Thermal Effect on Vibration of Non-homogeneous Orthotropic Rectangular Plate Having Bi-directional Parabolically Varying Thickness

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ABSTRACT— A mathematical model is developed with an aim that scientists and design engineers can make a use of it with a practical approach, for the welfare of the human beings. Effect of thermal gradient is studied on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Using Rayleigh- Ritz procedure, frequency and deflection function are calculated for first mode of vibration for different values of thermal gradient, taper constants and non-homogeneity parameter.

Keywords—Thermal gradient, vibration, non-homogeneous, orthotropoic, rectangular plate, bi-directional, parabolically

1. INTRODUCTION

Study of effect of vibration is of immense importance. They can't be restricted only in the field of science but our day-to-day life is also affected by it. Whether it be a constructive aspect e.g. aircraft, space shuttle, satellite or design engineering to the destructive aspect, e.g. tsunami, earthquake etc., none of these are remained untouched with the effect of vibrations. Study of vibration responses of an orthotropic rectangular plate with thickness variation under the effect of temperature is of great importance for design officers, engineers and also to industry peoples, as rectangular plates may be regarded as an approximation to the wings, blades and variation in thickness may also lead to reduction in weight of structure.

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The monograph written by Leissa [1] is the ample source for the study of plate vibration. His subsequent review articles [2,3] are best source for an extensive bibliography on the subject of plate vibration. Fauconneau and Marangoni [4] evaluated the natural frequency of a rectangular plate of uniform thickness under the effect of temperature.

Tomar and Gupta [5,6] studied the effect of temperature on the vibration of an orthotropic rectangular plate of variable thickness. Bhardwaj, Gupta and Choong [7] carried out vibration analysis for rectangular orthotropic quarter elliptic plate with thickness varying linearly along both the principal axes. Gupta, Lal and Sharma [8] studied the vibration responses for a non-homogeneous circular plate with non-linearly varying thickness.

Present study is dedicated to evaluate the thermal effect on the vibrations of non-homogeneous orthotropic rectangular plate with parabolically varying thickness in both directions. Frequency and deflection function are calculated for the first mode of vibration, for different values of temperature gradient, taper constants and non-homogeneity constant, using Rayleigh-Ritz procedure.

2. METHOD OF ANALYSIS

Let the rectangular plate, which is orthotropic in nature, is subjected to a steady one dimensional temperature distribution T along the x-axis, i.e.

$$T = T_0\left(1-\frac{x}{a}\right) \qquad (1)$$

where T_0 is the excess temperatures above the reference temperature at x=a .
For most orthotropic materials, moduli of elasticity (as a function of temperature) are described as, [6]:

$$E_x = E_1\left[1-\alpha\left(1-\frac{x}{a}\right)\right], E_y = E_2\left[1-\alpha\left(1-\frac{x}{a}\right)\right] \qquad (2)$$

$$, G_{xy} = G_0\left[1-\alpha\left(1-\frac{x}{a}\right)\right]$$

where α is thermal gradient parameter.
The governing differential eq^n of transverse motion of an orthotropic rectangular plate of variable thickness in Cartesian coordinate is [6]

$$\begin{aligned} &D_x W_{,xxxx} + D_y W_{,yyyy} + 2HW_{,xxyy} + 2H_{,x} W_{,xyy} \\ &+2H_{,y} W_{,xxy} + 2D_{x,x} W_{,xxx} + 2D_{y,y} W_{,yyy} \\ &+D_{x,xx} W_{,xx} + D_{y,yy} W_{,yy} + D_{1,xx} W_{,yy} \\ &+D_{1,yy} W_{,xx} + 4D_{xy,xy} W_{,xy} + \rho h W_{,tt} = 0 \end{aligned} \quad (3)$$

A comma followed by a suffix denotes partial differential with respect to that variable.
Deflection function for free transverse vibrations of the plate can be written as, in the form of Levy type solution,

$$W(x, y, t) = W(x, y)e^{ipt}$$

Assuming density 'ρ' to be linearly varying in x-direction, i.e,

$$\rho = \rho_0 \left(1 + \alpha_1 \frac{x}{a}\right) \quad (4)$$

where $\rho_0 = (\rho)_{x=0}$

where α_1 is non-homogeneity constant.
Plate under consideration is assumed to have parabolically varying thickness in both the directions, i.e.

$$h = h_0 \left(1 + \beta_1 \frac{x^2}{a^2}\right)\left(1 + \beta_2 \frac{y^2}{b^2}\right) \quad (5)$$

where $h_0 = h|_{\substack{x=0 \\ y=0}}$

where β_1 & β_2 are taper constants.
For plate executing transverse vibration of mode shape W(x, y), the Strain and Kinetic energies [6] are, respectively

$$V = \frac{1}{2}\int_0^a\int_0^b \begin{bmatrix} D_x (W_{,xx})^2 + D_y (W_{,yy})^2 \\ +2D_1 W_{,xx} W_{,yy} \\ +4D_{xy} (W_{,xy})^2 \end{bmatrix} dydx \quad (6)$$

and $$T_1 = \frac{1}{2} p^2 \int_0^a\int_0^b \rho h W^2 dydx \quad (7)$$

In order to apply Rayleigh-Ritz procedure, max Strain energy must be equal to max Kinetic energy i.e.

$$\delta(V - T_1) = 0$$

The two term deflection function is taken as,

$$W(x, y) = \begin{bmatrix} \left(\frac{x}{a}\right)^2 \left(\frac{y}{b}\right)^2 \left(1-\frac{x}{a}\right)^2 \left(1-\frac{y}{b}\right)^2 \times \\ \left[A_1 + A_2 \left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \end{bmatrix} \quad (8)$$

where A_1 and A_2 are constants. a & b are length and width of the plate, respectively.

For this problem i.e. for clamped plate, the boundary conditions are,

$W = W,_x = 0$ at x=0,a and

$W = W,_y = 0$ at y=0, b

Using eq^n (2), (4),(5),&(8) in eq^n (9), after calculating V and T_1 from eq^n (6) &(7),one has,

$$\delta(V_1 - \lambda^2 T_2) = 0 \quad (9)$$

where $\lambda^2 = \dfrac{12a^4 \rho_0 p^2 (1-\nu_x \nu_y)}{E_1 h_0^2}$, which is frequency parameter
Eq^n (9) contains two unknown parameters A_1 and A_2, that may be evaluated as,

$$\frac{\partial(V_1 - \lambda^2 T_2)}{\partial A_q} = 0 \quad q=1,2$$

and one get,

$$c_{q1}A_1 + c_{q2}A_2 = 0 \qquad q=1,2 \quad (10)$$

where c_{q1} and c_{q2} (q=1,2) involves parametric constants and frequency parameter.
For a non- zero solution, it is desired that coefficient of equation (10) must vanish.
In this way the frequency equation comes out to be,

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} = 0 \quad (11)$$

3. RESULT AND DISCUSSION

Frequency equation (11) is quadratic in λ^2, so it will give two roots. But it had already been observed that Rayleigh Ritz method gives best approximation for first mode of vibration as compared to further higher modes of vibrations [6], if two term deflection function is used. So the frequency parameter and deflection function, both are calculated for the first mode of vibration of the plate. The parameters for orthotropic material have been taken as [6]

E_2/E_1 =0.32, $\nu_x (E_2/E_1)$ =0.04, $(G_0/E_1)(1-\nu_x\nu_y)$ =0.09

Results are plotted in fig 1,2,3,4,&5. In *fig.1*, shows the variation of frequency parameter'λ'with the thermal gradient parameter 'α' for the following two cases:

α_1=0.0, β_1=0.0, β_2=0.0 and

α_1=0.0, β_1=0.2, β_2=0.6

Clearly, when β_1&β_2 increases, λ increases but with an overall decrease with respect to α.
Fig. 2, shows an overall decrease in frequency parameter λ with respect to non-homogeneity constant but with increase in α, β_1 &β_2, frequency is found to increase, for the following two cases:

$$\alpha=0.0, \beta_1=0.0, \beta_2=0.0 \quad \text{and}$$
$$\alpha=0.4, \beta_1=0.2, \beta_2=0.6$$

From Figs *3& 4*, it is clear that as taper constants increases frequency parameter λ increases but in case of β_1 this increase is more as compared to β_2, for the following cases:

$\alpha_1 = 0.0, \alpha = 0.0, \beta_2 \, or \, \beta_1 = 0.0$;
$\alpha_1 = 0.0, \alpha = 0.0, \beta_2 \, or \, \beta_1 = 0.6$;
$\alpha_1 = 0.0, \alpha = 0.4, \beta_2 \, or \, \beta_1 = 0.0$;
$\alpha_1 = 0.0, \alpha = 0.4, \beta_2 \, or \, \beta_1 = 0.6$;
$\alpha_1 = 0.8, \alpha = 0.0, \beta_2 \, or \, \beta_1 = 0.0$;
$\alpha_1 = 0.8, \alpha = 0.0, \beta_2 \, or \, \beta_1 = 0.6$;
$\alpha_1 = 0.8, \alpha = 0.4, \beta_2 \, or \, \beta_1 = 0.0$;
$\alpha_1 = 0.8, \alpha = 0.4, \beta_2 \, or \, \beta_1 = 0.6$;

In *Fig. 5*, shows the variation of 'λ' with 'X', for the following cases,
$\alpha = 0.0, \beta_1 = 0.0, \beta_2 = 0.0, \alpha_1 = 0.0, a/b = 1.5$
for Y= 0.2 and 0.4
$\alpha = 0.4, \beta_1 = 0.2, \beta_2 = 0.6, \alpha_1 = 0.8, a/b = 1.5$
for Y= 0.2 and 0.4
From figs it is clear that when 'X' increases from 0 to1, 'W' firstly increases from 0 to some max value and than decreases to 0. Here X= x/a & Y= y/b.

The genuineness of above results are confirmed by comparing them with those of Leisaa [1] and Tomar and Gupta [6], under following conditions, for the following cases:

$$\alpha_1 = 0.0, \alpha = 0.0, \beta_1 = 0.0 \beta_2 = 0.0$$
&
$$\alpha_1 = 0.0, \alpha = 0.2, \beta_1 = 0.0 \beta_2 = 0.0$$

4. CONCLUSION

From the above results it is clear that increase in thermal gradient and non-homogeneity, reduces the frequency of vibration, whereas increase in taper constants increases the frequency of vibration.

When a comparison is done between plates, one having linearly varying bi-directional and other having parabolically varying bi-directional thickness, a similar pattern of result is found but on viewing microscopically, it is found that for plate having parabolically varying bi-directional thickness, all the effects are occurring for lesser values of frequency parameter. Hence it is concluded that plates with parabolic variation in thickness are more stable as compared to those of linearly varying thickness, for bearing up of thermally induced vibration effects.

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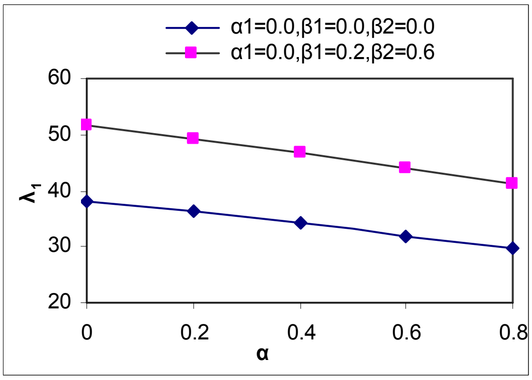

Fig.1: λ Vs thermal gradient α

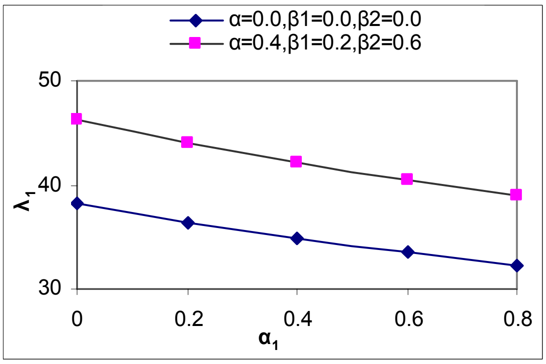

Fig.2: λ Vs non-homogeneity constant α_1

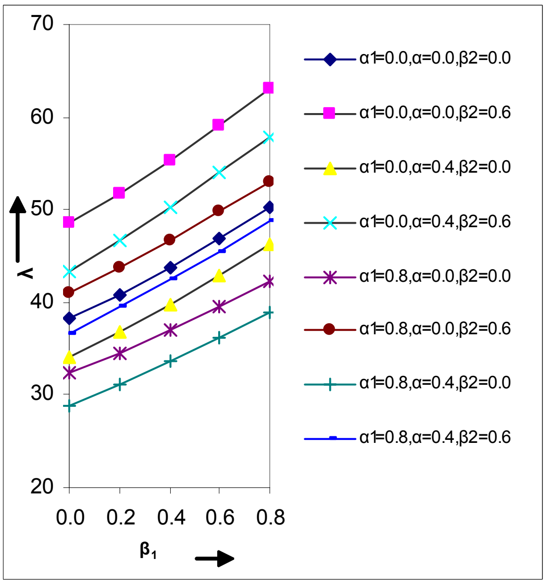

Fig.3: Frequency λ Vs taper constant β_1

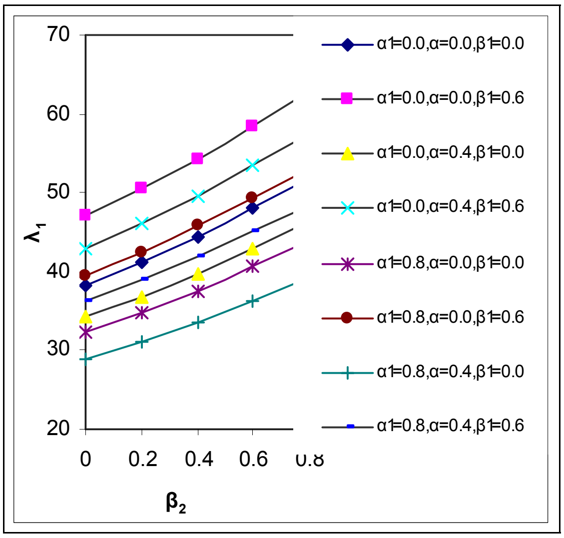

Fig.4: Frequency λ Vs taper constant β_2

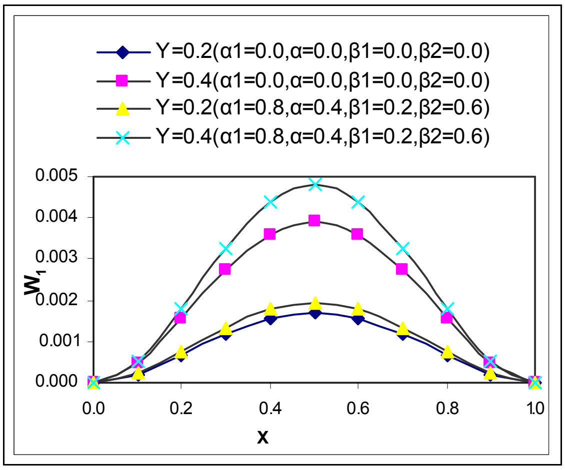

Fig.5: Deflection Vs X