Subpixel Edge Detection Based on Morphological Theory

Yu Lei and Nie Jiafa

***Abstract*—A high accuracy method is proposed for edge detection of noisy image to the subpixel extent. The specific characteristics of the target edge are fully extracted by morphological edge detection, in which structure elements are formed by means of neutral networks, as well the influences of the noises in images are diminished. Then, the edge is located in subpixel precision by adopting Zernike moments method. Experimental results show that the proposed method is stable and accurate, efficiently fulfill the requirements for high performance.**

***Index Terms*—edge detection, morphological theory, neutral network , subpixel**

I. Introduction

The image edges include abundant information that is very important for obtaining the image characteristic by image recognition. The main aim of edge detection is to extract edge information and for image analysis, target recognition and image coding in the following steps [1][2].

Mathematical Morphology is a science based on Set theory. Many experts and scholars have done a series of researches on its theories and applications, which results in its applications in many fields. In recent years, mathematical morphology is becoming one kind of new theory and new method in fact for digital image processing and recognition.

Subpixel edge detection and localization in noisy images is a commonly needed tool for artificial vision applications; particularly for on-line dimensional control of manufactured parts. A lot of edge detection techniques with pixel resolution are well known and some of them are designed in order to be robust against image corruption. Y.Shan[3] proposed a method for subpixel edge location combined canny operator and second order moment. In [4], zernike moments (ZMs) operator is proposed for subpixel edge location.

In order to satisfy the requirements for edge location accuracies, and consequently the specific characteristics of the edge can be fully extracted, the morphological filter which is based on neutral networks is used to remove the noise, smooth the image and detect the edge. Then, high accuracy of subpixel edge location is obtained by zernike moments operator. It will perform the stable ability of mathematics morphology, and also high accuracy of orthogonal moment for subpixel image.

Manuscript received July 21, 2008.

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II. Morphological operations based on neutral networks

A. Basic of morphological operations

Morphology method could be divided to binary and gray. Its basic thought is to measure and detect corresponding shapes in an image in order to achieve the goal of analyzing and recognizing image by using structure elements with some given forms. The foundation of Mathematical Morphology is binary morphology, and its fundamental transforms include dilation, erosion, open, and close [1].

Because practical image is mostly grey image, binary morphology theory needs to be popularized to grey level morphology. Thus it can deal with the gray and color images effectively. Assuming $f(x,y)$ is input image function, $b(i,j)$ is structure element function. The following expression can be inferred [2].

Grey dilation and erosion definitions are as follow-ing respectively:

$$(f\oplus B)(x,y)=\max\left\{f(x-i,y-j)+\left|b(i,j)\in B, f(x-i,y-j)\in f\right.\right\} \quad (1)$$

$$(f\Theta B)(x,y)=\min\left\{f(x+i,y+j)+\left|b(i,j)\in B, f(x+i,y+j)\in f\right.\right\} \quad (2)$$

where x and y is the row number and column number of original input image matrix respectively, i and j is the row number and column number of structure element matrix respectively.

B. Training Structure elements by neutral networks

The window of morphological filter lies on the shape of structural elements. When using neural network method, we can first set B's initial shape. Then change the two-dimension image data in filter window into a set of multi-dimension vector data by using the common image processing structure and its corresponding scanning model. Notice that the sequence of each component in the structuring element B should correspond with that of the scanning points in filtering window. In fact, the value of component in B reflects the numerical distribution of B, while the sequence of component reflects the spatial distribution of B, i.e. the structural shape [7].

We can design a three-layer BP neural network model by dilation and erosion operators, as shown in Fig. 1. When hidden layer employs erosion and output layer employs dilation, this BP network is an open operation neutral network. But when hidden layer employs dilation and output layer employs erosion, the network is an close operation neutral network. In the picture, the symbol $\circ$ denotes open operation, and $\bullet$ denotes close operations.

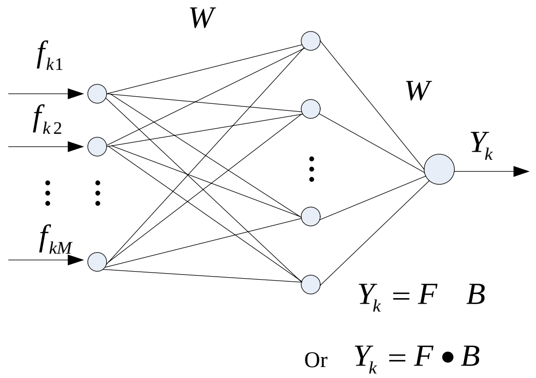


Fig. 1 three-layer Neutral Network model

In order to ensure the strict restriction to the input data of every layer by structure elements B, introduce a binary limited weight vector W into the open/close operation neutral networks. In erosion neutral network,

$$W(x,m) = \begin{cases} \mathbf{1}, & x+m \in P \\ \mathbf{0}, & others \end{cases} \tag{3}$$

In dilation neutral networks:

$$W(x,m) = \begin{cases} \mathbf{1}, & x-m \in P \\ \mathbf{0}, & others \end{cases} \tag{4}$$

For the purpose of training filter parameters in the paper, the most important thing is that the nonlinear projection output of morphological filter should approximate to the expected values of training samples. Assume that the dimension of sample vectors is M, the length of training samples is L. After inputting the kth training sample, the output is Y_k, the expected value is d_k. Then open/close morphological method based on BP neutral networks could be described as follows,

(1)Initialize the network weights $b_m(\mathbf{1} \le m \le M)$ and set the end value ε of error cost function E, select the pertinent learning rate η and momentum $\alpha\ (\varepsilon \ll \mathbf{1}, \mathbf{0} < \eta < \mathbf{1}, \mathbf{0} < \alpha < \mathbf{1})$.

(2)Generate the initial training and delimit the maximal epoch $T_{\max}$, $t = \mathbf{1}$.

(3)While ($E > \varepsilon$ and $t < T_{\max}$) DO

$$\delta_m = \frac{\mathbf{1}}{L}\sum_{k=\mathbf{1}}^{L}\left(Y_k - d_k\right) \times g\left(Y_k, b_m\right) \tag{5}$$

$$b_m(t+\mathbf{1}) = b_m(t) - \eta\delta_m + \alpha[b_m(t) - b_m(t-\mathbf{1})] \tag{6}$$

$$E = \frac{\mathbf{1}}{\mathbf{2}L}\sum_{k=\mathbf{1}}^{L} e_k^{\mathbf{2}} = \frac{\mathbf{1}}{\mathbf{2}L}\sum_{k=\mathbf{1}}^{L}(Y_k - d_k)^{\mathbf{2}} \tag{7}$$

Where, for the open network,

$$g\left(Y_k, b_m\right) = \begin{cases} -\mathbf{1}, & Y_k = f_{km} - b_m + b_j,\ j \ne m \\ \mathbf{1}, & Y_k = f_{km} - b_i + b_m, i \ne m \\ \mathbf{0}, & others \end{cases} \tag{8}$$

And for the close network

$$g\left(Y_k, b_m\right) = \begin{cases} -\mathbf{1}, & Y_k = f_{km} - b_m + b_i, i \ne m \\ \mathbf{1}, & Y_k = f_{km} - b_j + b_m,\ j \ne m \\ \mathbf{0}, & others \end{cases} \tag{9}$$

III. ZERNIKE MOMENTS OPERATOR

The results of edge detection are smoothened by the morphology method. The noises in the images could be removed. However, the accuracy of edge could not approach to the subpixel extent. Here, we just employ the existing Zernike moments operator.

The reason of using Zernike moments is the special property of circular polynomials of Zernike moments. Only three masks are used to calculate four parameters of every edge point, as shown in Fig. 2. k is the step height, h is the background gray level, l is the perpendicular distance from the center of the circular kernel and the edge makes an angle off with respect to the x-axis.

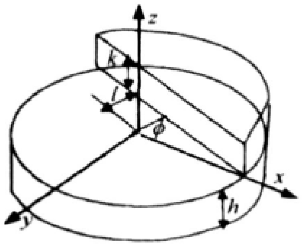


Fig.2 subpixel step edge model

A. Theory about Zernike moments

Zernike moments for an image $f(x,y)$ is defined as [5],

$$A_{NzMz} = \frac{Nz+\mathbf{1}}{\pi}\iint_{x^{\mathbf{2}}+y^{\mathbf{2}}\le\mathbf{1}} f(x,y)V_{NzMz}^{*}(\rho,\theta)dxdy \tag{10}$$

where $Nz+\mathbf{1}/\pi$ is a normalization factor and it is ignored in future discussion. In discrete form, A_{NzMz} can be expressed as

$$A_{NzMz} = \sum_{x}\sum_{y} f(x,y)V_{NzMz}^{*}(\rho,\theta), x^2+y^2 \le \mathbf{1} \tag{11}$$

It can be seen from (11) that in a discrete image, the neighborhood of that point should be mapped onto the interior of the unit circle for evaluating Zernike moments A_{NzMz} of an image point. The complex polynomials $V_{NzMz}(\rho,\theta)$ can be expressed in polar coordinates as

$$V_{NzMz}(\rho,\theta) = R_{NzMz}(\rho)e^{jMz\theta} \tag{12}$$

where $R_{NzMz}(\rho)$ is a radial polynomial defined as

$$R_{NzMz}(\rho) = \sum \frac{(-\mathbf{1})^s(Nz-s)!\ \ \rho^{Nz-\mathbf{2}s}}{s!(\frac{Nz+|Mz|}{\mathbf{2}}-s)!(\frac{Nz-|Mz|}{\mathbf{2}}-s)!}$$

If an image is rotated by an angle ϕ, the Zernike moments of the original image Anm and the Zernike moments of the rotated image has the following relationship:

$$A'_{NzMz} = A_{NzMz}(\rho)e^{-jv\theta} \tag{13}$$

B. Zernike moments operator edge detection

For calculating edge parameters l and ϕ, three masks $A_{\mathbf{00}}$, $A_{\mathbf{11}}$, $A_{\mathbf{20}}$, should be deduced. According to (12), the orthogonal complex polynomials can be written as: $V_{00}=1, V_{11}=x+jy, V_{20}=2x^2+2y^2-1$. In this work we the unit circle is divided into 7×7 homogeneous grids, masks are calculated when making integral for $V_{00}^*, V_{11}^*, V_{20}^*$ on the dashed area of every grid. Herein, assuming $f(x,y)$ to be constant over every pixel, convolving these masks with the image points can get Zernike moments[5].

According to (13), the relationship between Zernike moments of original image A_{00}, A_{11}, A_{20} and rotated image $A_{00}', A_{11}', A_{20}'$ can be given as $A_{00}'=A_{00}$, $A_{11}'=A_{11}e^{-j\phi}$, $A_{20}'=A_{20}$.

Furthermore, the following equations can be deduc-ed based on theory of Zernike moments,

$$A_{00}' = h\pi + \frac{k\pi}{2} - k\sin^{-1} l - kl\sqrt{1-l^2} \tag{14}$$

$$A_{11}' = \frac{2k\sqrt{\left(1-l^2\right)^3}}{3} \tag{15}$$

$$A_{11}' = \frac{2kl\sqrt{\left(1-l^2\right)^3}}{3} \tag{16}$$

When the edge is rotated an angle ϕ, it will be aligned parallel to y-axis so that

$$\iint_{x^2+y^2\le 1} f'(x,y)\,y\,dx\,dy = 0 \tag{17}$$

So

$$\phi = \tan^{-1}\left(\frac{\mathrm{Im}[A_{11}]}{\mathrm{Re}[A_{11}]}\right) \tag{18}$$

Solving (17) and (18), the edge parameter l can be given as:

$$l = \frac{A_{20}}{A_{11}'} \tag{19}$$

The subpixel location of image edge is

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + l \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \tag{20}$$

That means, the three Zernike moments A_{00}, A_{11}, A_{20} [6] could locate the edge to subpixel accuracy.

IV. Experimental results

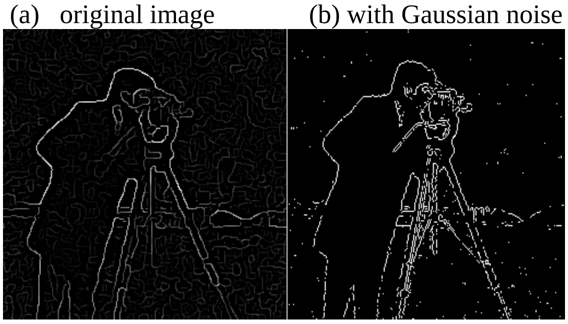
In this section, we take examples to show that the proposed method will eliminate the noise, and improve the detection performance (especially on the accuracy).

The tests can be divided into two parts. In the first part, the noisy images are filtered by the morphological method based on neutral network. In the second part, the subpixel edges are located by the Zernike Moments.

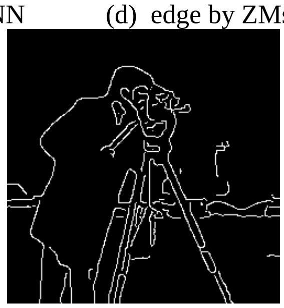
In the following tests, the initial values of the BP networks are the same. $\varepsilon = 0.01$, $\alpha = 0.3$, $\eta = 0.25$. The initial shape of $\boldsymbol{B}$ is a circle with $R=\sqrt{10}$,and the initial value are all zeros. $M=L=5$.

Fig. 3(a) shows the original image cameraman, Fig.3(b) shows the image with Gaussian white noise with variance σ = 0.02. Fig.3(c) is the result of Fig.3(b) detected by the morphology based on neutral network(M-NN), Fig.3(d) is the result of Fig.3(b) only by Zernike moments (ZMs), and Fig.3(e) is the result of Fig.3(c) by ZMs, that is the result of proposed method. It can be seen that the edge detected by M-NN has been smoothed, but not very thin. ZMs operator can sharpen the edge, because integrity is not sensitive to noise. Compared with the former two detections, the edge image by the proposed method is the greatest as expected, because it combined the virtues of two methods.

(a) original image (b) with Gaussian noise



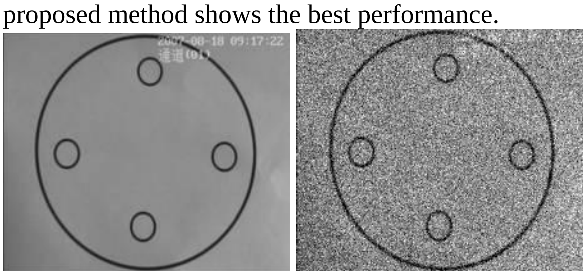
(c) edge by M-NN (d) edge by ZMs only



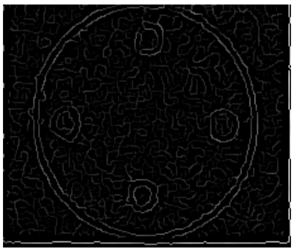
(e) edge by proposed method

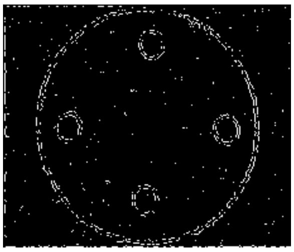
Fig.3 Edge detection of noisy image with different methods

The detection comparison of an experimental image taken by CCD camera in a testing laser communication system is as shown in Fig.4. The size of image is 231*194 in pixel. Gaussian white noise with variance σ = 0.02 is also added to the image (Fig.4 (b)). The edges of the image with noise have been located by using M-NN only, the Zernike moments-based subpixel location method only and the proposed method, respectively. It can be seen that from the aspects of the accuracy and continuity of the edge, the proposed method shows the best performance.



(a) original image (b) with Gaussian noise


(c) edge by M-NN


(d) edge by ZMs only

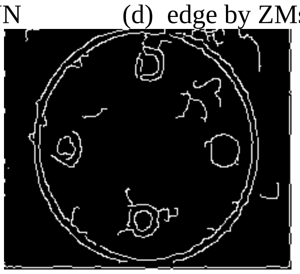

(e) edge by proposed method

Fig.4 Edge detection of noisy experimental CCD image with different methods

V. Conclusion

A high-accuracy edge detection method that combines the morphology with neutral networks and Zernike moments operator is proposed in the study. A three-layer neutral network is employed to determine the structure elements in the morphology method, so that the image can be smoothened and all probable edge points can be detected. Zernike moments operator is adopted to locate the edge to subpixel accuracy degree. The test results show that the proposed method can improve the performance of the edge detection with noise. And it has been employed in a testing laser communication system. And the future research will focus on how to realize the proposed method in real time system.

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