Proceedings of the World Congress on Engineering and Computer Science 2008
WCECS 2008, October 22 - 24, 2008, San Francisco, USA

Optimal Trajectory Generation of Space Robots

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Abstract—**This paper deals with an approach to generate an optimal trajectory of space robot. The space robot consists of a spacecraft body and robotic arm mounted on the body. For a self-contained and autonomous operation of the space robot, such as a capture of a target, it must have automatic control of the spacecraft position and attitude control and, at the same time, automatic generation of the trajectory for the robotic arm joints. In this study hypothetical joints were introduced to deal with the spacecraft position and attitude. Two-dimensional motion of the space robot were discussed without loss of generality. In such a case two prismatic joints are required to give the position of the spacecraft and a rotary joint is required to define its attitude. Jacobian for this space robot was established and inverse kinematics is solved for each step to advance the position, attitude of the spacecraft and the configuration of robotic arm to final target position. In order to solve the above, weighted minimization by using Jacobian and Lagrange multiplier method was applied. In addition to that, final configuration of the robot arm was optimized by incorporating a penalty function that measures the manipulability of the arm posture. Numerical simulations were conducted to demonstrate the effectiveness of this approach. For the proper choice of weighting coefficients optimized trajectories have been obtained.**

Key words—**Jacobian, Lagrange Multiplier, Optimization, Space Robot, Trajectory Control,.**

I. INTRODUCTION

In the future space robot applications self-contained and autonomous operations are prerequisite. Especially after recognition of a target to capture, optimized trajectories for the spacecraft position and joints of robotic arm must be generated automatically.

The space robot operation is divided into three phases. The first one is rendezvous with the target by orbital maneuvers. Then, the space robot approaches it by fly-around and finally captures it. This paper discusses the fly-around phase after the orbit adjustment.

In this study the planar motion is discussed without loss of generality. This is because operations will be usually divided into the in-plane and out-of-plane motions.

Let us assume that the robotic arm consists of two rotary joints. This is enough to position the end effector of the arm. And we define three joints, two for translational movement and one for spacecraft attitude motion. The first two joints are assumed to be prismatic and joint variables will be defined relative to the inertial reference coordinate system. The second joint is rotary which express the attitude angles relative to the above inertial frame. As noted, these three joints are hypothetical to give the translational and rotational position of the spacecraft.

Based on.the above consideration we can define the end effector position using the space robotic arm joint variables and hypothetical three joints variables. By differentiating these relationships we obtain Jacobian matrix, which show the relation between the small displacement of the end effector position and small joint variables variation. If we provide with the value of small displacement of the end effector, then, we may determine the joint variables using the inverse of Jacobian matrix. However this does not hold for our case in which five degrees of freedom are given to determine two degrees of freedom.

Therefore we have to consider a solution of the redundant robotic arm problem. In this study the Lagrange multiplier was applied to solve this redundant system. Firstly there are relations, two equations, between the small displacement of the end effector and the small variations of joint variables including the hypothetical ones. These are constraints when the inverse problem is solved. Then, Lagrange multiplier is introduced to incorporate these constraints. And we define a function to be optimized for the small variations of joint variables, which will be given as a quadratic function of these variations. The above procedure is an optimization problem with constraints.

When the robotic arm approaches a target, it is necessary to consider the final posture. In this respect the manipulability of the arm was optimized by introducing a penalty function. As shown later, this manipulability is given by the two joints of the robotic arm, not by the hypothetical joints.

Two kinds of optimization functions are incorporated as multi-objective functions problem. In this study this was solved by a simplest formulation.

Numerical simulations were conducted to demonstrate the applicability of our approach. Although there are many parameters that must be defined a priori, satisfactory results are obtained by a simple set of parameters.

II. MODEL OF SPACE ROBOT

As noted above, let us define a space robot model for the planar motion. The hypothetical joints are introduced to give two-dimensional positions of the spacecraft and an attitude of the spacecraft.

Fig.1 shows a concept of space robot. The hypothetical joints are measured with respect to an inertially fixed coordinate system. Two translational displacements are

Manuscript received July 15, 2008
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given by θ_1, θ_2 and spacecraft attitude angle is specified by θ_3, and θ_4, θ_5 denote angles of robotic arm rotary joints. The position of the end effector is given by (x, y) as shown in Fig.1. The translational movement and attitude control will be achieved by gas jet thrusters and reaction wheel. Therefore, for instance, propellant consumed by gas jet thrusters should be minimized from operational points of view. This validates our approach to optimize the operation of the space robot in which the propellant is a key factor of operational life.

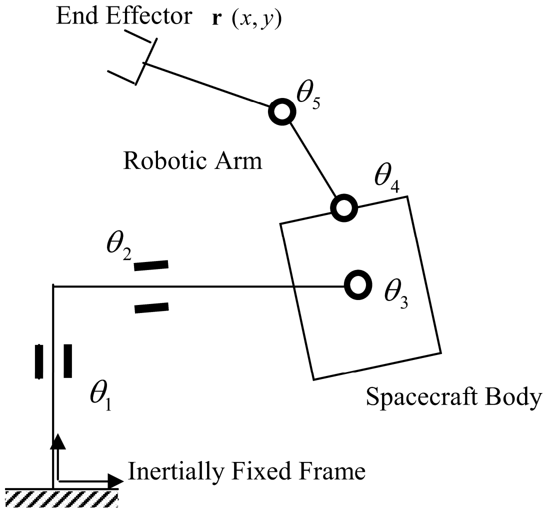

Fig.1 A Concept of Space Robot

III. MATHEMATICAL DEVELOPMENT

Based on the above definitions, let us proceed to the formulation of optimization problem.

The position vector of the end effector is denoted by

$$\mathbf{r} = \mathbf{f}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \tag{1}$$

where the right hand side of equation (1) is shown in Appendix (A-1).

By differentiating of both sides and using Jacobian matrix $J(\boldsymbol{\theta})$, we obtain the following infinitesimal relation;

$$\Delta\mathbf{r} = J(\boldsymbol{\theta})\Delta\boldsymbol{\theta} \tag{2}$$

where $\boldsymbol{\theta}$ describes a vector of joint variables. Jacobian matrix is also given in Appendix (A-2).

If we assume to minimize the joint travel, then, we treat this problem as follows.

Firstly, we define the minimization function

$$g = g(\Delta\boldsymbol{\theta}) \tag{3}$$

and as usual, in this study we adopt the following quadratic function;

$$g = \Delta\boldsymbol{\theta}^T P \Delta\boldsymbol{\theta} \tag{4}$$

where P is a weighting matrix among joint variables. In order to avoid the complexity of discussions here, we simply assume that P is a diagonal matrix.

$$P = \begin{bmatrix} P_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & P_{55} \end{bmatrix} \tag{5}$$

Then, we have minimization function as shown in Appendix (A-3).

Next $\Delta\mathbf{r}$ is given by the following;

$$\Delta\mathbf{r} = (\mathbf{r}_f - \mathbf{r}_0)/N = (\Delta x, \Delta y)^T \tag{6}$$

where $\mathbf{r}_0$ and $\mathbf{r}_f$ are initial position and final position, respectively. Each step $\Delta\mathbf{r}$ is given by linearly dividing the trajectory into N segments . N must be large enough not to violate the equation (2). Of course we are able to have different divisions depending on the curve if required. Between $\Delta\mathbf{r}$ and $\Delta\boldsymbol{\theta}$ we have two scalar equations from the vector equation (2). These two equations are the constraints which must be observed along the trajectory. Since these two equations are not enough to get solutions because of the rectangular nature of the matrix. So we introduce the Lagrange multiplier $\boldsymbol{\lambda}$. Then, we have an optimization problem:

$$\text{minimize } g(\Delta\boldsymbol{\theta}) + \boldsymbol{\lambda}^T[\Delta\mathbf{r} - J(\boldsymbol{\theta})\Delta\boldsymbol{\theta}] \, . \tag{7}$$

However, the above formulation is not good enough for the space robot operation. As easily shown, solutions from equation (7) include the stretched arm , i.e., straight posture. In order to avoid this kind of singularity we also introduce another measure, so-called manipulability. This manipulability assures that the robot would be able to work in a good situation right after the arrival at the target. The manipulability measure ω defined in our study is [1]

$$\omega = \sqrt{\left| J J^T \right|} \tag{8}$$

and as easily seen from the definition of the manipulability, joint variables included in this measure are just θ_4 and θ_5. Thus hypothetical joint variables do not contribute to the manipulability. Using the equation (8) optimum θ_4 and θ_5 can be calculated numerically by maximizing ω . Thus optimal values are defined as θ_4^* and θ_5^* .

By introducing the manipulability measure, we have two optimization functions given by equation (4) and equation (8), that is, multi-objective functions. At this point we simply define a single optimization function F by giving a weight α between joint variable increment and manipulability in the following manner:

$$F = \Delta\boldsymbol{\theta}^T P \Delta\boldsymbol{\theta} + \boldsymbol{\lambda}^T[\Delta\mathbf{r} - J(\boldsymbol{\theta})\Delta\boldsymbol{\theta}] + \alpha\left[(\theta_4^* - \theta_4 - \Delta\theta_4)^2 + (\theta_5^* - \theta_5 - \Delta\theta_5)^2 \right] \tag{9}$$

where α is a positive number.

The minimization of the function F is given by differentiating (9) by variables $\Delta\theta_1 \sim \Delta\theta_5$ and λ_1, λ_2, then we obtain seven equations as follows:

$$\begin{aligned}
\partial F / \partial\Delta\theta_1 &= 2P_{11}\Delta\theta_1 + \lambda_2 = 0 \\
\partial F / \partial\Delta\theta_2 &= 2P_{22}\Delta\theta_2 + \lambda_1 = 0 \\
\partial F / \partial\Delta\theta_3 &= 2P_{33}\Delta\theta_3 - \lambda_1(l_3 s_3 + l_4 s_{34} + l_5 s_{345}) \\
&\quad + \lambda_2(l_3 c_3 + l_4 c_{34} + l_5 c_{345}) = 0 \\
\partial F / \partial\Delta\theta_4 &= 2P_{44}\Delta\theta_4 - \lambda_1(l_4 s_{34} + l_5 s_{345}) \\
&\quad + \lambda_2(l_4 c_{34} + l_5 c_{345}) + 2\alpha(\Delta\theta_4 + \theta_4 - \theta_4^*) = 0 \\
\partial F / \partial\Delta\theta_5 &= 2P_{55}\Delta\theta_5 - \lambda_1 l_5 s_{345} + \lambda_2 l_5 c_{345} \\
&\quad + 2\alpha(\Delta\theta_5 + \theta_5 - \theta_5^*) = 0 \\
\partial F / \partial\lambda_1 &= \Delta\theta_2 - (l_3 s_3 + l_4 s_{34} + l_5 s_{345})\Delta\theta_3 \\
&\quad - (l_4 s_{34} + l_5 s_{345})\Delta\theta_4 - l_5 s_{345}\Delta\theta_5 - \Delta x = 0 \\
\partial F / \partial\lambda_2 &= \Delta\theta_1 + (l_3 c_3 + l_4 c_{34} + l_5 c_{345})\Delta\theta_3 \\
&\quad + (l_4 c_{34} + l_5 c_{345})\Delta\theta_4 + l_5 c_{345}\Delta\theta_5 - \Delta y = 0
\end{aligned} \tag{10}$$

These result in an equation of a matrix form as shown in (A-4). Inverting this matrix in the left hand side we obtain incremental vector of joint variables. For each step the same procedure as the above will be repeated.

From the above discussion we have to specify six weighting coefficients. These are numerically determined in the following section.

IV. NUMERICAL SIMULATIONS

Numerical data for a space robot model, which were used in our simulation, are as follows:

$$l_3 = 2m, l_4 = 2m, l_5 = 2m$$

$$\theta_4^* = 45^\circ \quad , \theta_5^* = 40^\circ \quad .$$

where θ_4^* and θ_5^* were determined by the calculation of the manipulability optimization independently.

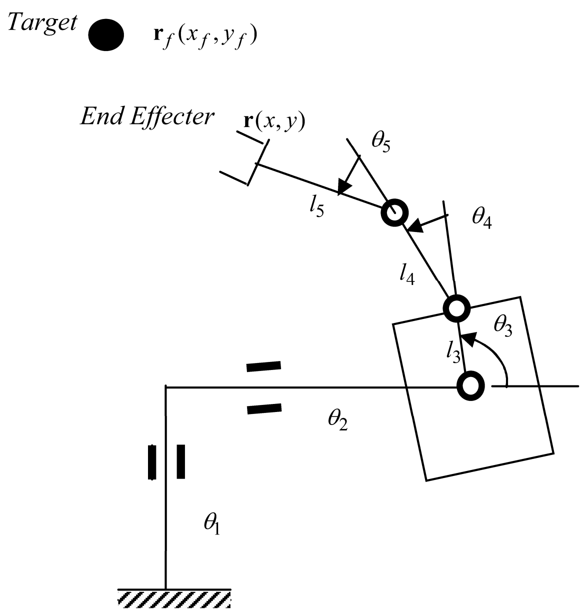

Fig.2 Geometry of the Space Robot System

The geometry of the system is illustrated in Fig.1. In our simulation the target is given as follows:

$$\mathbf{r}_f(x_f, y_f) = (7\sin(-60)^\circ, 7\cos(-60^\circ)) .$$

7m is the distance between the initial spacecraft position and the target position and -60° denotes the direction of the target.

As for α, weighting coefficient of penalty function in the equation (9), we introduced 5 m rule to define its value. A basic idea is that manipulability is not significant in the far region from the target. The penalty function should be heavily weighted as the robot approaches the target, that is, depending on the distance d. Based on this consideration we assumed the following values for α :

$$\alpha = 10^{-4} \; for \quad d \geqq 5$$

$$= n/50 \; for \quad d < 5 .$$

The reason why the smaller value is defined for $d \geq 5$ is that the singular configuration should be avoided even in the far region. In our case the singular configuration is that the arm is stretched out. And in the vicinity of the target we continuously changed the weighting coefficient at each step as shown above, provided that the maximum value is limited to be 1. In our simulation we divided the whole travel from the initial position to the target into 1000 steps and each step is interpreted 1 second for the convenient sake. This conversion is quite flexible.

Fig. 3 shows the trajectory of the prismatic joints, i.e., translational movement of the spacecraft, for the following weighting coefficients

$$P_{11} = 10, P_{22} = 10, P_{33} = 5, P_{44} = 1, P_{55} = 1$$

and initial posture of the robotic arm was

$$\theta_4 = 60^\circ, \quad \theta_5 = -30^\circ .$$

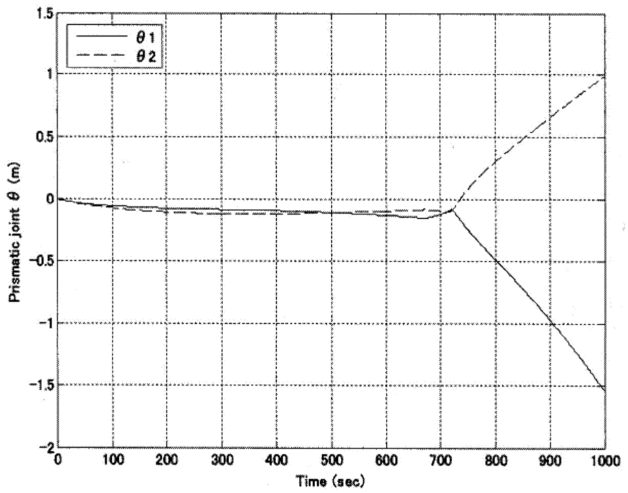

Fig.3 Trajectory for the Prismatic Joints

Apparently after reaching 5m both joints were hurrying to the final position. So, as shown in the latter chart, initial approaching were done by using the robotic arm, and then, in the final approach by using prismatic joints.

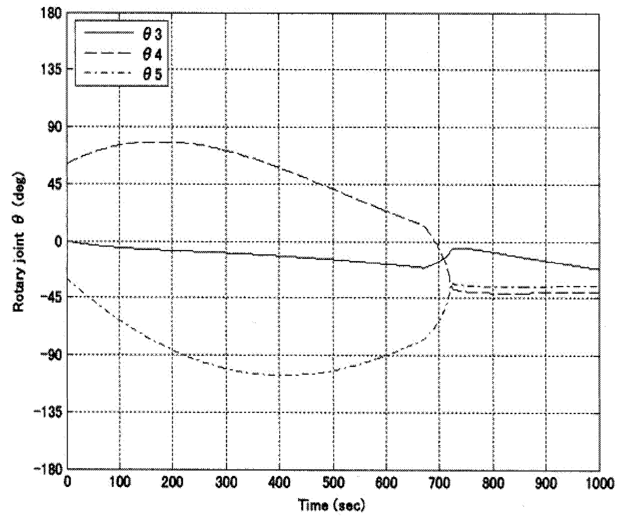

Fig.4 Trajectory for the Rotary Joints

Fig.4 illustrates the history of the three rotary joints, one for the spacecraft attitude angle. Within 5m two joints, θ_4 and θ_5, converged on the posture near the maximum manipulability. And the spacecraft attitude angle has not travel so much compared with other joints.

Fig.5 is a chrat of the manipulability measure along the trajectory shown above. The profile shows that relatively large value is maintained during the movements. Especially, after around 720sec the manipulability measure can be optimized by increasing the weight coefficients for robotic arm joints.

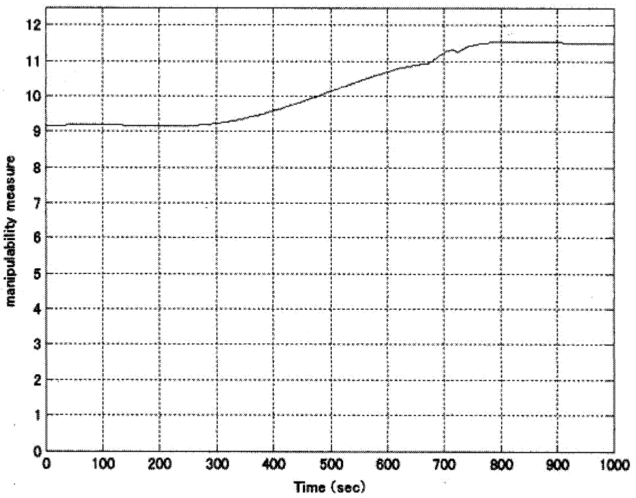

Fig.5 Manipulability Measure

V. CONCLUSIONS

In this paper we discussed autonomous generations of space robot trajectory, which includes the translational and rotational motion of the spacecraft. Such a scheme will be required for future space robot activities, for instance, deep space operations and orbital operation such as repairs of the failed spacecraft. A priori information, for instance, on surroundings and the capturing target, will of course be mandatory to establish an optimized autonomous system.

Numerical simulations were carried out to verify our approach. This formulation was multi-objective function optimization and therefore, there will be several methods to solve, however, in this study simple weight coefficient method was applied.

Satisfactory results were obtained by choosing appropriate set of weighting coefficients. Especially, with respect to α, weighting coefficient for manipulability measure, would be better to be changed as a function of the distance from the initial position. By this approach the best solutions regarding manipulability are available. Depending on the robot characteristics, weighting coefficient must be tuned. However, once we obtain a proper solution, it will be applicable to another case in which different target position is given, although other examples are not shown here due to the limited space.

The optimization problem discussed here still needs a lot of numerical efforts to obtain the best solution, but it is useful to generate the optimized trajectory.

There still remain issues to be solved. One of these will be the collision avoidance technique. Once the spacecraft recognizes the position of hindrance, then, the trajectory of the spacecraft can be designed into some segments that avoid the hindrance. And our approach may be applicable to each segment and step-by-step we will be able to have the optimized trajectory.

REFERENCE

[1] T. Yoshikawa,"Fundamental Theory of Robot Control" Corona Publishing,, 1988, pp.109-121

Appendix

$$\mathbf{f}(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)=\begin{bmatrix}\theta_2+l_3c_3+l_4c_{34}+l_5c_{345}\\ \theta_1+l_3s_3+l_4s_{34}+l_5s_{345}\end{bmatrix} \qquad \text{(A-1)}$$

$$J(\theta)=\begin{bmatrix}0 & 1 & -(l_3s_3+l_4s_{34}+l_5s_{345}) & -(l_4s_{34}+l_5s_{345}) & -(l_5s_{345})\\ 1 & 0 & (l_3c_3+l_4c_{34}+l_5c_{345}) & (l_4c_{34}+l_5c_{345}) & (l_5c_{345})\end{bmatrix} \qquad \text{(A-2)}$$

where $l_4\, and\, l_5$ are length of the links attached to each joint and l_3 is a hypothetical link between joint 3 and 4.

Trigonometric functions defined in the above are as follows;

$s_3=\sin(\theta_3), s_{34}=\sin(\theta_3+\theta_4), s_{345}=\sin(\theta_3+\theta_4+\theta_5)$

and the same definitions as the above are used for cosine function.

$$g(\Delta\boldsymbol{\theta})= P_{11}\Delta\theta_1^{\,2}+P_{22}\Delta\theta_2^{\,2}+P_{33}\Delta\theta_3^{\,2}+P_{44}\Delta\theta_4^{\,2}+P_{55}\Delta\theta_5^{\,2} \qquad \text{(A-3)}$$

$$\begin{bmatrix}2P_{11} & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 2P_{22} & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 2P_{33} & 0 & 0 & -(l_3s_3+l_4s_{34}+l_5s_{345}) & (l_3c_3+l_4c_{34}+l_5c_{345})\\ 0 & 0 & 0 & 2P_{44}+2\alpha & 0 & -(l_4s_{34}+l_5s_{345}) & (l_4c_{34}+l_5c_{345})\\ 0 & 0 & 0 & 0 & 2P_{55}+2\alpha & -(l_5s_{345}) & (l_5c_{345})\\ 0 & 1 & -(l_3s_3+l_4s_{34}+l_5s_{345}) & -(l_4s_{34}+l_5s_{345}) & -(l_5s_{345}) & 0 & 0\\ 1 & 0 & +(l_3c_3+l_4c_{34}+l_5c_{345}) & +(l_4c_{34}+l_5c_{345}) & +(l_5c_{345}) & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\theta_1\\ \Delta\theta_2\\ \Delta\theta_3\\ \Delta\theta_4\\ \Delta\theta_5\\ \lambda_1\\ \lambda_2\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 2\alpha(\theta_4^{*}-\theta_4)\\ 2\alpha(\theta_5^{*}-\theta_5)\\ \Delta x\\ \Delta y\end{bmatrix} \qquad \text{(A-4)}$$