Vibration of Visco-elastic Orthotropic Parallelogram Plate with Linearly Thickness Variation

Dr. A.K.Gupta, Amit Kumar**
*Dr. D.V.Gupta***

ABSTRACT— A simple model presented here is to study the effect of linear thickness variation on vibration of visco-elastic orthotropic parallelogram plate having clamped boundary condition on all the four edges. Using the separation of variables method, the governing differential equation has been solved for vibration of visco- elastic orthotropic parallelogram plate. An approximate but quite convenient frequency equation is derived by using Rayleigh-Ritz technique with a two-term deflection function. Time period and deflection function at different point for the first two modes of vibration are calculated for various values of taper constants, aspect ratio and skew angle(as θ shown figure1(a)).

Keywords—vibration,visco-elastic orthotropic ,parallelogram plate, linear thickness.

1. Introduction

Sufficient work [1 – 4] is available on the vibration of a rectangular plate of variable thickness, but none of them done on parallelogram plate. Singh and Saxena [5] have considered transverse vibration of skew plates with variable thickness. Bhatnagar and Gupta[6] have studied thermal effect on vibration of visco-elastic elliptic plate of variable thickness. Nair and Durvasul [8] studied vibration of skew plate. Sakiyama, Haung, Matuda and Morita [10] investigated free vibration of orthotropic square plate with a square hole. Li [11] has analyzed vibration of rectangular plate with general elastic boundary supports. Recently, Gupta and Khanna [9] have considered vibration of visco-elastic rectangular plate with linearly thickness variations in both directions.

The main object of the present investigation is to determine the effect of skew angle (θ) and taper constant on vibration of visco-elastic orthotropic parallelogram plate having clamped support on all the four edges. The hypothesis of small deflection and linear orthotropic visco-elastic properties are made. The Rayliegh-Ritz technique has been used to obtain the frequency equation of the plate. It is assumed that the visco-elastic of the plate are of the "Kelvin Type". Time Period and Deflection for the first two mode of vibration are evaluated for different values of aspect ratio, taper constant and skew angle (θ) and results are presented graphically.

* Department of Mathematics, M.S.College, Saharanpur-247001, India
** Department of Mathematics, College of Engineering, Roorkee-247667 India

2. PARALLELOGRAM PLATE AND EQUTION OF MOTION

The parallelogram plate R be defined by the three number a, b and θ as shown in figure 1(a) with $\xi = x - y\tan\theta$, $\eta = y\sec\theta$. The special case of rectangular plate follows by putting θ=0.

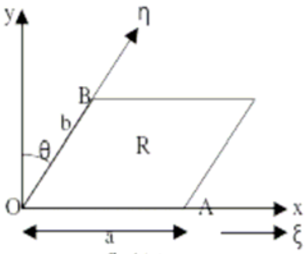


fig1(a)

For free transverse vibration of the parallelogram plate, w(ξ, η, t) can be expressed as

$$w(\xi,\eta,t)=W(\xi,\eta)T(t) \tag{1}$$

where T(t) is the time function and W is the maximum displacement with respect to time t.
The expressions for the strain energy, V_{max}, and kinetic energy,T_{max}, in the parallelogram plate when executing transverse vibration of mode shape W(ξ ,η) are [7]

$$V_{max}=(1/2)\int\int [\,D_\xi(W_{,\xi\xi})^2+D_\eta(W_{,\xi\xi}\tan^2\theta-2W_{,\xi\eta}\tan\theta\sec\theta+W_{,\eta\eta}\sec^2\theta)^2+2D_1W_{,\xi\xi}(W_{,\xi\xi}\tan^2\theta-2W_{,\xi\eta}\tan\theta\sec\theta+W_{,\eta\eta}\sec^2\theta)+4D_{\xi\eta}(-W_{,\xi\xi}\tan\theta+W_{,\xi\eta}\sec\theta)^2]\cos\theta\, d\eta d\xi \tag{2}$$

and

$$T_{max}=(1/2)\rho p^2\int_0^a\int_0^b (hW^2\cos\theta)\, d\eta d\xi \tag{3}$$

A comma followed by a suffix denotes partial differential with respect to that variable.

Assuming thickness variation of parallelogram plate linearly in ξ-direction only, as

$$h=h_0\{1+\beta(\xi/a)\} \quad (4)$$

where β is the taper constant in ξ-direction and $h_0=h|_{\xi=0}$

The flexural rigidities (D_ξ and D_η) and torsion rigidity ($D_{\xi\eta}$) of the plate can now be written as

$$(D_\xi/E_1)=(D_\eta/E_2)=(D_{\xi\eta}/G_0)/(1-\nu_\xi\nu_\eta)=h_0^3(1+\beta\xi/a)^3/12(1-\nu_\xi\nu_\eta) \text{ and } D_1=\nu_\xi D_\eta=\nu_\eta D_\xi \quad (5)$$

3. SOLUTIONS AND FREQUENCY EQUATION

The Rayleigh-Ritz technique requires maximum strain energy be equal to the maximum kinetic energy. So it is necessary
for the problem consideration that

$$\delta(V_{max}-T_{max})=0 \quad (6)$$

for arbitrary variations of W satisfying relevant geometrical bouandary conditions.

For a parallelogram plate clamped (c) along all the four edges, the boundary conditions are

$$W=W_{,\xi}=0 \text{ at } \xi=0,a \ \& \ W=W_{,\eta}=0 \text{ at } \eta=0,b \quad (7)$$

and the corresponding two-term deflection function is taken as [3]

$$W=[(\xi/a)(\eta/b)(1-\xi/a)(1-\eta/b)]^2[A_1+A_2(\xi/a)(\eta/b)(1-\xi/a)(1-\eta/b)] \quad (8)$$

which is satisfied Eq.(13),

Now assuming the non-dimensional variable as

$$X=\xi/a,\ Y=\eta/b,\ \hbar=h/a,\ \hat{W}=W/a \quad (9)$$

$$E^*_1=E_1/(1-\nu_\xi\nu_\eta),E^*_2=E_2/(1-\nu_\xi\,\nu_\eta),E^*=\nu_\xi E^*_2=\nu_\eta E^*_1 \quad (10)$$

and Component of E_1^* , E^*_2 , E^* and G_0 are E^*_1 , $E^*_2\sec\theta$,$E^*\sec\theta$ and $G_0\sec\theta$ respectively ξ- and η- direction .

Using Eqs.(5) ,(9) and (10) in Eqs.(2) and (3), then substituting the values of T_{max} & V_{max} from Eqs.(2) and (3) in Eq(6), one obtains

$$(V_1-\lambda^2T_1)=0 \quad (11)$$

$$V_1=\int_0^1\int_0^{b/a}(1+\beta X)^3[\{\sin^4\theta+(E^*_2/E^*_1)\sin^4\theta+2(E^*/E^*_1)\sin^2\theta\cos^2\theta+4(G_0/E^*_1)\sin^2\theta\cos^2\theta\}\hat{W}^2_{,XX}+(E^*_2/E^*_1)\hat{W}^2_{,YY}+4\{(E^*_2/E^*_1)\times\sin^2\theta+(G_0/E^*_1)\cos^2\theta\}\hat{W}^2_{,XY}+2\{(E^*_2/E^*_1)\sin^2\theta+(E^*/E^*_1)\cos^2\theta\}\hat{W}_{,XX}\hat{W}_{,YY}-4\{(E^*_2/E^*_1)\sin^3\theta+2(E^*/E^*_1)\sin\theta\cos^2\theta+2(G_0/E^*_1)\sin\theta\times\cos^2\theta\}\hat{W}_{,XX}\hat{W}_{,XY}-4\{(E^*_2/E^*_1)\sin\theta\,\hat{W}_{,YY}\hat{W}_{,XY}]dYdX \quad (12)$$

and

$$T_1=\int_0^1\int_0^{b/a}[(1+\beta X)\hat{W}^2]dYdX \quad (13)$$

Here $p^2=(E^*_1h_0^2/12a^2\rho\cos^5\theta)\lambda^2$ (14)

But equation (11) involves the unknown A_1 and A_2 arising due to the substitution of $W(\xi , \eta)$ from eq (8). These two constants are to be determined from eq(11),as follows:

$$\partial(V_1-\lambda^2T_1)/\partial A_n=0, \qquad n=1,2 \quad (15)$$

Equation (15) simplifies to the form

$$b_{n1}A_1+b_{n2}A_2=0, \qquad n=1,2 \quad (16)$$

where b_{n1} , b_{n2} (n = 1, 2) involve parametric constants and the frequency parameter .

For a non-trivial solution, the determinant of the coefficient of equation (16) must be zero. So one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} =0 \quad (17)$$

Form Eq. (17), one can obtains a quadratic equation in p^2 from which the two values of p^2 can found. After determining A_1 & A_2 from Eq.(16) , one can obtain deflection function W. Choosing $A_1=1$, one obtains $A_2=(-b_{11}/ b_{12})$ and then W comes out as

$$W=[XY(a/b)(1-X)(1-Ya/b)]^2[1+(-b_{11}/b_{12})XY(a/b)(1-X)(1-Ya/b)]. \quad (18)$$

4. DIFFERENTIAL EQUATION OF TIME FUNCTION AND ITS SOLUTION

The governing differential equation of time function of an orthotropic parallelogram plate of variable thickness one direction, is [11]

$$T_{,tt}+p^2(\check{n}/G)T_{,t}+p^2T=0 \quad (19)$$

Let us take intial conditions as

$$T=1 \text{ and } T=0 \text{ at } t=0 \quad (20)$$

Using intial conditions from Eq.(20) in solution of diff.eq.(19),one obtains

$$T(t)=e^{rt}[\cos(st)+(-r/s)\sin(st)] \quad (21)$$

where $r = -(p^2\check{n} / 2G)$ and $s= p(G^2 - p^2\check{n}^2) /2G$.

Thus , deflection w may be expressed , by using Eq.(21) and (18) in Eq.(1), to give

$$w = [XY(a/b) (1- X) (1- Ya/b)]^2[1+(- b_{11}/ b_{12}) XY(a/b) (1- X) (1- Ya/b)]x[e^{rt}\{ \cos(st) +(- r/s) \sin(st) \}] \quad (22)$$

Time period of vibration of the plate is given by

$$K=2\pi/p, \quad (23)$$

where p is frequency given by Eq.(17).

5. RESULT AND DISCUSSION

Time period and deflection are computed for viscoelastic orthotropic parallelogram plate whose thickness varies linearly
for different values of angle(θ), taper constant(β) and aspect ratio(a/b) at different points for first two mode of vibrations. The orthotropic
material parameters have been taken as [7] $E^*_2/E_1^*=0.01$, $E^*/E_1^*=0.3$, $G/E_1^*=0.0333$, $\check{n}/G=0.000069$, $E_1^*/\rho=3.0\times10^5$ and $h_0=0.01$

meter. These results are plotted in fig(1.1) ,(1.2) ,(1.3) and (1.4).

Fig(1.1) shows the graph of time period(K) for different values of taper constant (β) and fixed aspect ratio (a/b=1.5) for two values of angle (θ) i.e. θ=0 and θ=45 for first two mode of vibration. It can be seen that the time period (K) decrease when taper constant (β) increase for two mode of vibration at θ=0 and the time period (K) increase then slightly decease when taper constant (β) increase for two mode of vibration at θ=45.

Fig(1.2) shows the graph of time period(K) for different values of aspect ratio (a/b) and fixed taper constant (β=0 and β=0.6) for two values of angle (θ) i.e. θ=0 and θ=45 for first two mode of vibration. It can be seen that the time period (K) increase when aspect ratio (a/b) increase for two mode of vibration.

Fig(1.3) and (1.4) show the graph of deflection(w) for different values of X and fixed taper constant (β=0and β=0.6) ,Y=0.2, Y=0.4 and aspect ratio (a/b=1.5) for two values of angle (θ) i.e. θ=0 and θ=45 for first two mode of vibration. It can be seen that deflection (w) start from zero to increase and then decrease to zero for first two mode of vibration except for second mode at Y=0.4 ,β=0 or 0.6. At Y=0.4 and β=0 for second mode of vibration deflection (w) start from zero to increase and then decrease and then increase and then decrease and finally become to zero and at a Y=0.4 and β=0.6 for second mode of vibration deflection (w) start zero to decrease and then increase and finally become to zero for different value of X .

REFERENCES

[1] Leissa, A.W. Recent studies in plate vibration: 1981-1985, part-II, complicating effects, The Shock and Vibration Dig., Vol.19 , No.3, pp.10-24, 1987.

[2] Laura, P.A.A. , Grossi, R.O. and Carneiro, G.I. 'Transverse vibrations of rectangular plates with thickness varying in two directions and with edges elastically restrained against rotation', J. Sound and Vibration, Vol.63, No.4, pp.499-505, 1979.

[3] Tomar , J.S. and Gupta , A.K. 'Effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions', J. Sound and Vibration, Vol.98, No.2, pp.257-262, 1985.

[4] Sobotka, Z. 'Free vibration of visco-elastic orthotropic rectangular plates', Acta.Technica, CSAV, No.6, pp. 678-705, 1978.

[5] Singh, B. and Saxena, V. 'Transverse vibration of skew plates with variable thickness', J. Sound and Vibration, Vol. 206, No.1, pp.1-13, 1997.

[6] Bhatnagar, N.S. and Gupta, A.K. 'Thermal effect on vibration of visco elastic elliptic plate of variable thickness', Proc. Of International Conference On Modelling and Simulation, Melbourne, pp.424–429, 1987.

[7] Leissa, A.W. 'Vibration of plates', NASA SP-160, U.S. Govt. Printing office, 1969.

[8] Nair, P.S. and Durvasula, S. 'Vibration of skew plate', J. Sound and Vibration, Vol.26, No.1, pp.1-20, 1973.

[9]Gupta, A.K. and Khanna, A. 'Vibration of visco-elastic rectangular plate with linearly thickness variations in both directions ', J. Sound and Vibration, Vol.301, pp.450-457, 2007.

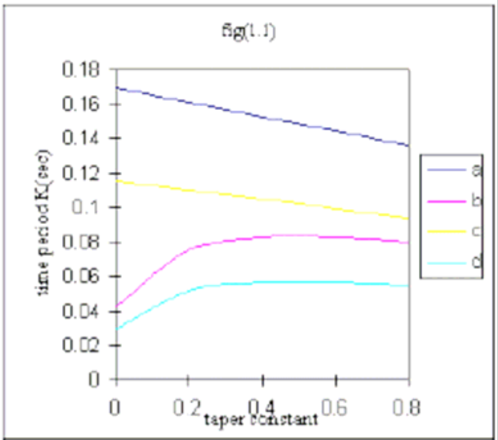
[10] Sakiyama, T. ,Haung, M.,Matuda, H. and Morita, C. 'Free vibration of orthotropic square plate with a square hole', J. Sound and Vibration, Vol. 259, No.1, pp.63-80, 2003.

[11] Li, W. L. , 'Vibration Analysis of rectangular plate with general elastic boundary supports', J. Sound and Vibration, Vol. 273, No.2, pp.619-635, (2004).

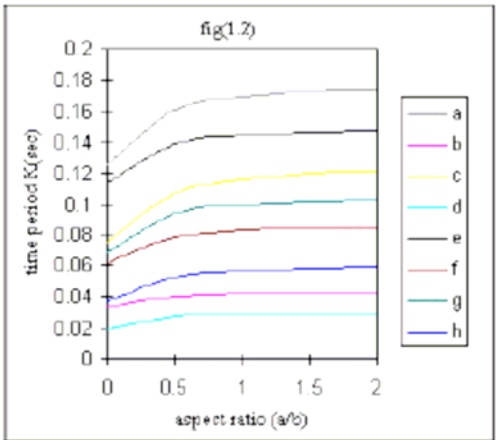
APPENDIX: LIST OF SYMBOLS

a length of orthotropic parallegram plate ,
b width of plate ,
ξ ,η co-ordinate in the plane of plate,
h(ξ ,η) plate thickness at the point (ξ ,η) ,
E_ξ , E_η Young's moduli in the ξ- and η-direction, respectively
ν_ξ , ν_η Poisson's ratio ,
D_ξ , D_η flexural rigidity in the ξ- and η-direction, respectively
$D_{\xi\eta}$ torsional rigidity,
G shear modulus,
ň visco-elastic constant,
ρ mass density per uint volume of the plate material ,
t time,
p radian frequency of vibration,
w(ξ ,η,t) transverse deflection of the plate, at the point (x,y) ,
W(ξ ,η) deflection fuction,
T(t) time fuction,
β taper constant,
K Time period .

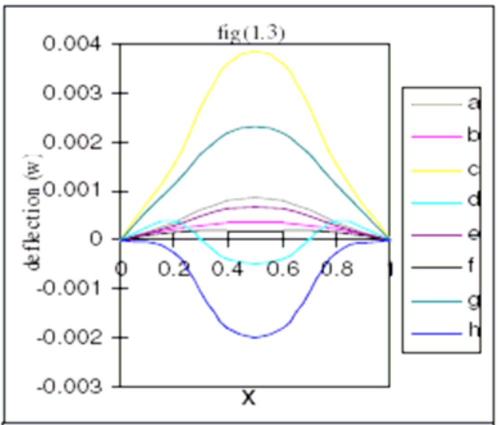
ISBN:978-988-98671-6-4



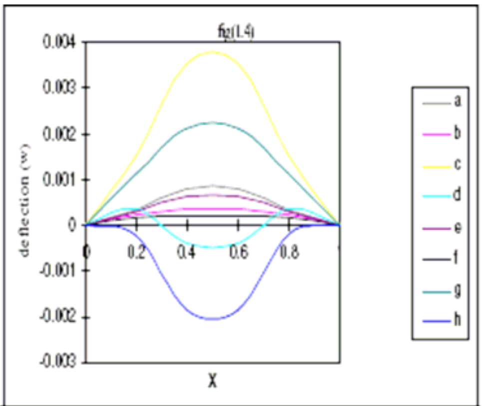
fig(1.1): Time period verse taper constant(β).
a-first mode(at θ=0) ,
b-second mode (at θ=0),
c-first mode(at θ=45),
d-second mode (at θ=45),
and aspect ratio (a / b =1.5)
for all X , Y.



fig(1.2):Time period verse aspect ratio(a/b).
a-first mode(at θ=0,β=0) , b-second mode(at θ=β=0),
c-first mode(at θ=45,β=0) , d-second mode(at θ=45 , β=0),
e-first mode(at θ=0,β=0.6) and f-second mode(at θ=0, β=0.6),
g-first mode(at θ=45,β=0.6) and h-second mode(at θ=45, β=0.6),
for all X,Y



fig(1.3): Deflection(w) verse X for θ=0.
a-first mode(at Y=0.2,β=0) ,b-second mode (at Y=0.2 , β=0),
c-first mode(at Y=0.4,β=0) ,d-second mode (at Y=0.4 , β=0),
e-first mode(at Y=0.2,β=0.6) ,f-second mode (at Y=0.2 , β=0.6),
g-first mode(at Y=0.4,β=0.6) ,h-second mode (at Y=0.4 , β=0.6)
and aspect ratio (a / b =1.5)



fig(1.4): Deflection(w) verse X for θ=45.
a-first mode(at Y=0.2,β=0) ,b-second mode (at Y=0.2 , β=0),
c-first mode(at Y=0.4,β=0) ,d-second mode (at Y=0.4 , β=0),
e-first mode(at Y=0.2,β=0.6) ,f-second mode (at Y=0.2 , β=0.6),
g-first mode(at Y=0.4,β=0.6) ,h-second mode (at Y=0.4 , β=0.6)
and aspect ratio (a / b =1.5)

ISBN:978-988-98671-6-4