The Double-Helix Pattern of Prime Number Growth

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Abstract—**This paper is the result of analyzing the growth pattern of the first 500 prime numbers. The growth rate was found to be related to two characteristics – two parallel threads and the almost viral impact of the multiples of 6 on these two threads. When corresponding cause and effect of these impacts are overlaid, a double-helix structure is the result. This concept and modeling approach is submitted with the hope of providing a model that helps connect the research benefits of bioinformatics and the potential impact of the prime number structure of the Riemann Hypothesis when applied by professionals in their scientific computing fields.**

Index Terms—**Double-helix, gap, growth, prime numbers.**

I. Introduction

Why should we look at the unresolved growth characteristics and relationships between the prime numbers? The most obvious answer is just co increase our understanding of the prime number behavior. We also need to somehow connect the sciences not just to understand prime numbers, but to see if their characteristics can help us solve natural incompressible problems, such as viruses.

In a March 2007 New York Times interview with Dr. Terence Tao covering his ground-breaking work in prime numbers, it was mentioned that a "larger unknown question is whether hidden patterns exist in the sequence of prime numbers or whether they appear randomly" [1]. Therein lays the scope of our challenge, broad as it may be. The intent of this paper is to provide such a pattern for modeling and attacking the characteristics and relationships between the prime numbers.

The method of analysis was straightforward. The incremental growth of the first 500 prime numbers was reviewed and the almost symmetric patterns were noticed. These patterns apparent had center elements that would be expected with palindromes. The first attempt placed these into a 10-step DNA sequence fit. Within the sequence were elements that, if taken away, would have made the pattern symmetric.

The presence of these seemingly extra elements (the number 6) gave us two indicators to solving our basic structure. One indicator was that removing certain select "6"s would have made the selected sequences into cleaner palindrome DNA sequences. The other indicator was that only multiples of 6 occur consecutively.

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All multiples of 6 were temporarily removed and the sequence was re-evaluated. The simplified sequence showed a definite a pattern of overlapping palindromes, with no consecutive, contiguous occurrence of any number. Closer examination of each palindrome explained the pattern. The string is an alternating sequence of elements from two sets; the set starting with 2 and incremented by 6, {2, 8, 14, 20, 26, …, 6n+2}, and the set starting with 4 and incremented by 6, {4, 10, 16, 22, 28, …, 6n+4}.

Two columns (threads) were formed from these two sets and the multiples of 6 were re-inserted in their original order to see how the multiples of 6 affected the basic 2-thread and 4-thread. The physical structure was a flattened spring/coil with the multiples of 6 as elements between the left and right 2-thread and 4-thread sides of the coil. The pattern showed a cause and effect rate of 5 coil steps, with very little variation. The 2-4 thread coil structure was twisted to overlay the cause and effect points. The result was a double helix structure.

This paper presents a logical model using the sequence of prime numbers that reveals patterns that not only explain the larger irregular incremental jumps between prime numbers, but also the interconnected purpose and relationship between prime number increments. It is the hope of the author that these findings will have a potential significance to quantum mechanics, physics, number theory, bioinformatics, and other computing fields to solve our most difficult, yet common, problems as a society. We will now walk-through the logic and decision points made to discover this prime number relationship.

II. Why Start with a DNA-like Sequence?

Since we normally look at the prime numbers as strictly increasing numbers, we naturally think of a programmed growth behavior. We consider an analog approach because neither the rate nor the ratio of the increase is linear. We also consider a complex variable or fractal rate, which makes us thinks of a polar-coordinate or spiral approach. Ideally, we expect a resulting prime number DNA-like sequence solution to explain or solve multidimensional problems [2].

We start by taking the individual incremental steps between the first 500 prime numbers [4]. Next, we assume the prime numbers have some connected spiral or fractal behavior [3]. Now, we look for short reversal sequences that resemble a recursive logic entry and exit routine. What better length to start with than 10 and 20 step sequences? Do we see sets of reversing sequences that lead us further? Yes.

We notice three almost reversed sequences of 20. Our three sample sequence sets of 20 give us two indicators that lead us to resolve the basic patterns of the prime number growth and relationship model. Table 1 has these three sequences in a format that overlaps the reversal sequence; it shows the forward first half of the sequence overlaid on the reverse second half of the sequence. This is done to demonstrate the two deterrents to having a perfectly clean recursive entry and exit, or palindrome, behavior.

Table 1. Ten-step reversal sequences noting unexpected inserts

Direction	Steps	Forward and Reverse Sequences									
Forward	5 to 14	4	2	4	2	4	6	2	6	4	2
Reverse	24 to 15	6	4	6	2	4	6	2	6	6	4
Forward	30 to 39	4	14	4	6	2	10	2	6	6	4
Reverse	49 to 40	4	12	12	2	4	2	10	2	6	6
Forward	56 to 65	6	6	2	6	4	2	10	14	4	2
Reverse	75 to 66	6	6	8	6	4	2	10	6	14	4

Our first deterrent is the darkened cells with white text. These values almost appear to have been inserted. Even more strange is that these "inserted" numbers are multiples of "6". Removing these numbers and shifting the items left would result in palindromes, with the only nonequivalent characteristic being the seemingly well-panned value differences shown in the dark grey cells.

Our second deterrent is that two of these seemingly inserted multiples of 6 are the only numbers allowed to be side-by-side duplicates of "6-6" (in steps 24 to 15) and "12-12" (in steps 49 to 40). With a couple deductive steps, these multiples of 6 lead us directly to their controlling power over a double-helix structure.

III. Double-Six Pair Indicators

Upon examining all the individual step increments of the first 500 consecutive prime numbers, we notice that only the multiples of 6 have duplicates side-by-side. What does this mean? Are the double-6 points touching endpoints to sequences? Do they indicate sums of either vertically or horizontally overlaid patterns? We could go into the dozens of failed approaches and the hundreds of attempted combinations from this analysis…but why? Just those two questions sufficiently summarize the complexity (and pain) of the effort expended. So, what did work? We simplified the problem…we temporarily removed all multiples of 6 and reevaluated the sequential growth of the 500 prime numbers. Yes, we realize that we are temporarily overlooking those prime numbers whose gap is a multiple of 6, but this is just a temporary removal .

A. *Overlapping Palindrome Theme*

At first thought it may sound absurd to remove items from the sequence to determine the core patterns of the sequence, but it is no different than dealing with one variable at a time or a partial derivative with respect to one plane at a time.

We use Table 2 to display how a centered approach can be taken to examine the resulting overlapping and growing palindromes, now that the multiples of 6 are removed. The shaded cells are examples of palindrome centers. Some palindromes have transition cells between successive palindromes. The growth of certain repeated non-palindrome sequences becomes evident as a result of this organization; Lines 18, 26, 29, 33 all starting with the "14-4-2-4" are an obvious example.

Table 2. Off-center growth and overlapping of palindromes

Line	4	3	2	1	0	1	2	3	4	5	6
1			2	4	2	4	2	4			
2			2	4	2	4	2				
3		4	2	4	8	4	2	4			
4			2	4	14	4	2				
5			10	2	4	2	10				
6			2	4	2	4	2				
7			4	2	10	2	4				
8	2	10	14	4	2	4	14	10	2	4	
9			8	4	8	4	8				
10				10	2	10					
11			2	4	8	4	2				
12			4	8	4	8	4				
13			2	10	2	10	2				
14			4	2	10	2	4	2	4		
15			8	10	8	10	8				
16			4	8	4	8	4	14			
17			10	2	10	2	4	2	10		
18			14	4	2	4	14	4	2	4	
19					20						
20			4	8	10	8	4	14	4	8	
21	4	2	10	2	10	2	10	2	4	2	4
22	8	22	2	10	8	10	8				
23			4	2	10	2	4				
24			2	4	2	4	2				
25					34	8	10				
26		14	4	2	4	8					
27			4	2	10	2	4	2	4		
28			8	4	8	4	8	4			
29		14	4	2	4	2	10				
30					20						
31			4	2	4	2	10	2	10		
32				8	4	2	10	8	16		
33		14	4	2	4	2	10	2	16	2	
34					22						
35			8	4	2	4	8				
36		10	2	10	14	10	2	4	2	10	
37		2	16	2	4	2	10				
38				8	4	8					
39		16	2	4	8	16	2	4	8		
40			4	2	22	2	4				
41			14	4	2	4	14	4	8	4	

Yet, there are still small sequences and items that do not fit; these are shown in darkened cells with white text. But, as much as we may want to head down a definite path of working with palindromes, or possible world of zero determinants, this is not the best course. We need to go the next decision point with some guidance from our next clue.

The clue – the patterns that we do not see are more significant than the patterns that we do see. We do not see any pairs of the same number; no "2-2", no "4-4', no "8-8", no "10-10", no "14-14", no "16-16", no "20-20", etc. The only exception is for the prime numbers less than 5, where there exists a single contiguous step pair of 2.

What does this mean? It obviously means there were no combinations of "2-6-2" or "4-6-4" when we had our multiples of 6 included. However, we did have combinations of "6-2-6", "6-6-2-6", "6-4-6", and "6-6-4-6". Here is one of the main key concepts: "6" is a controller; no single unique type of number can sandwich, squeeze, or control the number "6".

Table 3. Relationship between 6 and parallel 2-4 threads

#	Step & Action	4 to 2			2	2 to 4			4	4 to 2				Step & Action
1	From 5 to 7				2				4					From 7 to 11
2					2				4					
3					2				4					
4	4-Sat 6			6	2	6			4					
5					2				4					
6	5-Sat		6	6	2	6			4					
7					2			6	4	6				5-Sat
8	5-Yield/Prop				8				4					
9					2				4					
10					2				4					
11	5-Yield/Prop				14				4	6				
12					2				10					5-Yield-R
13	5-Shift R				2	6		6	4	6	6			5-Sat 6
14					2				10					6-Continue
15					2				4					
16	5-Absorb-L				2	12		12	4					

Now what? We have two questions: One, what is the next dimension to the logical model of data exists that makes the number "2" (or "4") non-contiguous? Two, how does the number "6" (and multiples of 6) exercise control over the palindrome values and sequences?

B. The Parallel 2 and 4 Threads

The structure that occurs from answering the first question leads us to the answer for the second question. We clearly notice the initial "2-4-2-4-2" pattern of growth, that is, until we meet the number "8" in line 3. But the number "8" is where "2" would normally be, and, in line 4 the number "14" is where "2" would normally be. In line 5, we see the same behavior with regard to the number "4", where the number "10" is where "4" would normally be. Now we have the pattern – this is a continuously alternating sequence of "2-4" impacted by the multiples of 6. So, we have two core threads, a 2-thread and a 4-thread. These can be considered two unique threads with elements from either one of two sets, the set starting with 2 and incremented by 6, {2, 8, 14, 20, 26, …, 6n+2}, or the set starting with 4 and incremented by 6, {4, 10, 16, 22, 28, …, 6n+4}. Our next step is to attempt to answer our second question explicitly – exactly how the multiples of 6 control our two core threads.

IV. How "6" Controls Double 2-4 Thread

We re-insert the multiples of 6 and place each occurrence of the 2-thread and 4-thread in a vertical column. We can best describe the physical model as a sequential traversal of a somewhat flattened coil. At 180-dgrees apart, the 2-thread is on the left arch of the coil, and the 4-thread is on the right arch of the coil; the multiples of 6 are elements on the transition segment connecting the threads. Table 3 contains a sample using the sequential progression of the 46 prime number increments after the number 5. Our selected set of numbers is the start of the 2-4 thread. Our sample has 16 items on the 2-thread, 16 items on the 4-thread, 7 items in the 2-thread to 4-thread zone, and 7 items in he 4-thread to 2-thread zone.

Cells in the table use either a shaded cell or a shaded grid-filled cell to correspond to the suggested relationship between positions of the multiples of 6 and the incremental amounts in the 2-thread and the 4-thread. Do any instructions or general rules result? Yes: we have saturation, yielding, propagating, absorbing, and shifting at a general rate of five coil steps. Some increments continue or maintain sequences at the step rate of 6. The step rate for continuing patterns may be the next dimension of analysis for the future modelers. As we review Table 3, we need to give a brief definition of each instruction:

- Saturating: This is squeezing the 6-value into the thread element when a thread element has multiples of 6 as both a predecessor and as a successor. The thread element is incremented by a maximum value up to the sum of the multiples of 6 on one side. The result of saturating mostly appears at 5 coils steps later.
- Yielding: This allows the releasing of an incremented value from a 2 or 4 thread element. Yielding is often accompanied by Propagating to the right. The result of yielding mostly appears at 5 coils steps later.
- Propagating: This is the act of reproducing one or more single (or combined) multiples of 6 at a previously incremented 2 or 4 thread element. The result of propagating mostly appears at 5 coils steps later.
- Absorbing: This is an increment of a 2 or 4 thread element the same as saturating except it allows for incrementing to the left. This possibility of early absorption also leads us to consider an early overlapping or intersecting path of the coil as it traverses about a helix formation.
- Shifting: This is an almost fluid or torque behavior of a multiple of 6. Although most shifts appear to be right, it is also possible that they go left due to the intersections with the actual wrap-around sate of the coil. Combined instructions may also be the source of shifting.
- Continuing: This is a designated periodicity with no change. Both 2 and 4 threads start with a continuing behavior. It is mostly obvious after a 2-4 thread member is incremented and that value is maintained. The result of continuing can appears at 5 or 6 coils steps later.

Wait a minute! How can the first saturation pattern be a 4-step exception to the general 5-step coil cause-effect rule? The beginning steps of a sequence are not always at the full rate of the series. We even see that principle with the gaps between the first three prime numbers (1, 3, and 5), which have adjacent gaps of 2 prior to the initial 2-4 thread trend starting with gaps after 5. Circuits, motors, and even people need power-up and warm-up cycle. This 4-step occurrence could also be considered as being 5 steps from its 4-thread origin. In Table 3, this is the right (4-2 transition) of line 3, instead of the left of line 4 (the same 4-2 transition).

Table 4 gives us an even more drastic example of multiple instructions. Not only is the moving total of 42 from spiral 89 to spiral 94 a very convincing example, but the absorption from spiral 87 to spiral 92 (12 to 14) and the saturation from spiral 90 to spiral 95 (12-10-12 to 16) are indicators of a possible natural or mechanical dynamic.

Table 4. Example of combined periodic influence on 2-4 threads

#	2	2 to 4					4	4 to 2			
87	2						10	12			
88	2						10				
89	8	6	6	6	18	6	4				
90	2					12	10	12			
91	8						16				
92	14			6			4				
93	2						4				
94	2						10	12	6	6	18
95	2						16				

Can the cause and effect of these instructions be similar to the affect of RNA, viruses, oncogenes, or other incompressible physical properties [5]? The author would not be the right person to say definitively. Although that would be the hope of discovering this new structure, the extent and usefulness of this structure needs to be examined by professionals in the appropriate fields of study.

This 5-step relationship leads us to attempt to define or develop a physical model for these corresponding actions and resulting transformations by the multiples of 6. When we overlap these assumed cause and effect relationships, we twist the coil such that the 5-coil steps occur in 360-degree, or 2-PI. Now our two base threads form two helix structures from this rate of overlapping.

V. Two Possible Double Helix Models

These 5-coil step control instructions point us to two possible basic structures: an internal-external double helix structure around a core and an alternating torque model across a core. Either model may be best, depending on goal of the computational scientist. The model is obtained by overlapping the cause and effect between the actions of our instructions (saturating, etc).

A. *Internal-External Helix Structure*

Fig. 1 gives us an example of the double helix spiraling externally around a core for lines 6 to 15 of Table 3. The dotted arrows are the progression from one level down to the next level (from left to right). This structure keeps the 2-thread (now in helix form) in the center, closest to the core. This also creates two cylinder shells, in some ways possibly similar to a cancerous virus shell or the spiral action of virus shells [6].

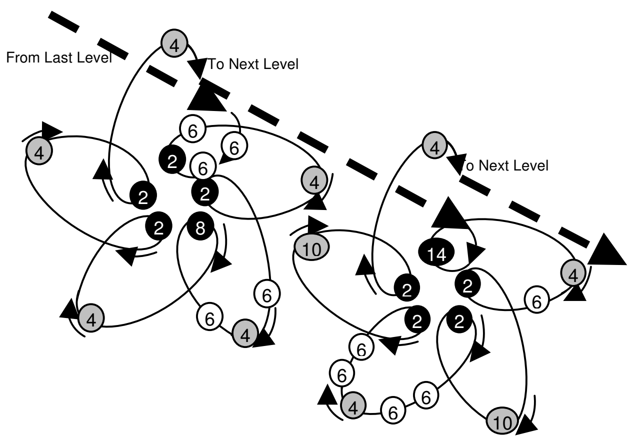

Figure 1. Five-Stage Outside-spiraling Double Helix

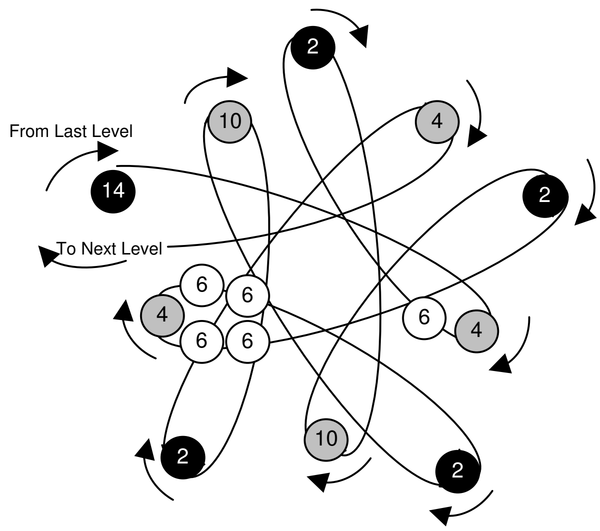

Figure 2. Five-Stage Cross-torque Spiraling Double Helix

B. *Alternating Cross-Torque Model*

Another option we have is a cross-torque model. Due to the complexity of the cross-torque model, Fig. 2 only has the right-most spiral of Fig. 1 (lines 11 to 15 of Table 3). In Fig. 2, the 4-thread elements were placed within the 5-point shape of the 2-thread to illustrate a point. We do not necessarily need to have smaller values in the center of the physical model, as the case would be if we were displaying the values on a number line. This also allows us to visually consider the concept of counter-balance, where the longer moment to arm torque value of the 2-thread would balance with the shorter moment to arm torque of the 4-thread.

One parallel to a cross torque model is attained by considering the actions taken to tighten a wheel with two sets of 5-lug-nuts onto its hub. One lug-nut is tightened (2 thread element) and then the opposing lug-nut (4-thread element) that is located at 180-degrees clockwise is tightened. The next clockwise 2-thread lug-nut gets tightened, immediately followed by the next opposing clockwise 4-thread. This analogy obviously breaks down because we repeat the sequence on the same set of lug-nuts on a wheel, in contrast to continually getting a new 2-4 cross-pair with our 2-4 thread coils.

While the lug-nut analog is very basic, our desire is to equate the behavior of primes to physical models. It might even be better to consider the a flywheel or counterbalance catapult affect of leveraging of force and momentum as with a recent method developed for maximizing an electric motor drive system [7]. Somehow that torque is transferred to the next level.

C. *Cylindrical Shells of the Double-Helix Structures*

Table 5 is side view of the 2 and 4 threads as they progress clockwise. It should be noted that the direct correlation between the two 2-thread and the 4-thread elements is displayed by their relative location in the shells. This simple example for analyzing the cylinder shells may provide a little insight on the relative rate of change for the threads. The shaded cells in Table 5 show 3X3 zero determinant vectors.

Table 5. Inside view of shells for the 2 and 4 threads

Left to Right Inside View of Clockwise Shell										
2 - Thread						4 - Thread				
2	2	2	2	2		4	4	4	4	4
2	2	8	2	2		4	4	4	4	4
14	2	2	2	2		4	10	4	10	4
2	2	2	2	2		4	4	10	4	10
14	2	14	2	8		4	4	10	4	4
8	8	2	2	8		4	10	10	4	4
2	8	8	2	2		4	4	4	10	10
2	2	2	2	8		4	10	4	4	10
8	8	8	8	14		10	4	4	4	10
2	2	2	14	2		10	4	10	4	4
14	2	20	8	8		4	4	4	10	4
14	8	2	2	2		4	4	10	10	10
2	2	8	2	8		4	4	22	10	10
8	2	2	2	2		4	10	4	4	4
2	8	14	2	8		34	10	4	4	4
2	2	2	8	8		10	4	4	4	4
8	14	2	2	20		4	4	4	10	4
2	2	2	8	2		4	10	10	4	10
8	14	2	2	2		16	4	4	10	16
2	8	2	8	2		22	4	4	10	10
14	2	2	2	2		10	4	10	16	4
2	8	8	2	8		10	4	16	4	16

These vectors are overlapping 3X3 zero determinant matrices. Although it is not exactly clear what is happening, this may indicate the 2-thread rate of change may be almost twice that of the 4-thread. The 2-thread makes 11 steps before another column is started that could be included into the zero determinant matrix. The 4-thread starts another column in 6 steps. This may indicate that more items are being incremented in the 2-thread, which deters the repetition of sequences that lead to a 3X3 zero determinant vector. There has already been some research that indicates this may be a potential concept worth exploring, since "studies by mathematicians and physicists have identified a close association between the distribution of prime numbers and quantum mechanical laws" [8].

Yet, instead of hyper-focusing on these two suggested models, instead we need to ask "how can the overlapping theme of the prime number threads be used to meet our/your computational needs?" Will this parallel the expected properties from the behavior of L-functions over a complex plane [9]? Maybe. Unfortunately, the author can only try to imagine how this approach can be used and what models will best serve the general computing needs of others. Apart from directing solutions, what should we reconsider regarding the properties of prime numbers?

VI. New Prime Number Mindset

Instead of thinking of the incremental growth of prime numbers as a value on a number scale, we need to think of them more in terms of an information container in a relational structure. The real applied value for that type of information container could be virus, cancer, density, mass, molecular, momentum, torque, etc. We need to consider the type of control that the increments with multiples of 6 have on the increased and decreased growth of prime numbers. We can do this by using a modulo-6 modeling approach.

A. *Modulo 6 Driven Modeling*

While much progress has been made by using modulo functions with the prime numbers [10], using modulo functions on the individual progressive growth steps may be the best way to summarize our discovery of a 2-4 thread in this paper. Two basic modulo 6 steps help us. First, by perform a modulo 6 operation on the first 500 prime numbers and we get our core 2-4 thread with only "2"s and "4"s, a clean alternating structure. This also gives us an overlapping 2X2 zero determinant matrix vector as a core structure. We can get a specific coil location by counting the number of "2"s. Second, divide the string in a single coil by 6 to determine the amount of torque between the 2-4 elements. This will always be an integer since the operation uses the "2—6m—4—6n" combination to get the single coil step's weight.

Is there a third modulo-6 dimension or characteristic that this new model brings us? Maybe. For anyone doing further research into the rates of updates or the zero determinant matrix vectors, a third modulo 6 operation might help solve the next step. There is also no written rule that all coil spirals around the double helix are at the rate of 5 steps, maybe a better physical model is to use 6 steps. I do not know. Along with the physical structure being controlled with a modulo 6 framework, we should probably take a few moments to correlate it to the work done by Riemann.

B. *Rethinking the Riemann Hypothesis*

If we are convinced that the intent of Riemann was to understand the distribution of the prime numbers in order to understand their behavior, we might consider this prime number modeling approach as a different aspect to Riemann's intended goal. The goal is to understand all the behavioral aspects of prime numbers. That is, we apply an added advantage we may have over initial ground-breaking work in prime numbers done by Riemann. We have evidence of the computational impact and contribution of the DNA structure.

When Riemann was looking for a distribution of the primes, he related their distribution to the zeta function, where the zeros lay on the critical line around the value of ½ on the complex plane [11]. The use of our double thread helix is a little twist to that concept. Our 2-4 thread has a core modulo 6 ratio of ½ and it twists along the complex plane at the rate of the matched cause and effect of the multiples of 6. Hopefully this modeling approach can give us a good link between the characteristics of prime numbers connecting point between several scientific and mathematical disciplines.

VII. Conclusion

The growth of the first 500 prime numbers is related to the affects to a double-threaded physical model (with a 2-thread and a 4-thread). The cause and effect increments by multiples of 6 to these threads lead us to a double helix structure with a generic rate of 5 coil steps per 360-dgrees.

When the multiple of 6 growth gaps are removed, the 2-4 growth thread occurs in a linear form as an alternating sequence of elements from two sets; the set starting with 2 and incremented by 6, {2, 8, 14, 20, 26, …, 6n+2} and the set starting with 4 and incremented by 6, {4, 10, 16, 22, 28, …, 6n+4}. Modulo 6 can be used to find the common structure.

Dividing any single step coil's string by 6 provides an integer weight or torque for a specific coil.

Two basic models were presented: an internal-external double helix structure around a core and an alternating torque model across a core. Either model may be best, depending on goal of the computational scientist.

This paper is presented with the hopes that these findings will have a potential significance to number theory, quantum mechanics, physics, bioinformatics, and other computing fields to solve our most difficult and common problems as a society.

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