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Pricing Game under Imbalanced Power Structure

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Abstract— **The issue of channel power in supply chain has recently received considerable attention in literature. Several different channel interactions between manufacturers and retailers and the relative power of supply chain members have been examined. Most of these practices considered all manufacturers to be Stackelberg leaders or followers over/under retailers. In this paper we add to the existing literature by proposing a model for analyzing the effect of asymmetric power within the supplier group on channel performance. We study and derive the unique Nash Equilibrium solutions for two noncooperative games in a two-supplier-one-retailer power-imbalanced supply chain with random price dependent demand in which suppliers offer substitutable products to a common retailer under periodic review policy. It is demonstrated that in imbalanced power case, the common retailer makes more profit than the two manufacturers do and also the profit among suppliers, is divided uneven. That is, the more powerful agent receives larger percentage share of selling his products. The whole supply chain converges to the integrated channel as the substitution degree of products increases.**

Index Terms— **Game theory, Imbalanced power manufacturers, Price competition, Substitutable demand**

I. INTRODUCTION

Supply chains are generally comprised of individual agents who are often guided by conflicting objective functions. In such contexts, the issue of agents' relative channel power becomes very important. Channel power here refers to an agent's relative ability to control the decision making process in the supply chain. Specifically, the more powerful firm moves first in a Stackelberg game. Several studies have been done on analyzing the interactions between manufacturers and retailers in literature. For example, Choi [1] examines the channel profits for manufacturers and retailer where interactions are either vertical Nash, or if they are Stackelberg leader-follower when either the manufacturer or the retailer plays the price leader role. Each of these three games has different implications for profits made by manufacturers and retailers, and consequently for the relative power of the channel members. Kadiyali et al. [2] generalize Choi's Model and study and empirically test interactions within a multi-supplier-single-retailer channel by allowing for a continuum of possible channel interactions between manufacturers and a retailer. However, the supplier-supplier interactions are subsumed within the retailer's interaction with each supplier. Yue Dai et al. [3] add to this literature of channel competition by applying a game theoretic approach to analyze a single-period distribution system with one-supplier-two-retailers when the supplier may have infinite or finite capacity. They study both the decentralized and centralized inventory control problems and derive equilibrium solutions for multiple competing firms when the demand is a known linear function of price.

Most of these previous supply chain interaction models are typically either two-stage Stackelberg games or one-stage non-cooperative games with all suppliers sharing an equal or balanced decision-making power. That is, all suppliers are assumed to act as either Stackelberg leaders or followers over/under the retailer, or all supply chain parties move simultaneously in decision-making. Xinjie Shi in his study [4] relaxes this symmetric power assumption and examines situations when suppliers have an unequal decision making power over each other so that one or more suppliers can exercise Stackelberg leadership over the other suppliers. He extends and generalizes the results by Choi and analyzes five additional possible channel configurations for supply chain under power imbalanced by modeling them as either two-stage or three-stage single period Bertrand Stackelberg games, where the demand is a linear deterministic function of products' retail prices. He examines the influence of each agent's decision making power on the strategic interactions and performance within a multi-supplier-one-retailer supply chain.

In this paper we extend previous literature by allowing for a multi period horizon supply chain with imbalanced power manufacturers. Stochastic price sensitive demand functions are built for two substitutable products. Given these demand functions, we obtain optimal pricing rules for manufacturers and the retailer. Since the pricing strategy of one firm affects the demand streams of other firms, there exists a strategic interaction among the agents' decisions; therefore game theory is applied to analyze this problem.

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We confine our analysis to pricing strategies considering parties' relative power in channels and study how manufacturers' channel power is related to demand conditions facing various substitutable degree and cost parameters of products. Our objective is to provide a method to measure how channel profits are divided between manufacturers and the retailer.

II. THE MODELING FRAMEWORK

Consider a supply chain with three risk neutral firms, two manufacturers and one retailer. The supplier i is paid a wholesale unit price w_i by the retailer who in turn charges a retail price r_i from the end customers, where the supplier index i $(i = 1,2)$ denotes the specific supplier, and thus the specific product i sold to the retailer. The product is manufactured at a unit cost of c_i, and incurs h_i^S (h^R) per unit in holding costs per period at the manufacturer i (retailer) and p per unit in loss-of-goodwill costs for each lost sale at the retailer. The manufacturer i utilizes outsourcing at a cost of b per unit, if a retailer order cannot be satisfied in full from on-hand inventory. To avoid the trivial case when it is optimal for the retailer to buy nothing, we enforce the constraint $r_i > w_i$ and to avoid unrealistic outsourcing costs we require $r_i + p \geq b$.

The length of time between successive shipments (replenishment cycle), is denoted by t. We use k to indicate cycle number $(k \in \{0,1,\dots,N\})$. We assume a finite time horizon where the suppliers make periodic shipments to the retailer. Price decisions are made once at the beginning of the time horizon and they remain unchanged throughout its duration. Lead time between supplier and retailer is negligible.

The retailer faces a random price dependent demand function $D_i^{r_i,r_j}$ for the product i over replenishment cycle k of length t periods where $D_i^{r_i,r_j}$ represents the demand for product i at price r_i given that the price of the other product j is r_j. Randomness in demand is price independent and can be modeled either in an additive or a multiplicative fashion. In our model we use the additive form. The demand function is modeled as a linear duopoly function: $D_i^{r_i,r_j} = l_i^{r_i,r_j} + \varepsilon$ where

$$l_i^{r_i,r_j} = a_i - mr_i + n(r_j - r_i), with\ a_i, m, n > 0, \quad for\ i,j = 1,2, i \neq j \tag{1}$$

is a decreasing function that captures product differentiation and the dependency between the demand for product i and the price of both products, and ε is a random variable defined on the range $[A, B], 0 \leq A < B$ and assumed to follow the normal distribution, while $F(.)$ represents the normal cumulative distribution function of ε, $f(.)$ the normal probability density function and μ and σ the mean and standard deviation of ε, respectively.

The coefficient a_i represents the product market base [5] that is invariant to the retail prices, m represents product i's demand sensitivity on its own retail price, and n denotes the degree of product substitution, which accounts for the effect of retail price differences of the two substitutable products. Thus $n = 0$ represents the case when the two products are completely independent, and as n increases, the degree of product substitution and consequently the competition between the two products, increases.

The base stock level of product i for manufactures and the retailer are defined as $Y_i^S = l_i^{r_i,r_j} + y_i^S$ and $Y_i^R = l_i^{r_i,r_j} + y_i^R$ where $l_i^{r_i,r_j}$ responds to the deterministic part of the demand which is dependent on the retail prices for both products $i\ and\ j$, while $y_i^S\ and\ y_i^R$ respond to the uncertainty in demand. Separating deterministic and stochastic parts of base stock level is a novel way that results to better understanding the model structure.

The two manufacturers can be asymmetric in size (imbalanced power system) or symmetric (balanced power system). In the asymmetric case assume that Supplier 1 is larger than supplier 2. This size difference affects the Stackelberg pricing positions that suppliers can have, as illustrated in the following. Furthermore, it is assumed that information is symmetric across the supply chain, and each supply chain agent objective is to maximize his/her own profit.

III. BENCHMARK SYSTEM

In this section, we formulate the problem of the centralized scenario as our benchmark where a single decision maker chooses the retail prices of both products that maximizes expected supply chain profit over the duration of the time horizon. The base model used in this section, is similar to as in Bogdan C. Bichescu [6].

Let Π_i^{SC} be the supply chain profit from product i and $\Pi^{SC} = \sum_{i=1}^{2} \Pi_i^{SC}$ the whole supply chain profit from both products in cycle k. We further define $x_i^R = \left(Y_i^R - D_i^{r_i,r_j}\right)^+ \ and\ x_i^S = \left(Y_i^S - D_i^{r_i,r_j}\right)^+$ as the retailer's and supplier i's on-hand inventory respectively.

$$\Pi_i^{SC} = r_i \min\{Y_i^R, D_i^{r_i,r_j}\} - h^R t \left(Y_i^R - D_i^{r_i,r_j}\right)^+ - p(D_i^{r_i,r_j} - Y_i^R)^+ - \frac{h_i^S}{2} t \left(Y_i^S + x_i^S\right) - b \left(Y_i^R - x_i^R - Y_i^S\right)^+ - c_i \left(Y_i^S - x_i^S\right) - S_c \tag{2}$$

$$\mathrm{E}\left(\Pi_i^{SC}\right) = Y_i^R (r_i - b) - (r_i - b + h^R t) - E\left[(Y_i^R - D^t)^+\right] - pE\left[\left(D^t - Y_i^R\right)^+\right] - h^S t Y_i^S + \left(b - c_i + \frac{h^S}{2} t\right) min\left\{Y_i^S, Y_i^R - E\left[\left(Y_i^R - D^t\right)^+\right]\right\} - S_c \tag{3}$$

The objective function is: $\text{Max}_{r_i,r_j\geq 0}\{\sum_{k=0}^{N} E(\Pi^{SC})\}$

To keep our model symmetric in relative cost parameters we assume $c_1 = c_2 = c$, $a_1 = a_2 = a$, $h_1^S = h_2^S = h^S$ and $t = 1$, which also allows for meaningful comparisons of the results with those of previous studies.

Proposition 1: For the centralized system, the supply chain profit function is jointly concave in $r_1 and\, r_2$ and the optimal retail prices for both products are

$$r_i^* = \frac{\left(a + \mu + m\left(c + \frac{h^S}{2}\right) + z\sigma(1 - \Phi(z)) - \sigma\phi(z)\right)}{2m} \tag{4}$$

when $min\{Y_i^S, Y_i^R - E[(Y_i^R - D_i)^+]\} = Y_i^S$, otherwise the outsourcing cost (b) appears in the equation instead of c.

Proof. The Hessian matrix in both cases is

$$\begin{vmatrix} \frac{\partial^2 E(\Pi^{SC}(r_i,r_j))}{\partial r_1^2} & \frac{\partial^2 E(\Pi^{SC}(r_i,r_j))}{\partial r_1 \partial r_2} \\ \frac{\partial^2 E(\Pi^{SC}(r_i,r_j))}{\partial r_2 \partial r_1} & \frac{\partial^2 E(\Pi^{SC}(r_i,r_j))}{\partial r_2^2} \end{vmatrix} = \begin{vmatrix} -2m - 2n & 2n \\ 2n & -2m - 2n \end{vmatrix} \tag{5}$$

thus the supply chain total profit is jointly concave in r_1 and r_2. The optimal retailer prices are obtained by solving first order conditions of $E(\Pi^{SC})$, where

$$E\left[\left(Y_i^R - D_i^{r_i,r_j}\right)^+\right] = E\left[\left(\left(l_i^{r_i,r_j} + y_i^R\right) - \left(l_i^{r_i,r_j} + \varepsilon\right)\right)^+\right] = \int_0^{y_i^R} \left(y_i^R - u\right)dF(u) = y_i^R F\left(y_i^R\right) - \int_0^{y_i^R} u\, dF(u) = y_i^R F\left(y_i^R\right) - \mu t F\left(y_i^R\right) + \sigma\sqrt{t} f\left(y_i^R\right) \tag{6}$$

$$E\left[\left(D_i^{r_i,r_j} - Y_i^R\right)^+\right] = \left(\mu t - y_i^R\right)\left[1 - F\left(y_i^R\right)\right] + \sigma\sqrt{t} f\left(y_i^R\right) \tag{7}$$

Because the unmet demand penalty is high, both manufacturers and the retailer seek to avoid it when possible by setting $y_i^S = \mu t + \acute{z}\sigma\sqrt{t}$ and $y_i^R = \mu t + z\sigma\sqrt{t}$ respectively where $\acute{z}$ and z characterize the on-hand service level for the firms and are determined by firms considering their own penalty costs in order to best responding to the uncertainty in demand. Consequently $F(y_i^R) = \Phi\left(\frac{y_i^R - \mu t}{\sigma\sqrt{t}}\right) = \Phi(z)$ and $f(y_i^R) = \phi\left(\frac{y_i^R - \mu t}{\sigma\sqrt{t}}\right) = \phi(z)$.

IV. DECENTRALIZED SYSTEM

Consider a decentralized supply chain in which each manufacturer decides on its wholesale price, and the retailer determines the retail prices for two products in order to maximize following individual expected profits over the time horizon.

$Retailer's\ profit\ from\ product\ i$: Π_i^R

$$= r_i\, min\left\{Y_i^R, D_i^{r_i,r_j}\right\} - w_i\left(Y_i^R - x_i^R\right) - h^R t\left(Y_i^R - D_i^{r_i,r_j}\right)^+ - p\left(D_i^{r_i,r_j} - Y_i^R\right)^+ \tag{8}$$

$$E(\Pi_i^R) = Y_i^R(r_i - w_i) - (r_i + h^R t - w_i)E\left[\left(Y_i^R - D_i^{r_i,r_j}\right)^+\right] + pE\left[\left(D_i^{r_i,r_j} - Y_i^R\right)^+\right] \tag{9}$$

Retailer's total profit from both products: $\Pi^R = \sum_{i=1}^{2} \Pi_i^R$

$Supplier\ i's\ profi$: Π_i^S

$$= w_i\left(Y_i^R - x_i^R\right) - \frac{h^S}{2}t\left(Y_i^S + x_i^S\right) - b\left(Y_i^R - x_i^R - Y_i^S\right)^+ - c\left(Y_i^S - x_i^S\right) - S_c \tag{10}$$

$$E(\Pi_i^S) = (w_i - b)\left\{Y_i^R - E\left[\left(Y_i^R - D_i^{r_i,r_j}\right)^+\right]\right\} - h^S t Y_i^S + \left(b - c + \frac{h^S}{2}t\right)\min\left\{Y_i^S, Y_i^R - E\left[\left(Y_i^R - D_i^{r_i,r_j}\right)^+\right]\right\} - Sc \tag{11}$$

The suppliers' total profit: $\Pi^S = \sum_{i=1}^{2} \Pi_i^S$ and the whole supply chain profit: $\Pi^{SC} = \Pi^R + \Pi^S$.

Applying our proposed model for eight possible channel configurations suggested by Xinjie Shi [4], it is seen that the Nash equilibrium exists only in structures that retailer is the end follower thus in our study, we examine the two stage and three stage Bertrand Stackelberg games shown in Fig. 1.

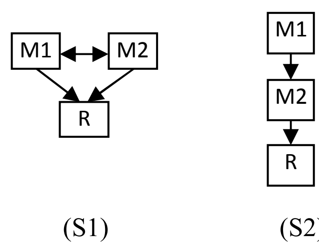

Figure 1. Balanced and Imbalanced Power Structures

A. BALANCED POWER STRUCTURE

In balanced power structure (S1), both manufacturers are assumed to have equal power over retailer. Under this Stackelberg formulation where the suppliers hold greater channel power, each manufacturer chooses the wholesale price using the response function of the retailer, conditional on the observed wholesale price of the competitor's product. So the manufacturers declare their decisions first and then the retailer follows with responding by respective retail prices. Because we assume that both players possess full information, the

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supplier can deduce the retailer's optimal response and plan accordingly.

Proposition 2: In the decentralized supply chain under balanced power structure, the retailer's reaction function given wholesale prices w_1 and w_2 is jointly concave in r_1 and r_2 and the optimal policy for the retailer is:

$$r_i^* = \frac{\left(a+\mu+mw_i^*+z\sigma\left(1-\Phi(z)\right)-\sigma\phi(z)\right)}{2m}, \quad i,j=1,2,\ \ i\neq j \tag{12}$$

Proof. The Hessian matrix is the same as in the centralized scenario and the condition for maximization is satisfied.

Proposition 3: Taking the retailer's reaction function into consideration the manufacturers' respective profit functions are concave in w_1 and w_2 and the Nash equilibrium wholesale price for product i is:

$$w_i^* = \frac{a+\mu+(n+m)\left(c+\frac{h^S}{2}\right)+z\sigma\left(1-\Phi(z)\right)-\sigma\phi(z)}{n+2m}, \quad i,j=1,2,\ \ i\neq j \tag{13}$$

Proof. The second derivative of Π_i^S with respect to w_i is negative $\left(\frac{\partial^2 E\left(\Pi_i^S(w_i)\right)}{\partial w_i^2} = -n-m \leq 0\right)$ and the optimal policy for manufacturers are derived from the first-order conditions of respective profit maximization problem. Substituting the reaction functions (13) in (12), the corresponding retail prices can be obtained.

B. IMBALANCED POWER STRUCTURE

In this structure , the leader manufacturer (M1) takes the retailer's and follower manufacturer's reaction functions into account for its own wholesale price decisions. The retailer's optimal policy is the same as (12).

Proposition 4: In the imbalanced supply chain model, the leader (M1) and follower (M2) manufacturers' profit functions are concave in w_i and the optimal wholesale prices are as shown in (14) and (15).

Proof. The second derivative of Π_2^S with respect to w_2 given w_1 and considering retailer's optimal policy is negative $\left(\frac{\partial E\left(\Pi_2^S(w_2)\right)}{\partial w_2^2} = -n-m \leq 0\right)$. Again by taking both retailer's and manufacturer 2's reaction function into consideration, the second derivative of $\Pi_1^S(w_1)$ with respect to w_1 is negative $\left(\frac{\partial E\left(\Pi_1^S(w_1)\right)}{\partial w_1^2} = -\frac{2m^2+4mn+n^2}{2(n+m)} \leq 0\right)$ thus the maximization conditions are satisfied and therefore there exists a unique Nash equilibrium.

V. NUMERICAL ANALYSIS

In the following we first clarify the proposed model by a numerical example. Then we do the general comparison among the prices and profits in two power structures and the benchmark system and analyze the effects of product differentiation changes on prices and profits. To facilitate further discussion, we set the problem parameters as depicted in Table I and obtain the respective equilibrium prices and expected profits.

As can be seen the whole supply chain as well as consumers benefit most from lower prices and larger profits in the centralized system when there is no leadership in channel and least when the suppliers are imbalanced in power (S2). In this case (S2) both suppliers' profits monotonically decrease, while the retailer's profit increases as the degree of product substitution n increases as illustrated in Fig. 2. Moreover, the total supply chain profit increases in n. When the two products are completely substitutable, the retailer gains the whole supply chain profit, which is the same as the case when an integrated supply chain is coordinated by the retailer. Consequently the difference in total supply chain profits in different structures becomes smaller as products are less differentiated and they are equal when products are perfect substitutable (Fig. 3).

Similar to Xinjie Shi study [4], it is observed that in imbalanced power supply chain (S2), the more powerful agent may not make the most profit (Fig. 2). Xinjie Shi states that although the powerful manufacturer (M1) gains less than the weaker firm (M2), the ratio of profit that supplier M1 shares from the sale of his own products is larger than that of M2 so there still is an intensive for him to be the channel leader. In addition to that, we also address the symmetric assumption in which we considered c_i, a_i and h_i^S to be equal for both suppliers. This assumption generally is not realistic as the economics of scale for the larger manufacturer can

$$w_2^* = \frac{a+\mu+nw_1^*+\left(c+\frac{h^S}{2}\right)(n+m)+z\sigma\left(1-\Phi(z)\right)-\sigma\phi(z)}{2(n+m)} \tag{14}$$

$$w_1^* = \frac{(2m+3n)\left(\mu+a+z\sigma\left(1-\Phi(z)\right)-\sigma\phi\right)+\left(c+\frac{h^S}{2}\right)(2m^2+2n^2+5nm)}{4m^2+8mn+2n^2} \tag{15}$$

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Variable	Value
$a_1 = a_2$	10^3
$c_1 = c_2$	0.5
$z = \acute{z}$	0.9
$h^S = h^R$	0.01
m	50
$\mu = p$	0
σ	10

	r_1	r_2	w_1	w_2	$E(\Pi^R)$	$E(\Pi_1^S)$	$E(\Pi_2^S)$	$E(\Pi^{SC})$
Benchmark System	10	10	-	-	-	-	-	10,071
Balanced Power (S1)	12.7	12.7	5.4	5.4	5,333	1,777	1,777	8,890
Imbalanced Power (S2)	13	12.8	6.1	5.6	5,006	1,806	1,951	8,764

Table I. (a) Parameter Values, (b) Equilibrium Prices and Expected Profits (Given n=100)

result to smaller costs and also the market share (a_i) of the stronger manufacturer is larger in general so the leader manufacturer's profit would be greater as a result.

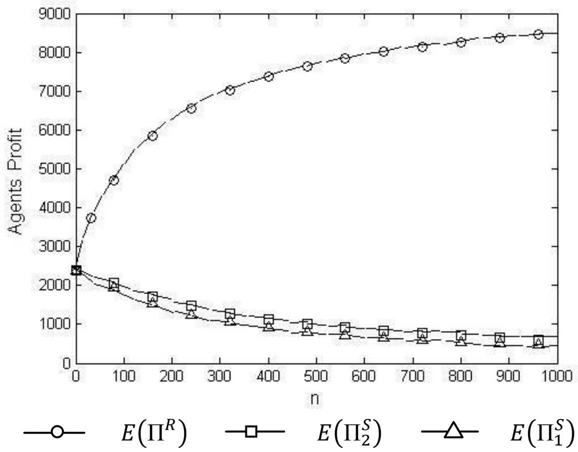

Figure 2. Expected Profits in Imbalanced Power Structure

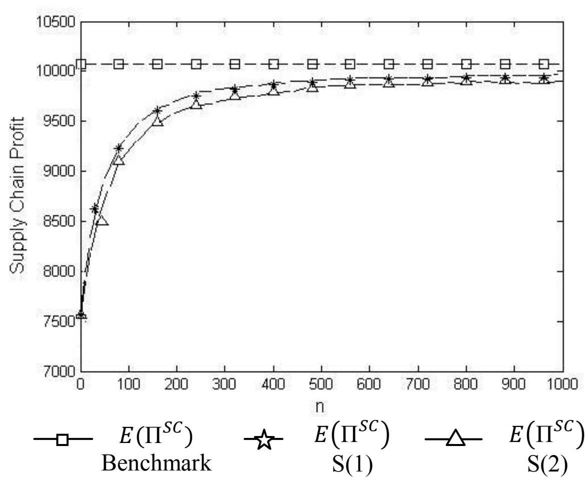

Figure 3. Total Supply Chain Profit in Three Structures

VI. CONCLUSION

In this paper, we study the pricing strategies of a two-supplier-single-retailer supply chain applying Game Theory methods. We derive the unique Nash equilibrium in the centralized benchmark system, the balanced and imbalanced power manufacturers' structures over the periodic review policy when the demand is a stochastic price sensitive function that depends on the selling prices for both products charged by the retailer. We also analyze the changes in equilibrium prices and profits as a result of changes in substitution degree of products.

Comparing three channel power structures, it is observed that the whole supply chain as well as consumers, benefit most in the benchmark centralized system with no leadership, and least in the imbalanced power manufacturers case (three stage Bertrand Stackelberg structure). It is demonstrated that by increasing substitution degree of products, the retailer gains a larger percentage of the total supply chain profit while manufacturers face decreasing fraction of total profit and when the degree of product substitution is sufficiently large the whole supply chain performs as an integrated system.

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