

$$0.5\overline{M}\,\overline{W}_0\left[\ddot{\varepsilon} - \varepsilon\left(\dot{\phi}\right)^2\right] = \overline{W}_r + \overline{W}_c \cos\phi$$
$$0.5\overline{M}\,\overline{W}_0\left[\varepsilon\ddot{\phi} + 2\dot{\varepsilon}\left(\dot{\phi}\right)\right] = \overline{W}_\phi + \overline{W}_c \sin\phi \qquad \textbf{(24)}$$

Using the stiffness and damping coefficients calculated above, non-dimensional mass parameter $\left(\overline{M}\right)$ and whirl ratio (Ω) have been calculated from the non dimensional form of the equation **(24)**. $\overline{M}$ thus found is the critical mass parameter above which the bearing is unstable and the corresponding Ω is the whirl ratio. The journal speed corresponding to the critical mass parameter is the threshold speed ω.

III. RESULTS AND DISCUSSIONS

The journal and bearing diameter, length of bearing, load, and speed are provided by the user. A grade of oil to be used is generally imposed which provides the viscosity, density and specific heat of the lubricant (Table I). The evaluated results in the non-dimensional and dimensional form are given in the tables II and III below respectively. Table IV shows the comparison of the steady state results with [14] which are in non dimensional form.

Standard Input Data (Table I)	
Length of bearing (mm)	100.0
Diameter of the bearing (mm)	100.0
Diameter of the shaft (mm)	99.56
Ellipticity Ratio	0.200
Radial load on the bearing (N)	500
Speed of the bearing (rpm)	200
Power law index	0.5
Density of the lubricant (kg/mm^3)	800
Specific Heat (J/kg K)	2000
Ambient Temperature	40
Viscosity at 40 degree (CP)	30
Viscosity at 100 degree (CP)	24

Results in Non-Dimensional Form (Table II)	
Eccentricity ratio	0.490
Load	2.562
Attitude angle (Deg)	59.491
Sommerfeld number	0.124
Side leakage	0.577
Stiffness SXX	10.261
Stiffness SYY	3.401
Stiffness SXY	- 0.197
Stiffness SYX	5.187
Damping CXX	12.908
Damping CYY	- 4.797
Damping CXY	-15.890
Damping CYX	10.100
Whirl ratio	0.581

Results in Dimensional Form (Table III)	
Eccentricity ratio(m)	1.078E-4
Load (N)	516.6830
*Flow (m**3/sec)*	8.31E-06
Avg Temperature(Deg C)	41.600
Viscosity(Pa.Sec)	2.98E-02
Stiffness SXX (N/m)	9405230
Stiffness SYY (N/m)	3117464
Stiffness SXY (N/m)	-18059.0
Stiffness SYX (N/m)	4754685
Damping CXX (N s/m)	451639.3
Damping CYY (N s/m)	-167949
Damping CXY(N s/m)	-556309
Damping CYX (N s/m)	353600.3
Whirl speed (rps)	15.224

IV. CONCLUSIONS

The basic assumption of Newtonian lubricant is inaccurate in the bearing analysis. Non-Newtonian effect considerably influences the performance of two-lobe bearings in terms of load carrying capacity, attitude angle, and flow and friction coefficient. A theoretical study on the effect of temperature and viscosity variation on the steady state characteristics of journal bearing has been made. This study provides steady state results for different L/D and eccentricity ratios in the form of empirical equations. From steady state and curve fitted results a computer aided design procedure is given. Finally, it is ensured that designed bearing operates stably.

The design example is for given oil, material properties and inlet temperature. A designer can generate similar data and obtain empirical relations for load, coefficient of friction, flow rate and stability using the foregoing method for different operating conditions, like oil and material properties.

REFERENCES

[1] O. Pinkus, Solution of Reynold's equation for arbitrarily loaded journal bearings, *J Basic Eng Trans* **3** (1961), pp. 145–152.

[2] F.K. Orcutt and E.B. Arwas, The steady state and dynamic characteristics of a full circular bearing and partial arc bearing in laminar and turbulent regimes, *Trans ASME, J Lub Tech* **89** (1967), pp. 143–153.

[3] S.C. Soni, R. Sinhasan and D.V. Singh, Analysis by the finite element method of hydrodynamic bearings operating in the laminar and super laminar regimes, *Wear* **84** (1983), pp. 285–296

[4] Newkirk, B.L., 1924, "Shaft Whipping," G.E. Rev., Vol. **27**, pp. 169-178.

[5] Newkirk, B.L., 1930, "Whirling balanced shafts," Third ICAM, Stockholm, Proc. **3**, pp. 105-110.

[6] E. Albert Yousif and M. Thamer Ibrahim, Lubrication of a slider bearing with oils containing additives and contaminants, *Wear* **81** (1982), pp. 33–45.

[7] J. Prakash and P. Sinha, Lubrication theory for micropolar fluids and its application to a journal bearing, *In J Eng Sci* **13** (1975), pp. 217–232.

[8] R. Narayanan, C.C. Narayanan and K.P. Nair, Analysis of mass transfer effects on the performance of journal bearings using micropolar lubricant, *In J Heat Mass Transfer* **30** (1995), p. 429.

[9] Dien.I.K. and Elrod.H.G. 1983 "A Generalized Steady-State Reynolds Equation for Non-Newtonian Fluids, With Application to Journal Bearings", Trans. ASME, J. of Lubrication Technology, **105**, 385—390.

[10] Booker, J.F., 1971, Dynamically loaded journal bearings: numerical application of the mobility method. Trans. ASME. J. Lubrication Technology, **93**, pp 168-176.

[11] Moes, H. and Bosma, R., 1981, "Mobility and impedance definitions for plain journal bearings." Trans. ASME. Jour. Lubrication Technology, **103**, pp 468—470.

[12] Majumdar. B.C., Brewe,D.E., and Khonsari,M.M. 1988,"Stability of a Rigid Rotor Supported on Flexible Oil Journal Bearings," ASME Journal of Tribology, **110**, pp 181—187.

[13] Abdul-Wahed N, Nicolas D, Pascal M. T. 1982, "Stability and unbalance response of large turbine bearings," Trans. ASME, **104**, pp 66-75

[14] Jagannath. K., K. Raghunandana, A. M. Chincholkar 2007, "Performance characteristics of two lobe bearings operating with non-Newtonian lubricants," Journal of Solid Mechanics and Materials Engineering, Vol. 1, No.4.