Wave Forecasting using computer models – Results from a one-dimensional model

S.K. Khatri and I.R. Young

Abstract— **The wave forecasting and hindcasting studies are often used for estimating wave parameters that are useful for the design of coastal structures, beach management planning and to predict natural disasters in order to take appropriate measures to avoid and reduce the damages.**

The common approach for such cases is the development of numerical models that are validated against the experimental measurements preferably field studies. The wave forecasting models incorporate representation of a number of source terms that contribute to the wave growth, some of these add energy to the system and others result in depletion of the energy. However, one important term among these is non-linear wave-wave interactions that have a positive and negative signature and thus results in both addition and withdrawal of energy from the wave spectrum. It plays a critical role in the evolution of wave spectrum and requires suitable techniques for its evaluation. It has always been a challenging task to compute this complex term.

This study presents results from a one-dimensional wave prediction model which incorporates a full form of non-linear wave-wave interactions and thus resulting in improved results as compared to the experimental studies.

Index Terms—**Non-linear wave interactions, Ocean waves, Wave forecasting, Wave models**

I. INTRODUCTION

The wave forecasting models have gone through a considerable development over more than 40 years. These have evolved from simple parametric forms to recently sophisticated models such as WAM, SWAN, WAVEWATCH III and so on. The increase in computing power and enhanced understanding of the physical processes responsible for wave evolution have resulted in reasonably sophisticated models that provide sufficiently reliable wave forecasting.

However, in order to develop more complex models, it is imperative to study the wave evolution using one-dimensional models which provide useful insight in important physical processes that help to develop reliable models for practical applications. This work is intended to provide results from such a wave model which is described in more detail in the following sections.

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II. DESCRIPTION OF MODEL

The wave modeling is a tool for predicting wave conditions as a future event in time and space. The success of a wave model depends upon the suitable representation of physical processes responsible for wave growth and numerics of the model. The wind-wave generation being a complex process is difficult to express mathematically due to many uncertainties inherent in the complex nature of these physical processes. A number of earlier wave models commonly known as “first and second generation models” mainly depend upon adhoc assumptions on the shape of wave spectra and the empirical form of the processes represented. The main constraint on developing the physically sound models had been the lack of understanding of the physical processes as well as the limited computer capabilities. But since last decade, there has been an increase in the understanding of the processes contributing to wave growth and the computer power, even today’s desktops. This has led to development of third generation wave models such as WAM [12], WAVEWATCH [18] and SWAN [1]. These models have been reasonably successful in operational predictions in various parts of the world. However, the need lies for more sophisticated representation of the processes, particularly the nonlinear wave interactions. In case of WAM, this has been achieved by using Discrete Interaction Approximation (DIA) which is a better representation than the earlier approaches; however, it uses a selected number of wave number combinations. The need to include more dynamic representation of this term has always been stressed [24]. One of such attempt was done by [9] in the form of EXACT-NL model. The model in its original form was only able to run on CRAY computers and for deep water only. It has been modified to run on workstations and is applicable for both deep and finite water depth. The essential parts of this model are:

A. Radiative Transfer Equation

The wave growth in terms of space and time is often expressed in the form of radiative transfer equation [7, 20]. In this equation the terms on left hand side represents evolution of spectrum which is equated to the summation of all terms that contribute to it. Mathematically, it is written as:

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$$\frac{\partial E(f,\theta)}{\partial t} + C_g \frac{\partial E(f,\theta)}{\partial x} = \Sigma S \quad (1)$$

where C_g is the group velocity and $E(f, \theta)$ is spectral energy in frequency (f) and direction (θ) domain. "S" represents source terms which include wind input (S_{in}), white cap dissipation (S_{ds}), nonlinear wave interactions (S_{nl}) and for finite water depth conditions the bottom friction (S_{bf}).

B. Wind Input

Wind input is expressed in a number of forms; these include those proposed by [10, 16, 17] among others. This term is mostly expressed as a function of inverse wave age and spectral energy. The modified EXACT-NL model uses form proposed by [16] and is written as:

$$S_{in} = \max\left[0, 0.025 \frac{\rho_a}{\rho_w}\left(\frac{U_{10}}{C_p}\cos\theta - 1\right)\omega E(f,\theta)\right] \quad (2)$$

where ρ_a and ρ_b are the air and water densities respectively, C_p is the phase speed of waves, U_{10} the wind speed at a height of 10 m and $\theta = 2\pi f$.

C. Whitecap Dissipation

Waves loose energy by formation of white caps and it is represented in wave models as a dissipative process. It is normally expressed in linear form and for the present case a form proposed by [13] is used:

$$S_{ds} = -C\varpi\left(\frac{\omega}{\varpi}\right)^n \left(\frac{\alpha}{\alpha_{PM}}\right)^m E(f,\theta) \quad (3)$$

where m, n and C are constants, the first two normally have a value of 2 and C is 3.33x10^{-5}. Further, ω is an integral wave steepness parameter and α_{PM} = 4.57x10-3 is theoretical value of α for a Pierson-Moskowitz spectrum [14].

D. Non-linear wave-wave interactions

It is this source term which is complex in nature and requires considerable mathematical and computing effort for its evaluation. Normally it is expressed in the form of Boltzmann integral given by [6] and is written as:

$$S_{nl} = \omega \iiint \begin{array}{l} G(k_1,k_2,k_3,k_4)\times\delta(k_1+k_2-k_3-k_4) \\ \times\,\delta(\omega_1+\omega_2-\omega_3-\omega_4)\times \\ \left[n_1 n_3(n_4-n_2) n_2 n_4(n_3-n_1)\right] dk_1 dk_2 dk_3 \end{array} \quad (4)$$

where $n_i = n(k_i)$ is the action density, k_i represent interacting wave numbers and $G(k_i)$ is the strength of interactions.

E. Bottom Friction

When waves travel in finite water depth these come in contact with ocean bottom and thus loose energy due to friction. The expression proposed by [3] has been used in the model, and is written as:

$$S_{bf} = -\frac{C_f}{g^2}\frac{\omega^2}{\sinh^2 kd} E(f,\theta) \quad (5)$$

where C_f is a friction coefficient that varies with the bed conditions and d is water depth.

III. EXPERIMENTAL STUDY

In order to obtain reliable estimates from the wave forecasting models, these need to be calibrated against experimental studies.

The wave growth under fetch limited condition ignoring depth limited conditions has been extensively studied, with well-known measurements reported by [5, 8] among others. Although attempts for such measurements in finite water depth have been made in past, notably those of [2, 4], however, the former was mainly concentrated on finding the depth limited asymptotes rather than providing insight on the evolution along fetch. The latter study although performed with more sophisticated instruments, but was based on measurements at a single location, and hence does not represent true fetch limited conditions.

An in-depth and well planned wave measurement study in finite water depth was carried out at Lake George [21, 22, 23]. The purpose of this study was to provide reliable and truly fetch-limited wave growth results. The Lake is 20 km long by 10 km wide and the depth is approximately uniform i.e. 2 m. A total of 8 measuring stations were located along the longest possible fetch available i.e. in North-South direction. The Zwart poles were used to record the wave height variation together with anemometers at a height of 10 m to record wind speed. The directional measurements were recorded at only one location using a spatial array of seven Zwart poles. The full details of the experiment are given in [21].

The above study reported empirical formulae for wave energy and peak frequency. The second part of this experiment has recently been completed with the aim of measuring the source term balance of wind-wave evolution. The formulae suggested in first study for energy and peak frequency growth are applicable both for deep and finite depth wind waves and were found in good agreement with the relationships proposed for deep water in Shore Protection Manual [15]. The proposed non-dimensional relationships can be written as:

$$\begin{array}{ll} \varepsilon = g^2 E / U_{10}^4, & \chi = gX / U_{10}^2 \\ \delta = gd / U_{10}^2, & \nu = f_p U_{10} / g \end{array} \quad (6)$$

where χ, ε, ν, and δ are non-dimensional fetch, energy, peak frequency and water depth respectively. X is fetch, E spectral energy, d water depth, f_p peak frequency and U_{10} wind speed at a height of 10 m.

According to this study, the non-dimensional energy (ε) is given as:

$$\varepsilon = 3.64\times10^{-3}\left\{\tanh A_1 \tanh\left[\frac{B_1}{\tanh A_1}\right]\right\}^{1.74} \quad (7)$$

where

$$A_1 = 0.493\delta^{0.75}, \; B_1 = 3.13\times10^{-3}\chi^{0.57} \quad (8)$$

and non-dimensional peak frequency (ν) is expressed as:

$$\nu = 0.133\left\{\tanh A_2 \tanh\left[\frac{B_2}{\tanh A_2}\right]\right\}^{-0.37} \quad (9)$$

where

$$A_2 = 0.331\delta^{1.01}, \; B_2 = 5.215\times10^{-4}\chi^{0.73} \quad (10)$$

It was shown that when plotted, a family of curves of ε versus χ for different non-dimensional water depths (δ) were obtained. These curves showed a behavior similar to deep water at short non-dimensional fetches, however, they grow as χ increases reaching an asymptotic level for each non-dimensional water depth. A similar behavior was observed for variation of peak frequency, the variation of ν versus χ showed a different curve for each non-dimensional water depth.

The role of bottom friction in development of wind waves was further discussed in detail. It was argued that although the bottom of Lake George consists of mud with no bed movement (no ripple formation), but the results of this study were in close agreement with the previous formulations presented by [4] and [19]. Hence, it was speculated that the active wave generation might not be affected by the bottom conditions and the results could be applicable universally.

IV. RESULTS AND CONCLUSIONS

A. *Effects of Phillip's parameter α*

It has been pointed out by Komen et al. (1984) that one of the factors that control the energy balance of the wave spectrum is the Phillips' parameter α. This parameter is used in the wave forecasting models when an initial spectrum is specified to set off the model. It is required because the Phillips' mechanism is responsible to initiate the wave generation and thereafter Miles's mechanism takes place to support the growth. However, the form of wind input used in the models ignores Phillips' mechanism and therefore the specification of input spectrum. In order to investigate the role of α, different values of this parameter were used. It was found that in order to produce growth curves which are in agreement with experimental observations, the values of α obtained from traditional relationships such as Lewis and Allos (1990) need to be multiplied by a factor 1.45. The results for energy growth curves using different values of i.e. 1.00, 1.30 and 1.45 are shown in figure 1.

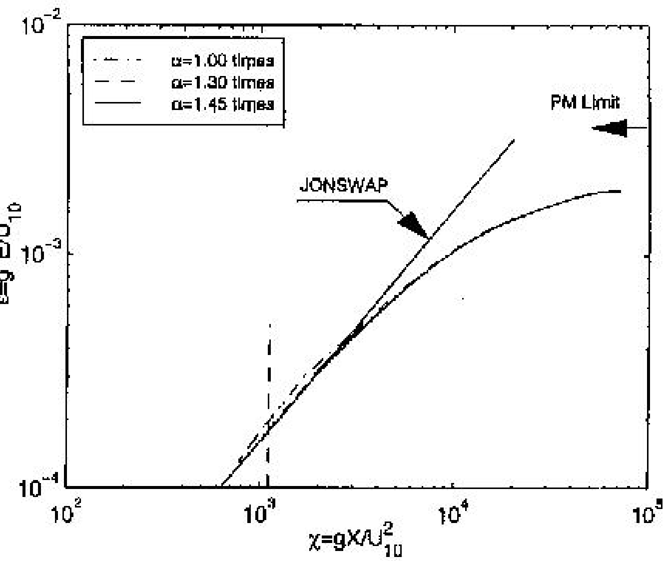

Figure 1: The energy growth curves using different values of Phillips' parameter α for the input spectrum. The other parameters are obtained from Lewis and Allos (1990) relationships.

B. *Non-dimensional energy against no-dimensional fetch*

Figure 2 shows a plot of non-dimensional energy versus non-dimensional fetch for non-dimensional water depths, δ equal to 1.0, 0.8 and 0.6. The model results are compared with the curves obtained for same parameters from the expressions proposed in Lake George experimental study [21]. Also shown is the curve for JONSWAP [8] for deep water as well as Pierson-Moskowitz limit [14] for fully developed wave spectrum. The plot shows that both model and experimental studies are in reasonable agreement particularly at short fetch. As the non-dimensional water depth δ increases, the curves progressively reach close to deep water results.

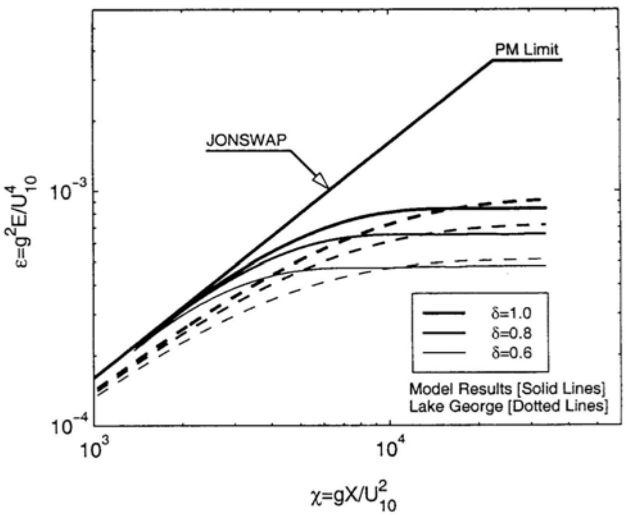

Figure 2: A comparison of non-dimensional growth curves obtained from the model and experimental study of Lake George [Young and Verhagen (1996a)] for different values of non-dimensional water depth, δ. The results for deep water study of JONSWAP [Hasselmaan et al. (1973)] and Pierson-Moskowitz (1964) for a fully developed spectrum are also shown.

C. *Non-dimensional peak frequency against non-dimensional fetch*

Figure 3 shows a plot of non-dimensional frequency ν against non-dimensional fetch χ for various values of non-dimensional water depth, δ.

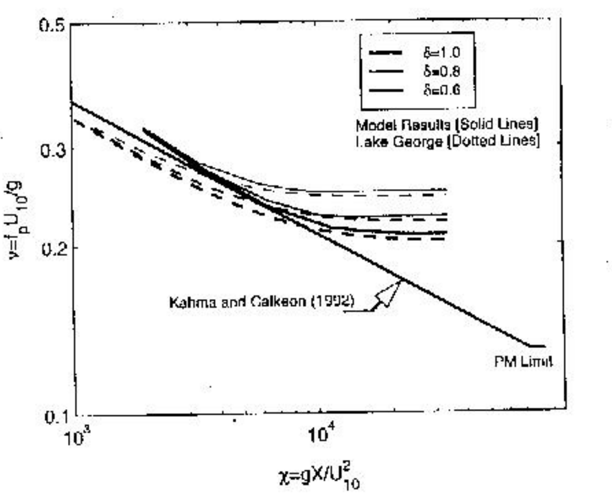

Figure 3: A comparison of peak frequency curves from the model and experimental study at Lake George [Young and Verhagen (1996a)] for different values of non-dimensional water depth, δ. Also are shown the deep water results using Kahma and Calkoen (1992) relationship and the Pierson and Moskowitz (1964) limit for a fully developed wave spectrum.

The results from model are compared with the results obtained from the proposed relationship in Lake George experimental study. Also shown is the curve obtained form the relationship proposed by Kahma and Calkeon (1992) together with the Pierson-Moskowitz limit for fully developed spectrum. It can be seen that both model and experimental results are in good consistency and as the non-dimensional water depth δ increases, the curves progressively get closer to the deep water curve of Kahma and Calkeon and towards PM value for fully developed spectrum.

V. CONCLUSIONS

The present study has concentrated on using a one-dimensional model for wave forecasting. A useful technique to incorporate full form of non-linear wave-wave interactions has been used which forms an important component of wave prediction models. In addition, the question of input spectrum to run the model has been revisited. It is found that contrary to the earlier belief that form of input spectrum has no bearing on the wave growth, it does affect growth rates particularly at short fetches.

Using above two improvements over the other models, the energy and peak frequency curves for different non-dimensional water depths have been obtained. These curves are compared with the fetch limited experimental observations carried at Lake George, Canberra Australia. It is found that these improvements in the model lead to better agreement between the model and experimental results.

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