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Magnetostatic Field Calculations Associated with Thick Solenoids in the Presence of Iron

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Abstract-**The effect of iron on the uniformity of the field produced by an axisymmetric thick solenoid is considered. Using a power series expansion of the vector potential in the radial and axial coordinates the potential is found and from this the magnetic induction B is derived. The solution to the vector potential and field components is also obtained using a Maclaurin series expansion in terms of the radial coordinate, ρ with numerical results using both methods of solution computed**

Key Words: **Time independent field, Maclaurin Series and Power Series expansion.**

I. Introduction.

In this paper magnetostatic field calculations associated with an axisymmetric conductor of rectangular cross section situated equidistant from two semi-infinite regions of iron of finite permeability are computed. The magnetostatic field associated with iron-free axisymmetric systems has been considered by Boom and Livingstone [1], Garrett [2] and many others. Caldwell [3], Caldwell and Zisserman [4] and [5] have carried out work which takes account of the effects of the presence of iron on such systems. The main advantages of introducing iron are:

i. Higher fields are provided for the same current, producing substantial power savings over conventional conductors.

ii. The field uniformity is improved even for superconducting solenoids by placing the iron in a suitable position. The geometry considered is shown in figure 1, a toroidal conductor V' of rectangular cross section having inner radius A, outer radius B and length L-2ε, is located equidistant between two semi-infinite regions of iron of finite permeability a distance L apart, the axis of the torus being perpendicular to the iron boundaries. The region V between the conductor and the iron is assumed insulating. Cylindrical polar coordinates (ρ, φ, z) are used, where ρ and z are normalized in terms of L.

Prior to Caldwell [3] the presence of iron in axisymmetric systems had been largely ignored see Loney [6] and Garrett [2] et al. In cylindrical coordinates Maxwell's equations give:

$$\underline{\nabla} \wedge \underline{B} = \begin{cases} 0 \;\; in \;\; V \\ -Ce_{\phi} \;\; in \;\; V' \end{cases}$$

where e_{ϕ} is a unit vector in the direction of increasing φ and C is a constant with

$$\underline{\nabla}.\underline{B} = 0 \text{ in V and V'} \tag{1}$$

Equation (1) suggests the introduction of a potential $\underline{A}$ such that $\underline{B} = \underline{\nabla} \wedge \underline{A}$, where $\wedge$ denotes the usual cross product of vectors, axial symmetry implies $B_{\rho} = -\frac{\partial A_{\phi}}{\partial z}; B_{\varphi}(\rho, z) = 0$; and $B_z = \frac{1}{\rho}\frac{\partial(\rho A_{\phi})}{\partial \rho}$.

By Maxwell's equation:

$$\underline{\nabla} \wedge \underline{B} = \underline{\nabla} \wedge (\underline{\nabla} \wedge \underline{A}) = \begin{cases} 0 \;\; in \;\; V \\ Ce_{\phi} \;\; in \;\; V' \end{cases}$$

thus with axial symmetry such that $\frac{\partial}{\partial \varphi} \equiv 0 \Rightarrow$

$$\frac{1}{\rho}\begin{vmatrix} e_{\rho} & e_{\phi} & e_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ -\frac{\partial A_{\phi}}{\partial z} & 0 & \frac{1}{\rho}\frac{\partial(\rho A_{\varphi})}{\partial \rho} \end{vmatrix} = \begin{cases} 0 \;\; in \;\; V \\ -Ce_{\phi} \;\; in \;\; V' \end{cases}$$

$$\Rightarrow \nabla_1^2 A_{\phi} = \begin{cases} 0 \;\; in V \\ Ce_{\phi} \;\; in V' \end{cases}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} - \frac{1}{\rho^2} + \frac{\partial^2}{\partial z^2}$

with boundary conditions for A_{ϕ}

$A_{\phi} = 0$ on $\rho = 0$

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$A_\phi \to 0$ as $\rho \to \infty$

$\frac{\partial A_\phi}{\partial z} = 0$ on $z=0$ and $z=1$

Using the integral representation of the vector potential this gives:

$\underline{A}(\underline{\rho}) = \int_{V'} \frac{j(\rho)}{|\underline{\rho} - \underline{\rho}'|} dv'$, hence for finite μ,

$$A_\phi(\rho, z) = \frac{\mu_0 j}{4\pi} \sum_{n=-\infty}^{\infty} K^{|n|} \int_a^b \int_0^{2\pi} \int_\varepsilon^{1-\varepsilon} \{ \frac{x \cos\vartheta dx d\vartheta dz'}{((z-z'-n)^2 + \rho^2 + x^2 - 2x\rho\cos\vartheta)^{1/2}} \} \tag{2}$$

where $K = \frac{\mu - 1}{\mu + 1}$, known as the image factor. Noting that $A_\phi(\rho, z)$ is an odd function in ρ and an even function in z then A_ϕ can be expanded as a power series about the z axis giving:

$$A_\phi(\rho, z) = \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \sum_{m=0}^{\infty} \rho^{2m+1} I_m(z) \tag{3}$$

where equation (2) gives

$$I_0(z) = \frac{1}{4}[[w \log_e |x + \alpha|]_a^b]_{z=\varepsilon}^{1-\varepsilon}$$

with w=z'-z-n and $\alpha^2 = x^2 + w^2$. Substituting expression (3) into equation (2) gives

$$\sum_{n=-\infty}^{\infty} K^{|n|} (\sum_{m=1}^{\infty} 4m(m+1)\rho^{2m+1} I_m(z) + \sum_{m=1}^{\infty} \rho^{2m+1} \frac{\partial^2 I_m(z)}{\partial z^2}) = 0$$

equating coefficients of $\mathrm{I_m}(z) \Rightarrow$

$$m(m+1) I_m(z) + \frac{\partial^2 I_{m-1}(z)}{\partial z^2} = 0, \quad m = 0, 1, 2, ...$$

$$\therefore I_m(z) = \frac{(-1)^m I_0^{2m}(z)}{2^{2m} m!(m+1)!}$$

and

$$A_\phi(\rho, z) = \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \sum_{m=0}^{\infty} \frac{(-1)^m I_0^{2m}(z)}{2^{2m} m!(m+1)!} \rho^{2m+1}$$

To relate this to the work of Garrett [2] let

$$a_1(x, w) = w \log_e |x + \alpha| \Rightarrow I_0 = \frac{j}{4}[[a_1(x, w)]_a^b]_{z=\varepsilon}^{1-\varepsilon} = \frac{A_1(z)}{2}$$

where $a_1(x, w) = \frac{j}{2}[[w \log|x + \alpha|]_a^b]_{z=\varepsilon}^{1-\varepsilon}$ (4)

and $A_m = \frac{j}{2}[[a_m(x, w)]_a^b]_{z=\varepsilon}^{1-\varepsilon}$ (5)

so that

$$A_\phi(\rho, z) = \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \sum_{m=0}^{\infty} \frac{(-1)^m (2m)! A_{2m+1} \rho^{2m+1}}{2^{2m+1} m!(m+1)!}$$

$$= \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \sum_{m=0}^{\infty} \frac{(-1)^m (2m-1)! A_{2m+1} \rho^{2m+1}}{(2m+2)!}$$

where (2m-1)!= 1.3.5…(2m-1), and (2m+2)!= 2.4.6…(2m+2),

with $A_{m+1} = \frac{1}{m} \frac{\partial^m a_1}{\partial z^m}$ (6)

so for the field components

$$B_z(\rho, z) = \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \sum_{m=0}^{\infty} \frac{(-1)^m (2m-1)! A_{2m+1}(z) \rho^{2m}}{(2m)!}$$

and

$$B_\rho(\rho, z) = \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \sum_{m=0}^{\infty} \frac{(-1)^{m+1} (2m+1)! A_{2m+2}(z) \rho^{2m+1}}{(2m+2)!}$$

Hence

$$A_\phi(\rho, z) = \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \left(\frac{\rho}{2} A_1 - \frac{\rho^3}{8} A_3 + \frac{\rho^5}{16} A_5 + ... \right)$$

$$B_z(\rho, z) = \mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \left(A_1 - \frac{\rho^2}{2} A_3 + \frac{3\rho^4}{8} A_5 + ... \right)$$

and

$$B_\rho(\rho, z) = -\mu_0 \sum_{n=-\infty}^{\infty} K^{|n|} \left(\frac{\rho}{2} A_2 - \frac{3\rho^3}{8} A_4 + \frac{5\rho^5}{16} A_6 + . \right)$$

The first five terms will be quoted, the remainder can be obtained from the recurrence relations equations (4), (5) and (6). So that

$$A_2 = \frac{j}{2}[[\frac{x}{(w^2 + x^2)^{1/2}} - \log_e(x + (w^2 + x^2)^{1/2})]_a^b]_{z=\varepsilon}^{1-\varepsilon}$$

$$A_3 = \frac{j}{2}[[\frac{-x}{(w^2 + x^2)^{1/2}} + \frac{xw}{(w^2 + x^2)^{3/2}}]_a^b]_{z=\varepsilon}^{1-\varepsilon}$$

$$A_4 = \frac{j}{12}[[\frac{x}{(w^2 + x^2)^{3/2}} - \frac{3xw}{(w^2 + x^2)^{5/2}} + \frac{xw}{(w^2 + x^2)^{3/2}}]_a^b]_{z=\varepsilon}^{1-\varepsilon}$$

$$A_5 = \frac{j}{48}[[\frac{3xw}{(w^2 + x^2)^{5/2}} + \frac{6xw}{(w^2 + x^2)^{3/2}} - \frac{15xw^3}{(w^2 + x^2)^{7/2}} - \frac{x}{(w^2 + x^2)^{3/2}} + \frac{3xw^2}{(w^2 + x^2)^{5/2}}]_a^b]_{z=\varepsilon}^{1-\varepsilon}$$

and

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$$A_6 = \frac{j}{240}[[\frac{-9x}{(w^2+x^2)^{5/2}} - \frac{9xw}{(w^2+x^2)^{3/2}} + \frac{15xw^2}{(w^2+x^2)^{7/2}} - \frac{xw^3}{(w^2+x^2)^{7/2}} - 105xw^4]_a^b]_{z=\varepsilon}^{1-\varepsilon}$$

II. The solution to the Magnetic Vector Potential using a Maclaurin Series in ρ.

In order to compare and validate the results of the previous section an independent solution for the magnetic vector potential and hence field components $B_\rho(\rho,z)$ and $B_z(\rho,z)$ must be derived. Using equation (2) the integral representation of the vector potential gives:

$\underline{A}(\underline{\rho}) = \int_{V'} \frac{j(\underline{\rho})}{|\underline{\rho}-\underline{\rho}'|} dv'$, hence for finite μ,

$$A_\phi(\rho,z) = \frac{\mu_0 j}{4\pi} \sum_{n=-\infty}^{\infty} K^{|n|} \int_a^b \int_0^{2\pi} \int_\varepsilon^{1-\varepsilon} \{ \frac{x\cos\vartheta dx d\vartheta dz'}{((z-z'-n)^2+\rho^2+x^2-2x\rho\cos\vartheta)^{1/2}} \} \quad (7)$$

where $K = \frac{\mu-1}{\mu+1}$, known as the image factor. By expanding $A_\phi(\rho,z)$ in a Maclaurin series in r it follows that:

$$A_\phi(\rho,z) = A_\phi(0,z) + \rho\frac{\partial A_\phi(0,z)}{\partial\rho} + \frac{\rho^2}{2!}\frac{\partial^2 A_\phi(0,z)}{\partial\rho^2} + O(\rho^3)$$

letting $I_n = \int_0^{2\pi} \frac{\cos\vartheta}{R} d\vartheta$, (8)

where

$$R = \{(z-z'-n)^2 + x^2 + \rho^2 - 2x\rho\cos\vartheta\}^{1/2}$$

$$A_\phi(\rho,z) = \frac{\mu_0 j\rho}{4}\Im + O(\rho^2) \quad (9)$$

where

$$\Im = \sum_{n=-\infty}^{\infty} K^{|n|}[[w\log_e(x+(w^2+x^2)^{1/2})]_{x=b}^a]_{w=w1}^{w2}$$

and $w1 = \varepsilon + n - z, w2 = 1 - \varepsilon + n - z$, which can be written as $A_\phi(\rho,z) = A_{\phi,1}(\rho,z) + O(\rho^2)$,

where $A_{\phi,1}(\rho,z)$ is given by expression (9), Similarly, it can be shown that:

$$B_\rho(\rho,z) = \frac{\mu_0 j\rho}{4}\sum_{n=-\infty}^{\infty} K^{|n|}[[w\log_e(x+\alpha^{1/2}) - x\alpha^{-3/2}]_{x=b}^a]_{w=w1}^{w2}$$

where $\alpha = w^2 + x^2$ and $w = z - z' - n$, and

$$B_z(\rho,z) = \frac{\mu_0 j}{2}\sum_{n=-\infty}^{\infty} K^{|n|}[[w\log_e(x+\alpha^{1/2})]_{x=b}^a]_{w=w1}^{w2} + O(\rho^2)$$

III. Calculating the Higher order terms.

For a coil of rectangular cross section situated equidistant from two semi-infinite regions of iron the magnetic vector potential A carrying a current density j is given by expression (7), thus expanding the vector potential in a Maclaurin series in r gives

$$A_\phi(\rho,z) = \frac{\mu_0 j}{4\pi}\sum_{n=-\infty}^{\infty} K^{|n|}\int_a^b xdx\int_\varepsilon^{1-\varepsilon}\{I_n|_{\rho=0} + \rho\frac{\partial I_n}{\partial\rho}|_{\rho=0} + \frac{\rho^2}{2!}\frac{\partial^2 I_n}{\partial\rho^2}|_{\rho=0} + O(\rho^3)\}dz'$$

where I_n is defined be equation (8)

Now $I_n|_{\rho=0} = 0$ and $\frac{\partial I_n}{\partial\rho}|_{\rho=0} = \frac{\pi x}{(w^2+x^2)^{3/2}}$,

where w=z-z'-n. Considering the higher order terms, it can be shown (see Pavlika [7]) that

$$\frac{\partial^3 I_n}{\partial\rho^3}|_{\rho=0} = -\frac{9x\pi}{\alpha^{5/2}} + \frac{45x^3\pi}{\alpha^{7/2}}$$

it follows that

$$\frac{\partial^5 I_n}{\partial\rho^5}|_{\rho=0} = \frac{225x\pi}{\alpha^{7/2}} + \frac{3150x^3\pi}{4\alpha^{9/2}} + \frac{44725x^5\pi}{8\alpha^{11/2}}$$

$$\frac{\partial^7 I_n}{\partial\rho^7}|_{\rho=0} = -\frac{11025x\pi}{\alpha^{9/2}} + \frac{297675x^3\pi}{4\alpha^{11/2}} - \frac{1091475x^5\pi}{8\alpha^{13/2}} - \frac{472925x^7\pi}{\alpha^{15/2}}$$

$$\frac{\partial^9 I_n}{\partial\rho^9}|_{\rho=0} = \frac{893025x\pi}{\alpha^{11/2}} - \frac{9823275x^3\pi}{\alpha^{13/2}} + \frac{31925644x^5\pi}{\alpha^{15/2}} - \frac{3990755x^7\pi}{\alpha^{17/2}} + \frac{16960498x^9\pi}{\alpha^{19/2}}$$

where $\alpha = w^2 + x^2$, thus in order to improve the accuracy of the vector potential and the field components these higher order terms must be considered. Performing the integrations with respect to x and z' gives

$$A_{\phi,2}(\rho,z) = \frac{\mu_0 j\rho^3}{4}\sum_{n=-\infty}^{\infty} K^{|n|}[[\frac{x}{8w\alpha^{3/2}}]_a^b]_{w_2}^{w_1}$$

$$= \frac{\mu_0 j\rho^3}{4}\sum_{n=-\infty}^{\infty} K^{|n|}\Im_2(x,w) \text{ , say}$$

$$A_{\phi,3}(\rho,z) = \frac{\mu_0 j\rho^5}{4}\sum_{n=-\infty}^{\infty} K^{|n|} *$$

$$[[\frac{x(2x^6+5w^2x^4+4wx^4(w^3+3)+w^3(w^3+27))}{192w^3\alpha^{7/2}}]_a^b]_{w_2}^{w_1}$$

$$= \frac{\mu_0 j\rho^5}{4}\sum_{n=-\infty}^{\infty} K^{|n|}\ \Im_3(x,w) \text{, say}$$

$$A_{\phi,4}(\rho,z) = \frac{\mu_0 j\rho^7}{4}\sum_{n=-\infty}^{\infty} K^{|n|}\ [[\Im_4(x,w)]_a^b]_{w_2}^{w_1}$$

Where

$$\Im_4(x,w) = \frac{-2w^{10}x + 274w^8x^3 + 22w^6x^5 + 97w^4x^7}{3072w^5\alpha^{11/2}} + \frac{44w^2x^9 + 8wx^{10}}{3072w^5\alpha^{11/2}}$$

and

$$A_{\phi,5}(\rho,z) = -\frac{\mu_0 j\rho^9}{4}\sum_{n=-\infty}^{\infty} K^{|n|}\ [[\frac{1}{A_0 w^7 x^7 \alpha^{15/2}} *$$

$$A_{\phi,5}(\rho,z) = -\frac{\mu_0 j\rho^9}{4}\sum_{n=-\infty}^{\infty} K^{|n|}\ [[\frac{1}{A_0 w^7 x^7 \alpha^{15/2}} *$$

$$\left\{\begin{matrix} 412406589178224x^{22} \\ +823457958700680x^2x^{20} \\ +406568751666210w^4x^{18} \\ -11011159703115w^6x^{16} \\ -611389980218160w^8x^{14} \\ -1765875128297280w^{10}x^{12} \\ -2661849239841792w^{12}x^{10} \\ -3217305707890560w^{14}x^8 \\ -2859827295902720w^{16}x^6 \\ -1559905797765120w^{18}x^4 \\ -479971014696960w^{20}x^2 \\ -63996135292928w^{22} \end{matrix}\right\}]_a^b]_{w_2}^{w_1}$$

where $A_0 = 355687429096000$

$$= -\frac{\mu_0 j\rho^9}{4}\sum_{n=-\infty}^{\infty} K^{|n|}[[\Im_5(x,w)]_a^b]_{w_2}^{w_1},$$

so that $B_{z,2}(\rho,z) = \mu_0 j\rho^2 \sum_{n=-\infty}^{\infty} K^{|n|}\Im_2(x,w)$

$$B_{z,3}(\rho,z) = \frac{3\mu_0 j}{2}\rho^4\sum_{n=-\infty}^{\infty} K^{|n|}\Im_3(x,w)$$

$$B_{z,4}(\rho,z) = 2\mu_0 j\rho^6\sum_{n=-\infty}^{\infty} K^{|n|}\Im_4(x,w)$$

$$B_{z,5}(\rho,z) = -\frac{5\mu_0 j}{2}\rho^8\sum_{n=-\infty}^{\infty} K^{|n|}\Im_5(x,w)$$

and

$$B_{\rho,2}(\rho,z) = \frac{\mu_0 j\rho^3}{4}\sum_{n=-\infty}^{\infty} K^{|n|}[[-\frac{x^3(x^2+4w^2)}{8w^2\alpha^{5/2}}]_a^b]_{w_2}^{w_1}$$

$$B_{\rho,3}(\rho,z) = \frac{\mu_0 j\rho^5}{4}\sum_{n=-\infty}^{\infty} K^{|n|} *$$

$$[[\frac{x(2w^8+9x^6w^2+12w^4x^4+145w^6x^2-140w^8)}{64w^4\alpha^{9/2}}]_a^b]_{w_2}^{w_1}$$

$$B_{\rho,4}(\rho,z) = \frac{\mu_0 j}{4}\rho^7\sum_{n=-\infty}^{\infty} K^{|n|}\frac{\partial}{\partial w}(\Im_4(x,w))$$

and $B_{\rho,5}(\rho,z) = \frac{\mu_0 j}{4}\rho^9\sum_{n=-\infty}^{\infty} K^{|n|}\frac{\partial}{\partial w}(\Im_5(x,w))$

IV. Deriving Two Integral Recurrence relations used to obtain the terms of the Maclaurin Series.

Here two integral recurrence formulae frequently used in obtaining the mathematical details of the Maclaurin series expansion will be derived. Let

$$J_m = \int_a^b \cos^m \vartheta d\vartheta$$

$$= \int_a^b \cos^{m-2}\vartheta d\vartheta - \int_a^b \cos^{m-2}\vartheta \sin^2\vartheta d\vartheta \quad .$$

$$= J_{m-2} - \int_a^b \cos^{m-2}\vartheta\sin^2\vartheta d\vartheta$$

Now $\frac{d}{d\vartheta}(\cos^{m-1}\vartheta) = (1-m)\sin\vartheta\cos^{m-2}\vartheta$

Thus

$$\int_a^b \cos^{m-2}\vartheta\sin^2\vartheta d\vartheta$$

$$= \frac{1}{1-m}\int_a^b \sin\vartheta\frac{d}{d\vartheta}(\cos^{m-1}\vartheta)d\vartheta$$

and hence

$$J_m = \frac{m-1}{m}J_{m-2} + \frac{1}{m}\left[\cos^{m-1}\vartheta\sin\vartheta\right]_a^b$$

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which is the required expression. The other recurrence relation that reduces the amount of manipulation of the terms arising is now derived. Consider

$$\frac{w^{m-2}}{(w^2+x^2)^{n-1/2}}-\frac{x^2w^{m-2}}{(w^2+x^2)^{n+1/2}}$$

$$=\frac{w^m}{w^2(w^2+x^2)^{n-1/2}}-\frac{x^2w^m}{w^2(w^2+x^2)^{n+1/2}}$$

$$=\frac{w^m}{(w^2+x^2)^{n+1/2}}\left[\frac{w^2+x^2}{w^2}-\frac{x^2}{w^2}\right]$$

$$=\frac{w^m}{(w^2+x^2)^{n+1/2}}$$

so that

$$\int\frac{w^m}{(w^2+x^2)^{n+1/2}}dw=$$

$$\int\frac{w^{m-2}}{(w^2+x^2)^{n-1/2}}-\frac{x^2w^{m-2}}{(w^2+x^2)^{n+1/2}}dw$$

which is the required expression. Results for $A_\phi(\rho,z), B_\rho(\rho,z)$ and $B_z(\rho,z)$ using the Maclaurin series with a=0.9, b=1.1, ε = 0.05 and $\mu_0 j$ = 100 were found to be in good agreement with the solution using the Power series expansion as shown in tables I, II, III, IV and V.

V. Conclusions

The two methods of solution were found to be in good agreement. The summations were performed from -200 to 200 with a change only in the fourth decimal place occurring when the number of terms in the summation was doubled. The effect of the permeability of the iron is shown in figures 2, 3, 4 and 5.

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VII. Tables

Table I. Values of $A_\phi(\rho,z)$ using the Power Series Expansion accurate $O(\rho^4)$.

ρ	Z	$\mu=10^3$	$\mu=10^2$	$\mu=10$	$\mu=1$
0	0.1	0	0	0	0
0.1	0.1	0.8957	0.8807	0.7590	0.3495
0.2	0.1	1.7911	1.7613	1.5177	0.7021
0.3	0.1	2.6852	2.6414	2.2797	1.0607
0.4	0.1	3.5812	3.5232	3.0396	1.4285
0.5	0.1	4.4730	4.4001	3.8051	1.8094
0.1	0.2	0.8976	0.8835	0.7655	0.3743
0.1	0.3	0.8985	0.8836	0.7710	0.3952
0.1	0.4	0.8992	0.8859	0.7735	0.4070
0.1	0.5	0.8992	0.8860	0.7747	0.4123

Table II. Values of $B_\rho(\rho,z)$ using the Power Series Expansion accurate $O(\rho^5)$.

ρ	Z	$\mu=10^3$	$\mu=10^2$	$\mu=10$	$\mu=1$
0.1	0.1	5.584-3	0.0127	0.0718	0.2815
0.2	0.1	1.131-2	0.0272	0.1472	0.5776
0.3	0.1	2.350E-2	0.0451	0.2297	0.9026
0.4	0.1	3.826-2	0.0680	0.3226	1.2710
0.5	0.1	5.896-2	0.0976	0.4297	1.6972
0.1	0.2	8.727-3	0.0141	0.0607	0.2316
0.1	0.3	8.493-3	0.0122	0.0443	0.1647
0.1	0.4	5.153-3	0.0070	0.0234	0.0855
0.1	0.5	0	0	0	0

Table III. Values of $B_z(\rho,z)$ using the Power Series Expansion accurate $O(\rho^4)$.

ρ	Z	$\mu=10^3$	$\mu=10^2$	$\mu=1$
0	0.1	17.9169	17.6163	6.9821
0.1	0.1	17.0149	17.6150	7.0022
0.2	0.1	17.9090	17.6111	7.0627
0.3	0.1	17.8990	17.6046	7.1634
0.4	0.1	17.8851	17.5964	7.3045
0.5	0.1	17.8672	17.5838	7.4860
0.1	0.2	17.9731	17.6545	7.5232
0.1	0.3	17.9722	17.6770	7.9258
0.1	0.4	17.9860	17.6995	8.1802
0.1	0.5	17.9866	17.7014	8.2672

Table IV. Values of $A_\phi(\rho, z)$ using the Elliptic Integrals of the 1st and 2nd kind, accurate $O(\delta^8)$.

ρ	Z	μ=10^3	μ=10^2	μ=10	μ=1
0	0.1	0	0	0	0
0.1	0.1	0.89171	0.881237	0.7575	0.3480
0.2	0.1	1.79492	1.762866	1.5140	0.6901
0.3	0.1	2.69390	2.645276	2.2679	1.0200
0.4	0.1	3.59465	3.528857	3.0178	1.3318
0.5	0.1	4.49779	4.414001	3.7624	1.6195
0.1	0.2	0.89781	0.882507	0.7641	0.3732
0.1	0.3	0.89595	0.883736	0.7692	0.3925
0.1	0.4	0.89919	0.884628	0.7725	0.4048
0.1	0.5	0.89942	0.884954	0.7737	0.4090

Table V. Values of $B_\rho(\rho, z)$ using the Elliptic Integrals of the 1st and 2nd kind, accurate $O(\delta^8)$.

ρ	Z	μ=10^3	μ=10^2	μ=10	μ=1
0.1	0.1	5.831E-3	0.0162	0.1041	0.0361
0.2	0.1	1.314E-2	0.0342	0.2119	0.0775
0.3	0.1	2.343E-2	0.0555	0.3673	0.1425
0.4	0.1	3.818E-2	0.0819	0.4520	0.1598
0.5	0.1	5.886E-2	0.1150	0.5913	2.0971
0.1	0.2	8.425E-3	0.0165	0.0851	0.2936
0.1	0.3	8.082E-3	0.0135	0.0606	0.2071
0.1	0.4	4.897E-3	0.0070	0.0315	0.0106
0.1	0.5	0	0	0	0

VIII. Figures

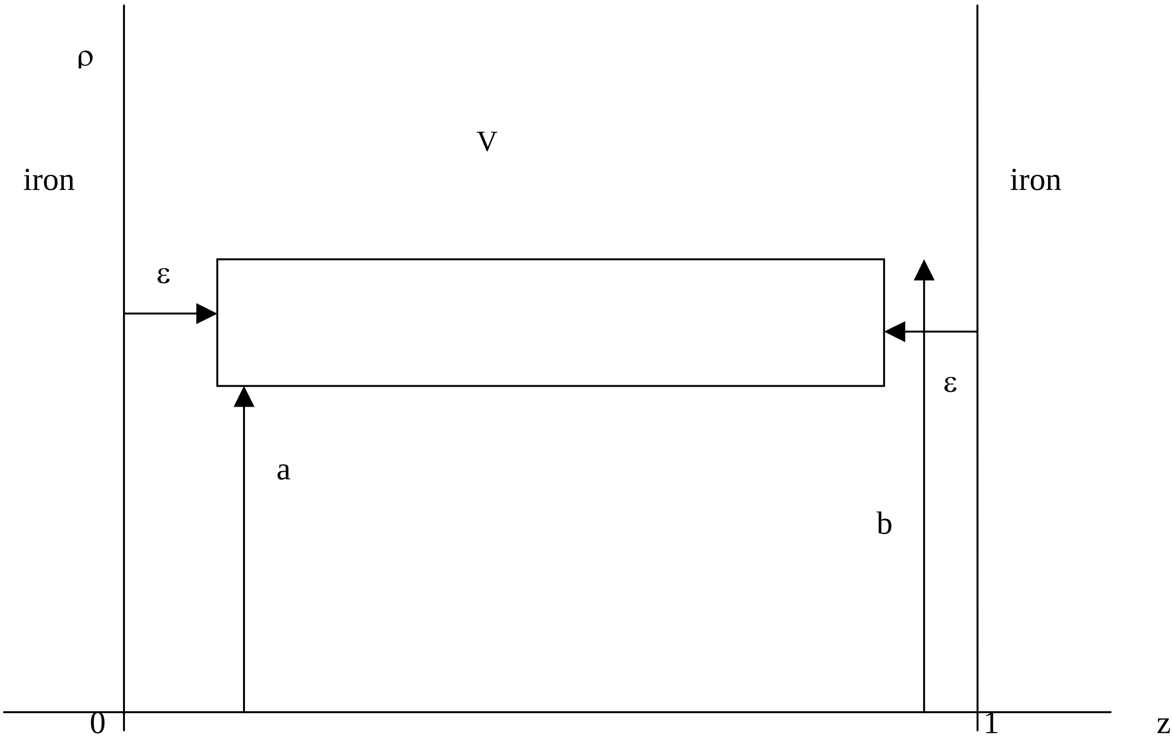

Fig 1. A toroidal conductor V' of rectangular cross section located midway between two semi infinite regions of iron of finite permeability. The region V is assumed to be insulating.

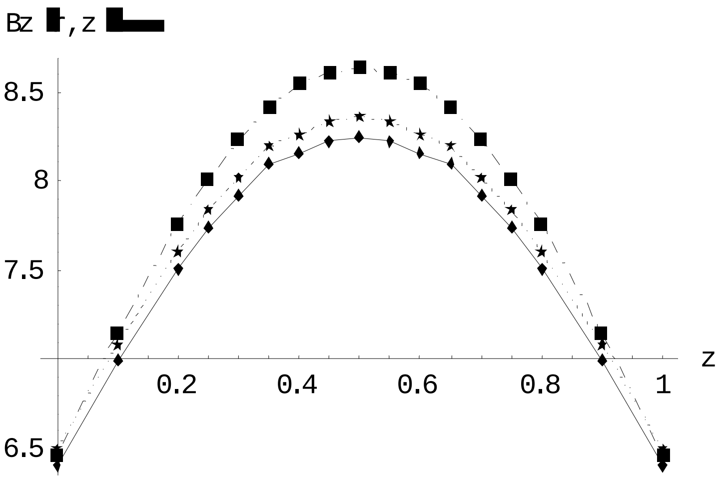

Fig 2. The variation of $B_z(\rho ,z)$ with ρ and z for two semi-infinite regions of iron of unit permeability. *+: ρ =0.3, ™ρ r=0.2, •: ρ =0.1*

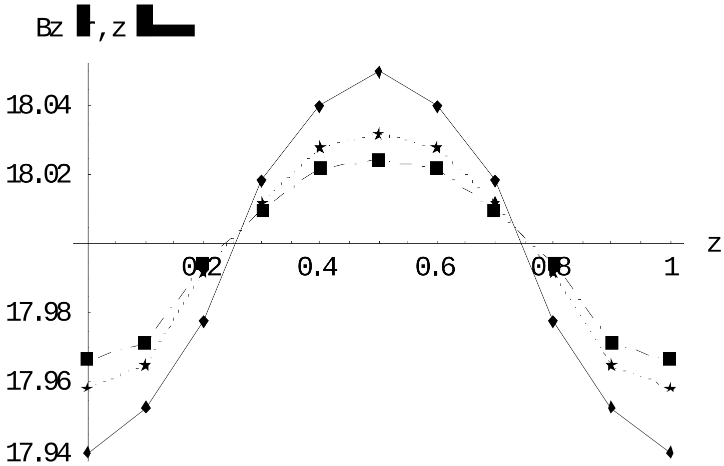

Fig 3. The variation of $B_z(\rho,z)$ with ρ and z for two semi-infinite regions of iron of infinite permeability. *+: ρ =0.1, ™: ρ =0.2, •: ρ =0.3*

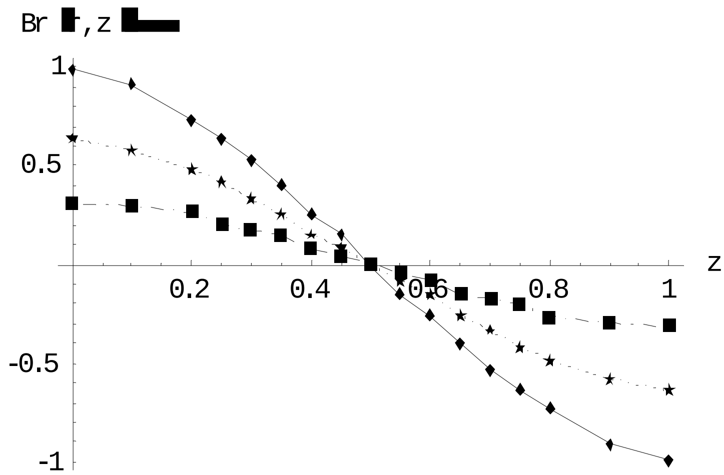

Fig 4. The variation of $B_\rho(\rho,z)$ with ρ and z for two semi-infinite regions of iron of unit permeability. *+: ρ =0.1, ™: ρ =0.2, •: ρ =0.3*

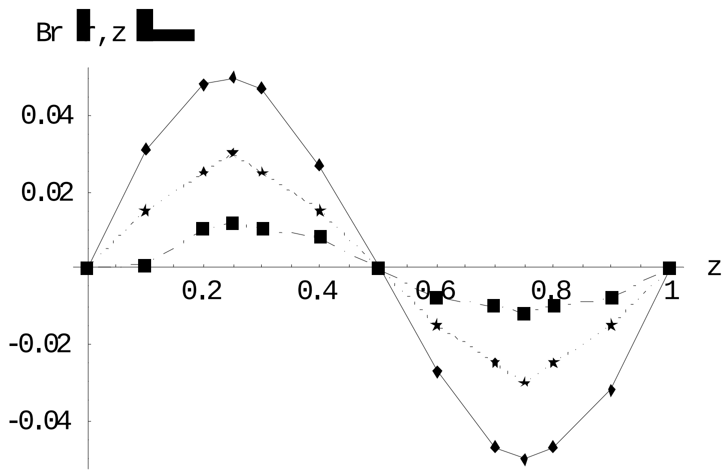

Fig 5. The variation of $B_r(\rho,z)$ with ρ and z for two semi-infinite regions of iron of infinite permeability. *+: ρ =0.1, ™: ρ =0.2, •: ρ =0.3*

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