

# Linear-Quadratic-Gaussian (LQG) Controller for Two link- robotic Manipulator Control

Leila Fallah Araghi , M. Habibnejad Korayem, Amin Nikoobin

**Abstract—** This paper presents a new method for the control of Two link- robotic manipulator systems using Linear-Quadratic-Gaussian (LQG), in This method Linear-Quadratic-Gaussian (LQG) that is a modern state-space technique for designing optimal dynamic regulators by kalman used for the control of Two link- robotic manipulator systems. The simulation result are very promising.

**Index Terms—** Two link- robotic manipulator systems, using Linear-Quadratic-Gaussian controller, optimal dynamic regulators.

## I. INTRODUCTION

In the recent years using intelligece control such as fuzzy control, Neural Network, Neuro Fuzzy and because that they can control nonlinear systems that would be difficult or impossible to model mathematically. In the recently years In Dynamic control of robot have been utilized in many researchers work in this area. Such as Lianfang Tian et al use a neural network approach for the motion control of constrained flexible manipulators robots [1], Yi , et al have investigated the robustness and stability of a fuzzy logic controller applied to a robotic manipulator with uncertainties such as friction, unmodeled dynamics, and external disturbance etc [2], Kumbla et al have implemented hierarchical control on robotic manipulator using fuzzy logic [3].

Bannerjee, et al have used a Fuzzy Logic Controller to achieve position control of a two-link manipulator [4]. Adams, et al [5] have used GA to optimize the membership functions and rule bases of a multi-stage fuzzy PID controller with a fuzzy switch ror robot control. Brudka, et al presented an intelligent robot control system which employs ultrasonic distance measurements, And for Consecutive stages of data processing they used to neural networks applications [6], Adamiv, et al used neural networks application for mobile robot control on predetermined trajectory of the road [7], Ya-Chen , et al used an Fuzzy neural adaptive controller to multiple-link robot control [8], Devendra P, et al used the proportional plus derivative (PD) control with the PD controller gain parameters optimized via Genetic Algorithm

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(GA) And Fuzzy Logic for control of Two link- robot [9], Z.G. Zhang, et al report on the design and stability analysis of a simple quadruped running controller that can autonomously generate steady running with good energy efficiency and suppress

such disturbances as irregularities of terrain [10], Dongbing Gu, et al presented a new path-tracking scheme for a car-like mobile robot based on neural

predictive control, they employed A multi-layer back-propagation neural network to model non-linear kinematics of the robot instead of a linear regression estimator in order to adapt the robot to a large operating range [11], Mathew L, et al studied on the implementation of several

Intelligent control techniques as applied to the balancing of the inverted wedge problem. These included a basic four-input direct fuzzy controller (including the use of the nonlinear input term) and an adaptive fuzzy control technique known as the FMRLC [12].

Lawrence J. Alder, et al developed a new approach to identification is That utilizes a nominal controller in order to isolate the dynamic effects related to the uncertain parameters from the other known system dynamics[12], John L Meyery, et al presented the results of positive position feedback (PPF) control and linear-quadratic Gaussian (LQG) control for vibration suppression of a flexible structure using piezoceramics [13],

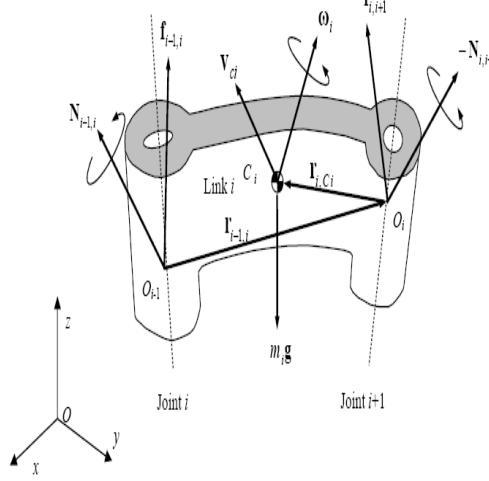
In this paper Two link- robotic manipulator systems is controlled, The problem statement is given in section II In section III LQG controllers is proposed. Simulation results are given in section IV. Finally section V includes conclusions,

## II. DYNAMIC EQUATION OF TWO LINK- ROBOTIC MANIPULATOR SYSTEMS [12]

In this section we derive the equations of motion for an individual link based on the direct method has been derived, i.e. Newton-Euler Formulation. The motion of a rigid body can be decomposed into the translational motion with respect to an arbitrary point fixed to the rigid body, and the rotational motion of the rigid body about that point. The dynamic equations of a rigid body can also be represented by two equations: one describes the translational motion of the centroid (or center of mass), while the other describes the rotational motion about the centroid. The former is Newton's

equation of motion for a mass particle, and the latter is called Euler's equation of motion.

Figure 1 shows all the forces and moments acting on link  $i$ .



III.

Fig. 1: Free body diagram of link  $i$  in motion

Let be  $V_{ci}$  the linear velocity of the centroid of link  $i$  with reference to the base coordinate frame  $O$ -xyz, which is an inertial reference frame. The inertial force is then given by

$-m_i V_{ci}$ , where  $m_i$  is the mass of the link and  $V_{ci}$  is the time derivative of  $V_{ci}$ .

Based on D'Alembert's principle, the equation of motion is then obtained by adding the inertial force to the static balance of forces in eq.(1) so that

$$f_{i-1,i} - f_{i,i+1} + m_i g - m_i V_{ci} = 0, \quad i = 0, 1, \dots, n \quad (1)$$

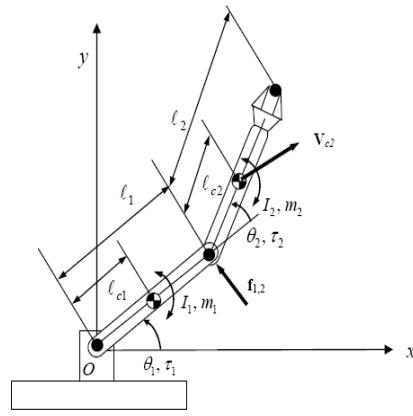
$f_{i-1,i}$  and  $-f_{i,i+1}$  are the coupling forces applied to link  $i$  by links  $i-1$  and  $i+1$ , respectively, and  $\mathbf{g}$  is the acceleration of gravity.

Adding these terms to the original balance of moments we have:

$$N_{i-1,i} - N_{i,i+1} - (r_{i-1,i} - r_{i,ci}) \times f_{i-1,i} + (-r_{i,ci}) \times f_{i,i+1} - I_i \omega_i - \omega_i \times (I_i \omega_i) = 0 \quad (2)$$

$N_{i-1,i}$  and  $-N_{i,i+1}$  are the moment applied to link  $i$  by links  $i-1$  and  $i+1$ , respectively.

If we consider two individual links robot, Let us obtain the Newton-Euler equations of motion for the two individual links.



IV.

Fig.2: Mass properties of two planar robot

From eq. (1) and (2), the Newton-Euler equations for link 1 are given by:

$$f_{0,1} - f_{1,2} + m_1 g - m_1 V_{c1} = 0 \quad (3)$$

$$N_{0,1} - N_{1,2} - (r_{0,1}) \times f_{0,1} + (r_{1,cl}) \times f_{1,2} - I_1 \omega_1 = 0 \quad (4)$$

Note that all vectors are  $2 \times 1$ , so that moment  $N_{i-1,i}$  and the other vector products are scalar quantities. Similarly, for link 2,

$$f_{1,2} + m_2 g - m_2 V_{c2} = 0 \quad (5)$$

$$N_{1,2} - (r_{1,c2}) \times f_{1,2} - I_2 \omega_2 = 0 \quad (6)$$

To obtain closed-form dynamic equations, we first eliminate the constraint forces and separate them from the joint torques, so as to explicitly involve the joint torques in the dynamic equations. For the planar manipulator, the joint torques  $\tau_1$  and  $\tau_2$  are equal to the coupling moments:

$$N_{i-1,i} = \tau_i \quad (7)$$

Substituting eq.(7) into eq.(6) :

$$\tau_2 - (r_{1,c2}) \times (m_2 V_{c2} - m_2 g) - I_2 \omega_2 = 0 \quad (8)$$

Similarly, eliminating  $f_{0,1}$  yields,

$$\tau_1 - (r_{0,1}) \times m_1 V_{c1} + (r_{0,1}) \times m_2 V_{c2} + (r_{0,1}) \times m_2 g + (r_{0,1}) \times m_2 g - I_1 \omega_1 = 0 \quad (9)$$

Next, we rewrite  $V_{ci}$ ,  $\omega_{ci}$  and  $r_{i-1,i}$  using joint displacements  $\theta_1, \theta_2$  which are independent variables. Note

that  $\omega_2$  is the angular velocity relative to the base coordinate frame, while  $\theta_2$  is measured relative to link 1. Then, we have

$$\omega_1 = \dot{\theta}_1, \omega_2 = \dot{\theta}_1 + \dot{\theta}_2 \quad (10)$$

The linear velocities can be written as

$$V_{c1} = \begin{bmatrix} -l_{c1} \dot{\theta}_1 \sin(\theta_1) \\ l_{c1} \dot{\theta}_1 \cos(\theta_1) \end{bmatrix} \quad (11)$$

$$V_{c2} = \begin{bmatrix} -(l_1 \sin\theta_1 + l_{c2} \sin\theta_1 + \theta_2) \dot{\theta}_1 - l_{c2} \sin\theta_1 + \theta_2 \dot{\theta}_2 \\ (l_1 \cos\theta_1 + l_{c2} \cos\theta_1 + \theta_2) \dot{\theta}_1 - l_{c2} \cos\theta_1 + \theta_2 \dot{\theta}_2 \end{bmatrix} \quad (12)$$

$$\tau_1 = H_{11} \ddot{\theta}_1 + H_{12} \ddot{\theta}_2 - h \dot{\theta}_2 - 2h \dot{\theta}_1 \dot{\theta}_2 + G_1 \quad (13)$$

$$\tau_2 = H_{22} \ddot{\theta}_2 + H_{21} \ddot{\theta}_1 - h \dot{\theta}_1 + G_2$$

That:

$$H_{11} = m_1 l_{c1}^2 + I_1 + m_1 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos(\theta_2)) + I_2$$

$$H_{22} = m_1 l_{c2}^2 + I_2 \quad (14)$$

$$G_1 = m_1 l_{c1} g \cos(\theta_1) + m_2 g \{l_{c2} \cos(\theta_1 + \theta_2) + l_1 \cos(\theta_1)\}$$

$$H_{11} = m_2 (l_{c2}^2 + l_1 l_{c2} \cos(\theta_2)) + I_2$$

$$h = m_2 l_1 l_{c2} \sin(\theta_2) \quad (15)$$

$$G_2 = m_2 g l_{c2} \cos(\theta_1 + \theta_2) \quad (16)$$

### III. LQG controller [14, 15, 16, 17]

In a typical control system design problem, inputs are a dynamic system to be controlled (plant), a description of the types of inputs to be encountered, and specifications regarding acceptable values of system errors, i.e., the

difference between desired and Actual responses. What is then required is to design a controller so that the plant controller combination satisfies the specifications. The LQG control is a modern state space technique for designing optimal dynamic regulators. It enables a trade off in performance and control effort, and takes into account process and measurement noise. Linear-Quadratic-Gaussian (LQG) control is a modern state-space technique for designing optimal dynamic regulators. The LQG regulator consists of an optimal state-feedback gain and a Kalman state estimator. The first design step is to seek a state-feedback law that minimizes the cost function of regulation performance, which is measured by a quadratic performance criterion with user-specified weighting matrices, and to define the tradeoff between regulation performance and control effort. The next design step is to derive a state estimator using a Kalman filter because the optimal state feedback cannot be implemented without full state measurement. Since the Kalman filter is an optimal estimator when dealing with Gaussian white noise, it minimizes the asymptotic covariance of the estimation error. In the optimal feedback control, a gain matrix is sought to minimize a specified quadratic performance criterion  $J(u)$  expressed as the integral of a quadratic form in the state vector  $x$  plus a second quadratic form in the control vector  $u$ , i.e.

$$J(u) = \int_0^\infty (x^T Q x + u^T R u) dt \quad (17)$$

The weighting matrices  $Q$  and  $R$  are user-specified and define the trade-off between Regulation performance (how fast  $x(t)$  goes to zero) and control effort [16,17]. The first

design step seeks a state feedback law  $u = -kx$  that minimizes the cost function  $J(u)$ . The minimizing gain matrix  $K$  is obtained by solving an algebraic Riccati equation [14]. This gain is usually called the LQ-optimal gain. Sometimes, the process seeking the LQ state feedback regulator, i.e. LQ-optimal gain matrix  $k$ , is called Linear Quadratic Regulator (LQR) design.

It is clear that the LQ-optimal state feedback  $u = -kx$  is not implemental without full state measurement. However, one can derive a state estimate  $\hat{x}$  such that  $u = -k\hat{x}$  remains optimal for output feedback problem. This state estimate is generated by Kalman filter [15] as shown below:

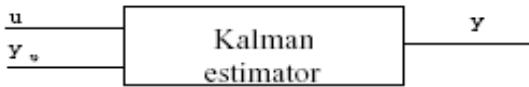


Fig.3: Block diagram of the Kalman estimator

Mathematically, the Kalman state estimator can be expressed by:

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u + L(y_0 - \hat{C}\hat{x}) \quad (18)$$

with two inputs, controls  $u$  and measurements  $y_0$ .

Determines the Kalman gain  $L$  in conjunction with dynamic systems through an

Algebraic Riccati equation. The Kalman filter is an optimal estimator when dealing with Gaussian white noise. Specifically, it minimizes the asymptotic covariance of the

estimation error  $\hat{x} - x$ .

To form the LQG regulator, simply connect the Kalman filter and LQ optimal gain  $K$  as shown in Figure 5-3.

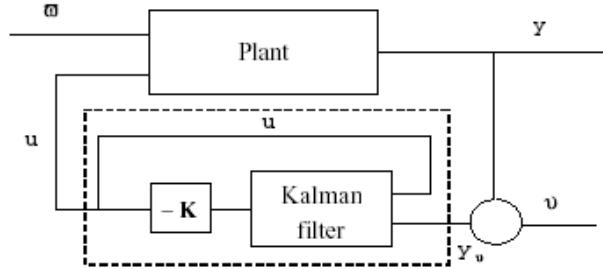


Figure.4 Block diagram of LQG regulator

This regulator has state space equations:

$$\dot{\hat{x}} = (\hat{A} - \hat{L}\hat{C} + \hat{B}k) \hat{x} + \hat{L}y_0 \quad (19)$$

$$u = -k\hat{x} \quad (20)$$

In addition to LQG design, a Disturbance Accommodate Control (DAC) design and a hybrid LQG/DAC control system are also studied in this chapter. Hence, nine cases will have to be discussed, which are three control systems, the LQG, the DAC and the hybrid LQG/DAC, in conjunction with three different control models, the MCC model, the SCC model and the SSCC model, respectively. In this paper LQG regulator has been used for two link- robotic manipulator systems. Figure 5 shows the structure of system that used in this system [16].

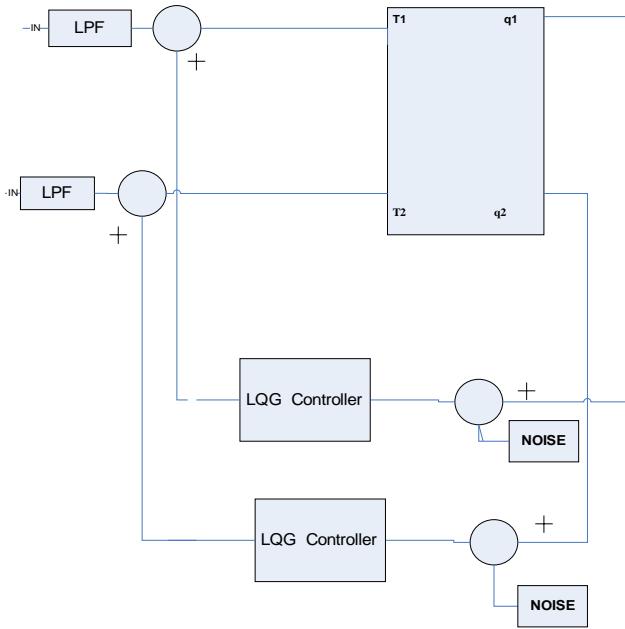


Fig. 5: structure of LQG regulator for Two link- robotic manipulator systems

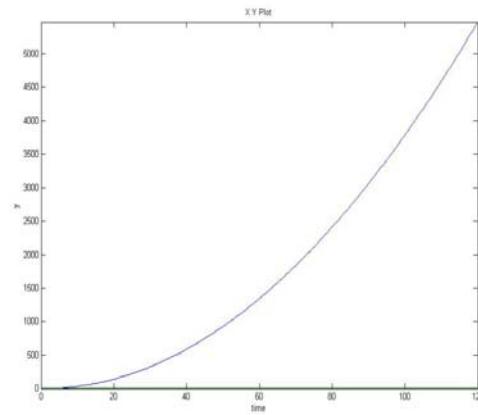


Fig. 7: the output of two link- robotic manipulator systems

As the first method for closed loop control of two link robot the *LQG regulator* controller has been used, figure 8 shows the simulation result

**IV. SIMULATION RESULT**  
 In the first the dynamic model of two link- robotic manipulator systems has been simulated using Matlab software 2007 as you see the system is unstable. Figure 6 shows the schematic of two link- robotic manipulator systems and figure 7 shows the simulation result for step input:

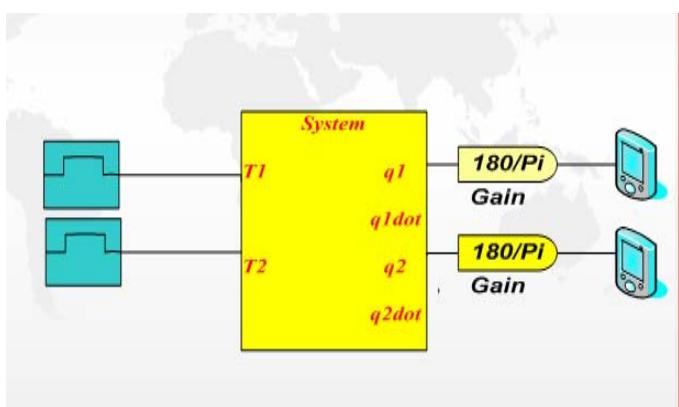


Fig. 6: schematic of the two link- robotic manipulator systems

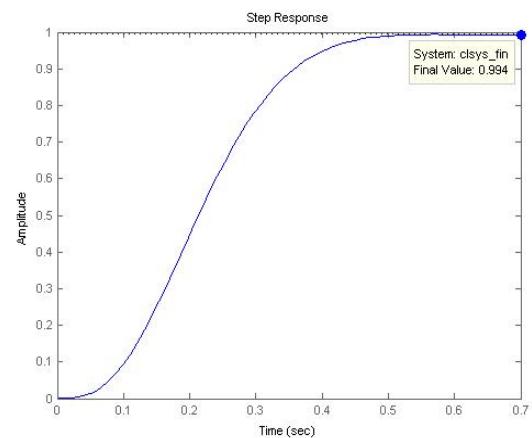
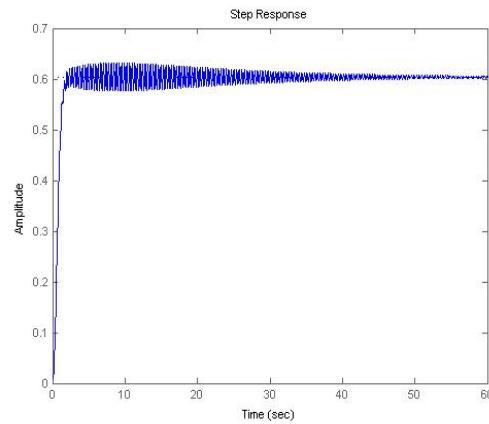


Fig. 8: the output of two link- robotic manipulator systems using LQG regulator,

## V. CONCLUSION

This paper presents a new method for the control of Two link-robotic manipulator systems using method Linear-Quadratic-Gaussian (LQG).

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