Production Process Optimization in Flexible Manufacturing System Using Petri Nets

E. Sharifi Tashnizi, S. N. Farahani, A. R. Fazeli Nahrekhalaji

Abstract— **This paper studies optimizing sharing resources priorities, keeping system job sequences and minimizing the total time of production lines in flexible manufacturing systems. Automatic lines in production cells, sharing resources and processing times of each, are studied and modeled by Petri nets. Subsequently, the practicability of model is validated by netlab software. Then, a nonlinear programming problem is applied to optimizing the PNS transitions of the FMS problem. Consequently, optimal times for each transition of the related PNS are achieved.**

Index Terms— **Flexible Manufacturing Systems, Petri Nets, Netlab Software.**

I. INTRODUCTION

Many companies have realized that in order to compete in today`s world market, they must rely on innovative developments in manufacturing technology [11].To increase productivity, companies are applying computer controlled machine tools, automated materials handling and storage systems. Due to the progress in manufacturing technology the Flexible Manufacturing concept has emerged [8,14].

The flexibility description in the FMS domain is more important. Different types of flexibility are considered: machine flexibility, process flexibility, product flexibility, route flexibility, production rate flexibility, development flexibility and transition flexibility [12].

One of the major goals of FMS_S is to reduce the total production time.In fact this paper minimizes time factor in FMS_S [7], while taking into consideration process and transition flexibility types.

II. MODEIING FMSS BY PETRI NETS

In this illustration, we design a flexible manufacturing system consisting of four machines M_1, M_2, M_3, M_4 and four robots named R_1, R_2, R_3, and R_4 as shown in the Fig.1.

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The system that we are going to model produces three types of product, A, B, C.

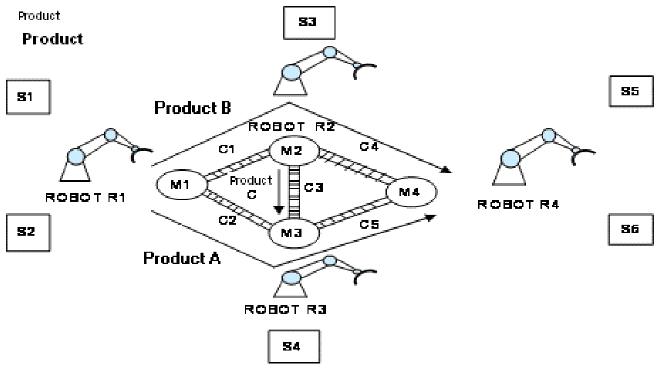

Fig.1. A typical FMS

There are some assumptions in the example of Fig.1:
Product A is processed by M_1, M_3, M_4, respectively.
Product B is processed by M_1, M_2, M_4 respectively.
Product C is processed by M_2, M_3 respectively.
The materials handling tools of production line A, include: R_1, C_2, C_5, R_4, respectively.
The materials handling tools of production line B, include: R_1, C_1, C_4, R_4, respectively.
The materials handling tools of production line C, include: R_2, C_3, R_3, respectively.
The processing time of each station is depicted in Table.1.[10].
As an example, the procedure to manufacture product A is depicted as follows:
R_1 takes raw stock from storage S_1 and loads M_1; M_1 starts machining; the conveyor C_2 transfers the intermediate product from M_1 to M_3 for further machining; the conveyor C5 transfers the intermediate product on M_3 to M_4; M_4 processing; finished product on M4 will be moved by R_4 to storage S_5.
Similar procedures are assumed for products B and C. At a time, each device works only on one product. In addition, it is requested that product B be delivered at dth clock. The problem is to choose the appropriate shared resources priorities plus the job sequences pattern so that minimize the total production time.

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Table.1.The processing time of each station.

Station	Processing time	On product
M_1	a_1 hrs.	A
M_1	b_1 hrs.	B
M_2	b_2 hrs.	B
M_2	c_2 hrs.	C
M_3	a_3 hrs.	A
M_3	c_3 hrs.	C
M_4	a_4 hrs.	A
M_4	b_4 hrs.	B
R_1	a_{R1} hrs.	A
R_1	b_{R1} hrs.	B
R_4	a_{R4} hrs.	A
R_4	b_{R4} hrs.	B
R_2	c_{R2} hrs.	C
R_3	c_{R3} hrs.	C
C_2	c_{C2} hrs.	A
C_5	c_{C5} hrs.	A
C_1	c_{C1} hrs.	B
C_4	c_{C4} hrs.	B
C_3	c_{C3} hrs.	C

As shown in Table.2, some places are defined to model the activity sequences for one part of product A, B and C [16]. Figure.2 shows the Petri net model of typical FMS with the places defined recently [2].

Table.2.Place labels for the example system [4]:

P_1	Move a raw part A from storage S_1 to M_1 by R_1.
P_2	Machining raw part A by M_1.
P_3	Move intermediate product A from M_1 to M_3 by conveyor C_2.
P_4	Machining intermediate product A by M_3.
P_5	Move intermediate product A from M_3 to M_4 by conveyor C_5.
P_6	Machining inter mediate product A by M_4.
P_7	Move finished product A from M_4 to finished products storage S_5 by R_4.
P_8	Move a raw part B from storage S_2 to M_1 by R_1.
P_9	Machining raw stock B by M_1.
P_{10}	Move intermediate product B from M_1 to M_2 by conveyor C_1.
P_{11}	Machining intermediate product B by M_2.
P_{12}	Move intermediate product B from M_2 to M_4 by conveyor C_4.
P_{13}	Machining intermediate product B by M_4.
P_{14}	Move finished product B from M_4 to finished products storage S_6 by R_4.
P_{15}	Move a raw part C from storage S_3 to M_2 by R_2.
P_{16}	Machining raw part C by M_2.
P_{17}	Move intermediate product C from M_2 to M_3 by conveyor C_3.
P_{18}	Machining intermediate product C by M_3.
P_{19}	Move finished product C from M_3 to finished products storage S_4 by R_3.
P_{20}	Conveyor C_2 available.
P_{21}	Conveyor C_5 available.
P_{22}	Machine M_1 available.
P_{23}	Robot R_1 available.
P_{24}	Machine M_3 available.
P_{25}	Machine M_4 available.
P_{26}	Robot R_4 available.
P_{27}	Conveyor C_1 available.
P_{28}	Conveyor C_4 available.
P_{29}	Machine M_2 available.
P_{30}	Robot R_2 available.
P_{31}	Conveyor C_3 available.
P_{32}	Robot R_3 available.

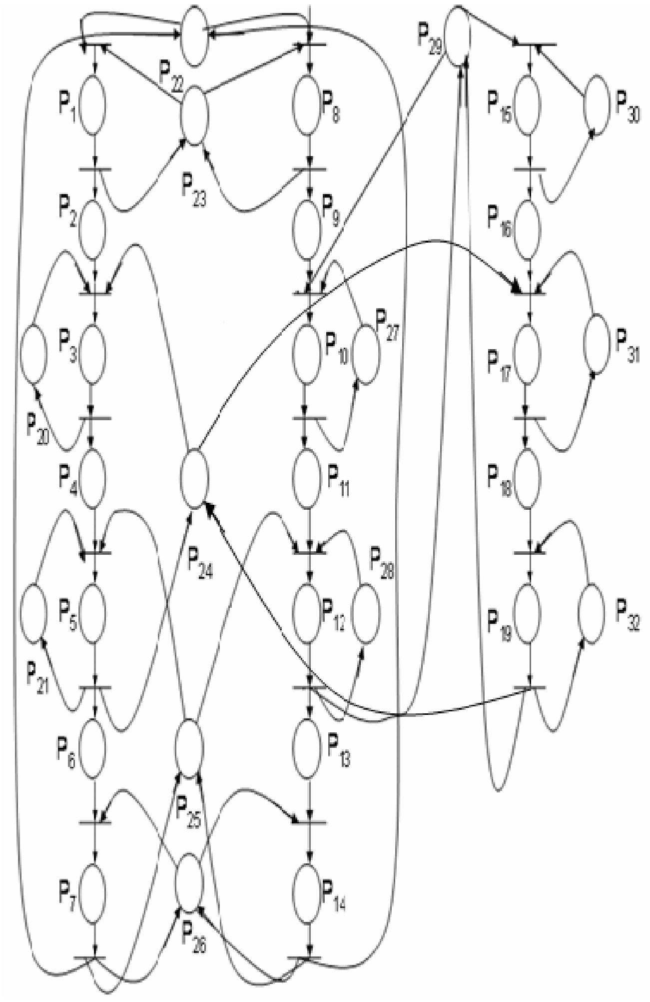

Fig.2.The obtained PNS related to the mentioned FMS in Fig.1

III. CHECKING THE VALIDATION OF THE CONSTRUCTED PNS

The correctness of system PNS model is checked in Netlab Software [6]. For analyzing the Petri nets properties, p- invariants, t- invariants, reachability graph and coverability graph must be calculated, at first [5].

A. Results of the net analysis

Dead transitions (RG): none.
Total deadlock (RG): none.
Reversibility (RG, condensed):
The net is reversible. Necessary conditions for invariants:
There exists a non-negative T-invariant. Therefore, the necessary condition for reversibility is satisfied, and the net may be reversible.
Partial deadlocks exist in the following sinks (RG, condensed): none.
Liveness (RG, condensed):
The net is live.Necessary conditions for invariants:
There exists a positive T-invariant.
Therefore, the necessary condition for liveness is satisfied, and the net may be live.
Boundedness (RG):
The net is bounded. Sufficient conditions for invariants:
There exists a positive P-invariant.

Therefore, the sufficient condition for boundedness is satisfied, and the net is bounded.
With regard to the analysis of properties, the above PN has the preliminary properties: liveness, reversibility and bounded ness [9].It is also deadlock free. It confirms that the PN model is feasible [5], [17].
In discrete and step by step execution of PNS, some transitions enable simultaneously [15].An important remark concerning the firing rule is that enabled transitions are never forced to fire. There is neither limitation nor priority in the enabled transitions firings [8].
Deciding about which activated transition to be fired, depends to the user.The question is: what the user`s criterion for choosing a transition would be and how a given conflict must be solved .
In the next section, a technique is explained that minimizes the total production time, while helping the user in best possible selection in case of conflicts.

IV. OPTIMIZING THE TOTAL PRODUCTION TIME IN FMS

Considering the related Petri net, at first we mark shared resources and arrangements of machines.
Let us define some variables in the Table.3.[3]:

Table.3. Defined variables for transitions optimization in PNS of the FMS problem.

Variable	The start transferring/processing time of	On product
X_{Aj}; j=1,3,4	M_j	A
X_{Bj}; j=1,2,4	M_j	B
X_{Cj}; j=2,3	M_j	C
$X_{A\text{-}R1}$	R_1	A
$X_{B\text{-}R1}$	R_1	B
$X_{A\text{-}R4}$	R_4	A
$X_{B\text{-}R4}$	R_4	B
$X_{C\text{-}R2}$	R_2	C
$X_{C\text{-}R3}$	R_3	C
$X_{A\text{-}C2}$	C_2	A
$X_{A\text{-}C5}$	C_5	A
$X_{B\text{-}C1}$	C_1	B
$X_{B\text{-}C4}$	C_4	B
$X_{C\text{-}C3}$	C_3	C

First condition: machine processing order satisfies.
By defining constraints as follows, work order on product A comes true .

$$X_{A\text{-}R1}+a_{R1} \leq X_{A1} \quad (1)$$

(e.g: the start time of transferring A by R_1 +moving duration by R_1 ≤the start time of processing A by M_1)

$$X_{A1}+a_1 \leq X_{A\text{-}C2} \quad (2)$$

$$X_{A\text{-}C2}+a_{C2} \leq X_{A3} \quad (3)$$

$$X_{A3}+a_3 \leq X_{A\text{-}C5} \quad (4)$$

$$X_{A\text{-}C5}+a_{C5} \leq X_{A4} \quad (5)$$

$$X_{A4}+a_4 \leq X_{A\text{-}R4} \quad (6)$$

By defining constraints as follows, work order on product B comes true .

$$X_{B\text{-}R1}+b_{R1} \leq X_{B1} \quad (7)$$

(e.g: the start time of transferring B by R1 +moving duration by R1 ≤the start time of processing B by M1)

$$X_{B1}+b_1 \leq X_{B\text{-}C1} \quad (8)$$

$$X_{B\text{-}C1}+b_{C1} \leq X_{B2} \quad (9)$$

$$X_{B2}+b_2 \leq X_{B\text{-}C4} \quad (10)$$

$$X_{B\text{-}C4}+b_{C4} \leq X_{B4} \quad (11)$$

$$X_{B4}+b_4 \leq X_{B\text{-}R4} \quad (12)$$

By defining constraints as follows, work order on product C comes true.

$$X_{C\text{-}R2}+c_{R2} \leq X_{C2} \quad (13)$$

(e.g: the start time of transferring C by R_2 +moving duration by R_2 ≤the start time of processing C by M_2)

$$X_{C2}+c_2 \leq X_{C\text{-}C3} \quad (14)$$

$$X_{C\text{-}C3}+c_{C3} \leq X_{C3} \quad (15)$$

$$X_{C3}+c_3 \leq X_{C\text{-}R3} \quad (16)$$

Second condition: shared resources can not work on two products, simultaneously (solving the conflicts).
The question is: how to define constraints that distinguishes which product the shared resources process, at first .
Answer: we must define binary variables.
The following constraints are "or " types:

Machine M_1:

$X_{A1}+a_1 \leq X_{B1}$ processes product A, at first.
Or
$X_{B1}+b_1 \leq X_{A1}$ processes product B, at first.

Machine M_2:
$X_{B2}+b_2 \leq X_{C2}$ processes product B, at first.
Or
$X_{C2}+c_2 \leq X_{B2}$ processes product C, at first.

Machine M_3:
$X_{A3}+a_3 \leq X_{C3}$ processes product A, at first.
Or
$X_{C3}+c_3 \leq X_{A3}$ processes product C, at first.

Machine M_4:
$X_{A4}+a_4 \leq X_{B4}$ processes product A, at first.
Or
$X_{B4}+b_4 \leq X_{A4}$ processes product B, at first.

Robot R_1:
$X_{A-R1}+a_{R1}\leq X_{B-R1}$ handles product A, at first.
Or
$X_{B-R1}+b_{R1}\leq X_{A-R1}$ handles product B, at first.

Robot R_4:
$X_{A-R4}+a_{R4}\leq X_{B-R4}$ handles product A, at first.
Or
$X_{B-R4}+b_{R4}\leq X_{A-R4}$ handles product B, at first.

Six "or " type constraints, as introduced thus far, must change into mathematical formalisms so that in each couple, one becomes surplus. For we don`t want both of them come true. Thus for each group of constraints we do define a binary variable.
The binary variable (y_1) is defined for the first couple of constraints.
y1=0; the former comes true and the latter surplus. The formalism changes so:

$$X_{A1}+a_1\leq X_{B1}+My_1 \quad (17)$$

$$X_{B1}+b_1\leq X_{A1}+M(1-y_1) \quad (18)$$

The binary variables y_2, y_3, y_4, y_5, y_6 are respectively defined for the other couples of constraints. M is a big number.
It is also asked to deliver product B at dth clock. In this case, a new constraint, as the following, is added to the problem:

$$XB_{-R4}+b_{R4}\leq d \quad (19)$$

It should be noted that we do not add the processing time, on product B, from the first machine in case there is an interruption between them. Therefore do only consider the time of the last workstation on B.
Objective functions are as follows:
Let us define:

$$X_{A-R4}+a_{R4}=\Theta_1 \quad (20)$$

$$X_{B-R4}+b_{R4}=\Theta_2 \quad (21)$$

$$X_{C-R3}+c_{R3}=\Theta_3 \quad (22)$$

Firstly, choose the maximum:
$$\text{Max}\ (\Theta_1, \Theta_2, \Theta_3) \quad (23)$$

Secondly, choose the minimum:
$$\text{Min Max}\ (\Theta_1, \Theta_2, \Theta_3) \quad (24)$$
$$\text{Max}\ (\Theta_1, \Theta_2, \Theta_3) = Y \quad (25)$$

The objective function is as below:
Min Y:
$$Y\geq\Theta_3;\ Y\geq\Theta_2;\ Y\geq\Theta_1 \quad (26)$$

V. NUMERICAL EXAMPLE

The obtained nonlinear problem can be solved through branch and band algorithm. This problem is multi objective and can easily be solved in WinQSB software.
The WinQSB software is capable of solving various types of nonlinear and linear problems. Any types of variables, binary, integer, nonnegative, can be defined and solved easily. Problem values re inserted either in table or normal form. The step by step solving is also arranged. There is an icon for analyzing, too.
Case study: Solving the optimization problem of typical FMS with the following assumptions:

$a_{R1}=1$; $a_{R4}=2$; $a_1=10$; $a_3=15$; $a_4=10$; $a_{C2}=3$; $a_{C5}=4$
$b_{R1}=1$; $b_{R4}=2$; $b_1=15$; $b_2=20$; $b_4=10$; $b_{C1}=4$; $b_{C4}=4$
$c_{R2}=2$; $c_{R3}=1$; $c_2=15$; $c_3=20$; $c_{C3}=3$.

Also consider "d " the delivery time of product B at 60th minute.
As already mentioned in equ.1-26, the general form of problem is formulated in equ.27.

Min Y (27)
$Y\geq X_{A-R4}+a_{R4}$
$Y\geq X_{B-R4}+b_{R4}$
$Y\geq X_{C-R3}+c_{R3}$
$X_{A-R1}+a_{R1}\leq X_{A1}$
$X_{A1}+a_1\leq X_{A-C2}$
$X_{A-C2}+a_{C2}\leq X_{A3}$
$X_{A3}+a_3\leq X_{A-C5}$
$X_{A-C5}+a_{C5}\leq X_{A4}$
$X_{A4}+a_4\leq X_{A-R4}$
$X_{B-R1}+b_{R1}\leq X_{B1}$
$X_{B1}+b_1\leq X_{B-C1}$
$X_{B-C1}+b_{C1}\leq X_{B2}$
$X_{B2}+b_2\leq X_{B-C4}$
$X_{B-C4}+b_{C4}\leq X_{B4}$
$X_{B4}+b_4\leq X_{B-R4}$
$X_{C-R2}+c_{R2}\leq X_{C2}$
$X_{C2}+c_2\leq X_{C-C3}$
$X_{C-C3}+c_{C3}\leq X_{C3}$
$X_{C3}+c_3\leq X_{C-R3}$
$X_{A1}+a_1\leq X_{B1}+My_1$
$X_{B1}+b_1\leq X_{A1}+M(1-y_1)$
$X_{B2}+b_2\leq X_{C2}+My_2$
$X_{C2}+c_2\leq X_{B2}+M(1-y_2)$
$X_{A3}+a_3\leq X_{C3}+My_3$
$X_{C3}+c_3\leq X_{A3}+M(1-y_3)$
$X_{A4}+a_4\leq X_{B4}+My_4$
$X_{B4}+b_4\leq X_{A4}+M(1-y_4)$
$X_{A-R1}+a_{R1}\leq X_{B-R1}+My_5$
$X_{B-R1}+b_{R1}\leq X_{A-R1}+M(1-y_5)$
$X_{A-R4}+a_{R4}\leq X_{B-R4}+My_6$
$X_{B-R4}+b_{R4}\leq X_{A-R4}+M(1-y_6)$
$X_{B-R4}+b_{R4}\leq d$
$X_{A-R1}, X_{A-R4}, X_{A-C2}, X_{A-C5}, X_{B-R1}, X_{B-R4}, X_{B-C1}, X_{B-C4}\geq 0,$
$X_{C-R3}, X_{C-C3}\geq 0$ C-R2≥ 0
$X_{Aj}\geq 0$; j=1,3,4
$X_{Bj}\geq 0$; j=1,2,4
$X_{Cj}\geq 0$; j=2,3
$y_i=0,1$; i=1,2,3,4,5,6

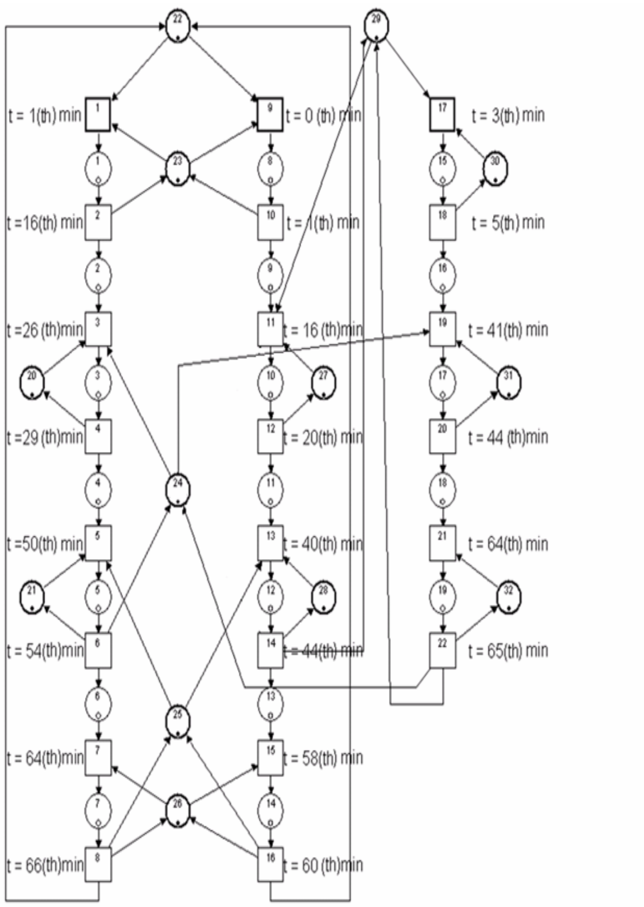

Fig.3.Determining optimal times for each transition of Fig.2

The final result, after substituting the values and solving the problem in QSB, is:
Solution Summary for FMS

	Solution Value	Decision Variable
1	$X_{A\text{-}R1}$	1.00
2	X_{A1}	16.00
3	$X_{A\text{-}C2}$	26.00
4	X_{A3}	29.00
5	$X_{A\text{-}C5}$	50.00
6	X_{A4}	54.00
7	$X_{A\text{-}R4}$	64.00
8	$X_{B\text{-}R1}$	0
9	X_{B1}	1.00
10	$X_{B\text{-}C1}$	16.00
11	X_{B2}	20.00
12	$X_{B\text{-}C4}$	40.00
13	X_{B4}	44.00
14	$X_{B\text{-}R4}$	58.00
15	$X_{C\text{-}R2}$	3.00
16	X_{C2}	5.00
17	$X_{C\text{-}C3}$	41.00
18	X_{C3}	44.00
19	$X_{C\text{-}R3}$	64.00
20	y_1	1.00
21	y_2	1.00
22	y_3	0
23	y_4	1.00
24	y_5	1.00
25	y_6	1.00
26	Y	66.00

Goal 1: Minimize $G_1 =$ 66.00

The obtained start times, give the optimal transitions firing times and also give the optimal total production time of typical FMS. The result of optimization problem is shown in its related PN_S in Fig.3.

VI. Conclusion

On the basis of the above analysis and the numerical results one can conclude that in discrete and step by step execution of PNS, and in some cases, some transitions representing the shared resources activities are enabled at the same time. But they can not be activated simultaneously [13]. Furthermore, the order in which activated transitions fire, is not fixed and it is non-deterministic. Therefore, for firing an enabled transition from among the others, user must select one of them. As a result, a decision must be made by the user.

One of the most important results of the present paper is achieving a technique for choosing the optimal choice. Besides, this optimization algorithm minimizes the total production time, while keeping job sequences and preventing the synchronization of shared resources. The issues mentioned above have been achieved via a nonlinear programming problem. As clarification and validation a numerical example has been shown through the study procedure.

Finally, the important conclusion of the present study is to increase the operating hours of FMSS through determining time for each transition in the related PNS.

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