The Varying Behavior of U.S. Market Persistence

James Preciado and Hedley Morris *

Abstract–**Many market models for pricing assets assume that the Efficient Market Hypothesis is true and that there is a random element in future returns. The randomness of the security is often simulated using geometric Brownian motion, which implies a constant persistence level estimated by a Hurst exponent of one half. However, after examining U.S. market indices, certain irregularities arise that challenge the assumption of constant persistence. Furthermore, there is the possibility of constructing a model for use in predicting future persistence levels.**

Keywords: Efficient Market Hypothesis, Persistence, Hurst exponent, Wavelets

1 Introduction

Persistent securities or markets are ones that trend in a certain direction. For example, following an upward movement, persistent securities or markets will tend to continue in an upward direction and, following a downward movement, will tend to continue in downward direction. Anti-persistent markets behave oppositely and trend back to a historic mean level. So, when there is an upward movement in an anti-persistent market, the next movement will tend to be downward. Similarly, a downward movement in an anti-persistent market will tend to be followed by an upward movement.

The Hurst Exponent serves as a numerical measurement of market persistence. A Hurst Exponent of one half implies the market exactly follows geometric Brownian motion and that the direction of future movements is entirely random. Anti-persistent, or mean reverting markets, have a Hurst exponent greater than zero and less than one half. Persistent markets have a Hurst exponent greater than one half and less than one [1].

In this paper, it will be shown that market persistence is not constant and the market indeed displays periods of anti-persistent behavior as well as increased persistent behavior. The analysis will be performed on both the S&P 500 and the Russell 2000. An attempt to model the persistence of both market indices will also be made.

*J. Preciado (James.Preciado@cgu.edu)is a graduate student at The Peter F. Drucker and Masatoshi Ito Graduate School of Management, Claremont Graduate University, CA 91711, USA and Hedley Morris (hedley.morris@cgu.edu) is with the School of Mathematical Sciences, Claremont Graduate University, CA 91711, USA.

2 Methodology

For any financial time series, the Hurst exponent can only be estimated since it is based on the assumption of the series being a pure fractal, which log returns are not. Instead of using the original rescaled range analysis to estimate the Hurst exponent, a wavelet transform will be used. The wavelet estimation is much more accurate as it has been shown that rescaled range analysis tends to overestimate persistence and is not well behaved when examining persistence over a short period of time. [2]. The wavelet spectral density is used to estimate persistence. The Hurst exponent is calculated from the slope of the regression line where wavelet octave is on the x-axis and the normalized power is on the y-axis. The wavelet transformation and subsequent estimation of the Hurst exponent was performed using the Rwave package in R. The modeling of persistence was also performed in R using the fSeries package. The data was obtained from a Bloomberg Terminal. Daily returns were calculated for the S&P 500 and the Russell 2000 starting January 2nd, 1987 and ending April 11th, 2008. Hurst exponents were also calculated for various sub-periods during this 20-year time frame

3 Results

The first task was to calculate the persistence of the S&P 500 and the Russell 2000 for the entire time period from January 2nd, 1987 through April 11th, 2008. The results obtained were consistent with previous research [3]. A Hurst exponent of .6244 was found for the S&P 500 while the Russell 2000 seemed to exhibit more persistent behavior with a Hurst Exponent of .6682.

Looking at the regression analysis plots above, the line fits fairly well with the data, which implies that the calculated slope, the Hurst exponent, is a good representation of the true market persistence. However, examining these plots further, one can notice a slight sinusoidal pattern through the regression line in both the S&P 500 and the Russell 2000. Using this evidence, one can further conjecture that the persistence of a market is not constant.

One possible explanation for non-uniform persistence is that differences in systemic market conditions could alter the behavior of stock prices. Particularly, times of economic stress could be a factor as to whether a stock will revert back to a mean or trend in a direction. To test this hypothesis, both the S&P 500 and the Russell 2000

ISBN: 978-988-98671-0-2

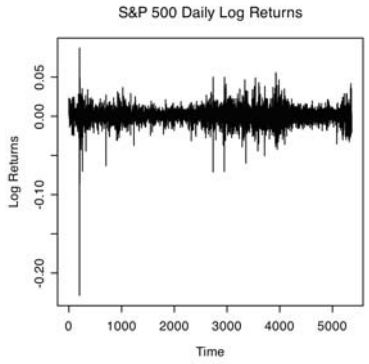

Figure 1: S&P 500 Daily log returns.

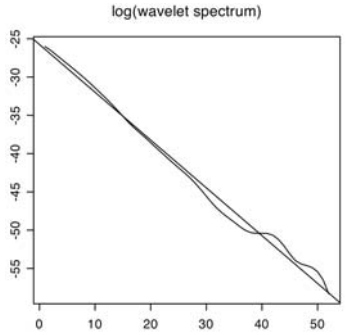

Figure 2: S&P 500 Wavelet Transform Hurst Estimation Plot.

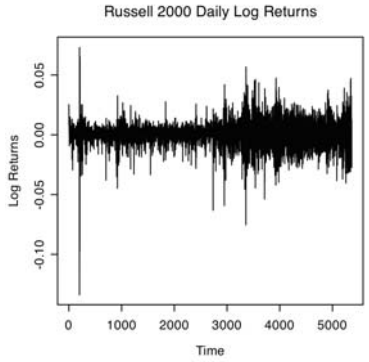

Figure 3: Russel 2000 Daily log returns.

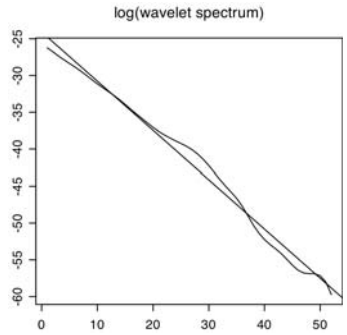

Figure 4: Russell 2000 Wavelet Transform Hurst Estimation Plot.

were broken up into six disjoint time periods, three during times of economic turmoil and three during times of relatively normal economic growth. In the following table, the first three time periods are years in which there was a recession or very little economic growth in the United States. The last three time periods contained neutral or bull markets.

Time Period	S&P 500	Russel 2000
January 1991–December 1992	.5906	.3989
January 2000–December 2002	.6670	.6479
August 2007–April 2008	.7936	.7752
January 1987–December 1989	.6598	.4312
January 1993–December 1999	.6933	.4993
January 2003–July 2007	.6946	.6657

Table 1: Persistence During Times of Economic Stress

The following are two sample wavelet plots for calculating the Hurst exponent during the time period of January 2000 through December 2002 for the S&P 500 and the Russell 2000.

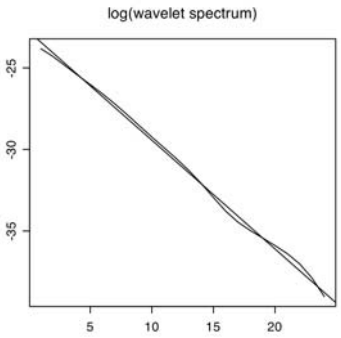

Figure 5: January 2000 through December 2002 S&P500.

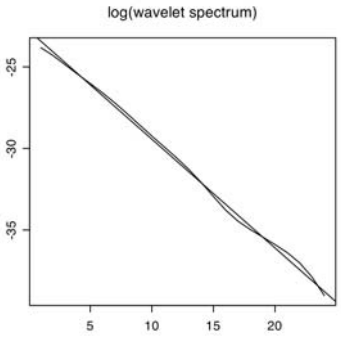

Figure 6: January 2000 through December 2002 Russell 2000.

As can be seen above, during the current period of economic downturn, large cap stocks (the S&P 500) and small cap stocks (the Russell 2000) were noticeably more persistent. However, in the recession of 1991 and 1992,

the S&P 500 and the Russell 2000 experienced their time of least persistence. Two conclusions can be made from this analysis. The first is that persistence is not constant. Secondly, economic stress clearly changes the persistence of stock markets, but it is unclear if it will push persistence above or below historical levels. The last part of this paper is dedicated to further exploring the idea that market persistence is not constant and to try and uncover a model for this persistence. In order to accomplish this, a Hurst exponent was calculated using the returns of the previous ninety trading days. So beginning May 13th, 1987, a time series of daily Hurst exponents, using data from the previous ninety days, was created. The idea behind this calculation is that these daily Hurst exponents will estimate the most current condition of market persistence. It is a more refined approach since the Hurst calculation plots display some pattern of deviation from the regression line. The following are two plots of these daily Hurst exponents for the S&P 500 and the Russell 2000 along with their 1st, 2nd, 3rd and 4th moments.

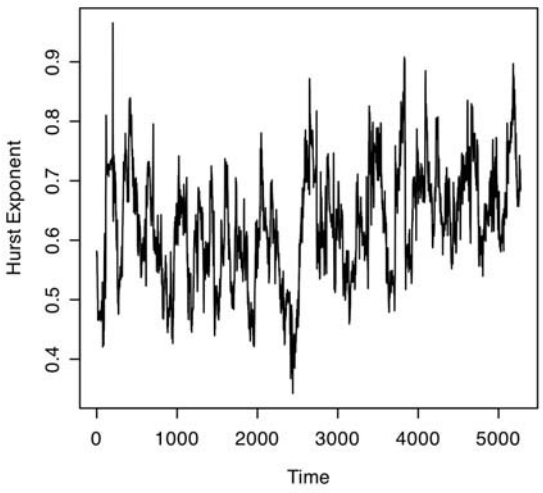

Figure 7: Daily Persistence of S&P 500(Mean = .62722 Variance = .00877 Skewness = -.00076 Kurtosis = -.40211).

As can be seen above in figures 7 and 8, during the current period of economic downturn, large cap stocks, the S&P 500, and small cap stocks, the Russell 2000, were noticeably more persistent. The recession of 1991 and 1992, though, is actually when the S&P 500 and the Russell 2000 experienced less persistence. Two conclusions can be made from this analysis. The first is that persistence is not constant. Secondly, economic stress clearly does change the persistence of stock markets but it is unclear if it will push persistence above or below historical levels. The last part of this paper is dedicated to further exploring the idea that market persistence is not constant and to try and uncover a model for this persistence. In order

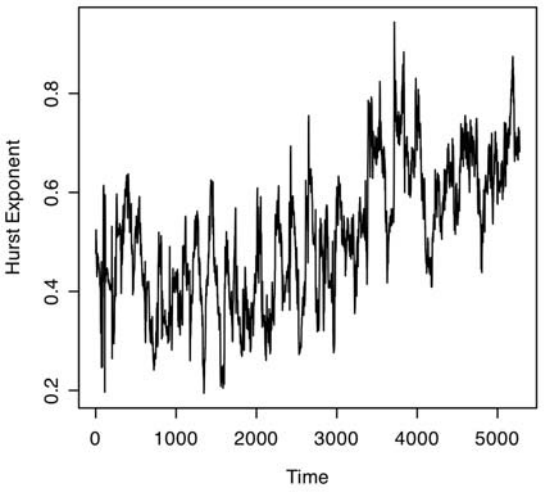

Figure 8: Daily Persistence Russell 2000 (Mean = .51189 Variance = .01909 Skewness = .13096 Kurtosis = -.73795).

to accomplish this, a Hurst exponent was calculated using the returns of the previous ninety trading days. So beginning May 13th, 1987, a time series of daily Hurst exponents, using data from the previous ninety days, was created. The idea behind this calculation is that these daily Hurst exponents will estimate the most current condition of market persistence. It is a more refined approach since the Hurst calculation plots display some pattern of deviation from the regression line. The following are two plots of these daily Hurst exponents for the S&P 500 and the Russell 2000 along with their 1st, 2nd, 3rd and 4th moments. Finally, an AR(4) - GARCH(1,1) model was

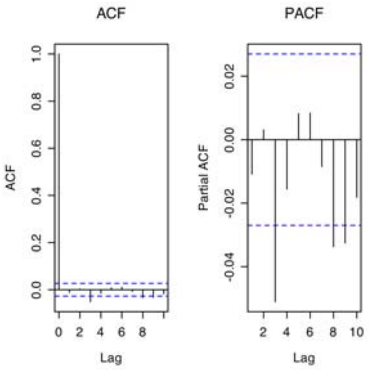

Figure 9: S&P 500 ACF and PACF.

fitted to the series of daily Hurst exponents from the S&P 500 and the Russell 2000. The type of model was determined by examining the following ACF and PACF graphs for the S&P 500 and Russell 2000 persistence series. The amount of significant lags in the PACF, as well as a lack

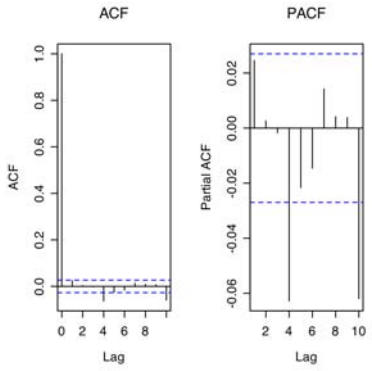

Figure 10: Russell 2000 ACF and PACF

of significant lags in the ACF for both indices, suggests using an AR model.

The following are the persistence models:

S&P500 AR(4):

$$H_t = .9815H_{t-1} + .0132H_{t-2} - .0544H_{t-3} + .0435H_{t-4}$$

GARCH(1,1):

$$\sigma_t^2 = .914 + .4025\epsilon_{t-1}^2 + .000221\sigma_{t-1}^2$$

Russell 2000 AR(4):

$$H_t = 1.0189H_{t-1} + .0208H_{t-2} - .0007H_{t-3} + .0640H_{t-4}$$

GARCH(1,1):

$$\sigma_t^2 = .938 + .3286\epsilon_{t-1}^2 + .000297\sigma_{t-1}^2$$

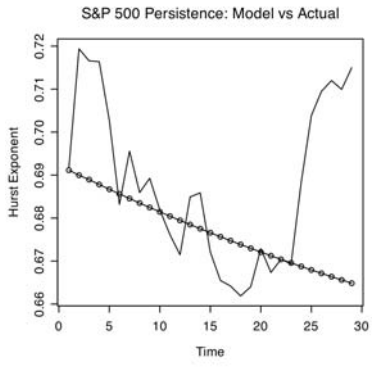

Figure 11: S&P Hurst exponent forecast.

In order to test the models, the actual daily persistence calculations were made, using the same methodology described earlier, for the time period of April 1st 2008 through May 11th 2008. The actual persistence was then plotted against the model estimates of persistence for both the S&P 500 and the Russell 2000. The actual persistence is the solid line while the model estimates for persistence are the lines highlighted with circles. As you can see, the model performs well for a few days into the future and then begins to underestimate persistence levels.

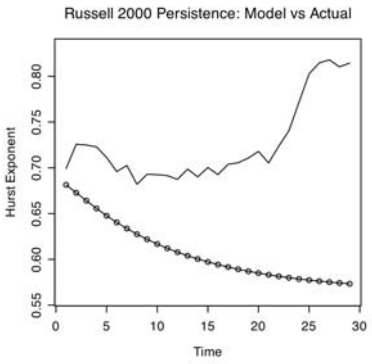

Figure 12: Russell 2000 Hurst exponent forecast.

These forecasts are very good over a five day period. The percentage errors are shown in table 2.

Date	S&P 500	S&P 500 Error	Russell 2000	Russell 2000 Error
4/1/2008	.6918	0.09%	.6991	2.54%
4/2/2008	.7194	4.08%	.7257	7.29%
4/3/2008	.7166	3.85%	.7249	8.38%
4/4/2008	.7164	3.99%	.7229	9.31%
4/7/2008	.7028	2.28%	.7113	8.94%
4/8/2008	.6832	0.35%	.6959	7.98%
4/9/2008	.6956	1.58%	.7025	9.81%
4/10/2008	.6859	0.36%	.6821	8.00%
4/11/2008	.6893	0.99%	.6930	10.26%
4/14/2008	.6819	0.07%	.6925	10.94%

Table 2: Percent Forecast Error for First Ten Days

4 Concluding Remarks

The strongest conclusions that can be drawn from this analysis is that the persistence of the United States' stock market is not constant. The series of Hurst exponents calculated using the most recent ninety days of returns changes dramatically with respect to time. Both the S&P 500 and the Russell 2000 experience periods of above normal persistence as well as go through periods of anti-persistence. In fact, when examining the charts more closely, there seems to be a slight upward trend in persistence for the S&P 500 and a more noticeable increase in persistence for the Russell 2000. The coefficients for the AR(4) - GARCH(1,1) models are similar for the S&P 500 and the Russell 2000. The like structure of the models shows that market persistence for large capitalization

stocks and small capitalization stocks behaves in much the same manner over short periods of time. These models provide a benchmark when substituting fractal Brownian motion for the traditional assumption of geometric Brownian motion in asset pricing. Previous research has shown these effects are small over short durations [3]; however, a more detailed look at market pricing of securities and derivatives could yield significant adjustments in values when correcting for actual persistence. One possible explanation for changing persistence in markets is broad economic stress; although it is ambiguous whether this causes more persistent behavior or anti-persistent behavior. Using other criteria, such as treasury yields or the VIX, as a time period screen could lead to more predictable patterns of the Hurst exponent and market persistence. In addition, data such as changes in analyst expectations and time until earnings reports could be used to model persistence in individual stocks. In conclusion, the Hurst exponent can be a powerful measurement of current market conditions and its importance in technical analysis and security pricing should be explored further.

References

[1] Couillard M. and Davison M.,"A comment on measuring the Hurst exponent of financial time series", *Physica A: Statistical Mechanics and its Applications*, Volume 348, 15 March 2005, Pages 404-418

[2] Kyaw N., Los C., Zong S., "Persistence characteristics of Latin American financial markets", *Journal of Multinational Financial Management*, Volume 16, Issue 3, July 2006, Pages 269-290.

[3] Jamdee S., Los C., "Long memory options: LM evidence and simulations", *Research in International Business and Finance*, Volume 21, Issue 2, June 2007, Pages 260-280