Relative Motion of Robots as a Means for Signaling

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***Abstract*—Gesturing is one of the most prevalent means of communication in the world around us. In this article, we describe a way to mimic this notion of gesturing to the signaling between mobile robots. One can decompose this mode of signaling into two parts. The first is the ability of the gesturing agent to generate a gesture as a motion. The other part is the ability of the receiving agent to perceive this gesturing motion and interpret it in accordance with a pre-determined gesture set. We consider the relative motion of two Dubins' vehicles in the plane and analyze the transmission of a signal from one robot to the other in the form of the relative motion between the two robots. We formulate the two tasks as a pair of non-linear control problems. We require the transmitting robot to track the states of the receiving robot, and track one of its motion modes. The transmitting robot further needs to overlay the signal to be transmitted on the other motion mode for the receiver robot to be able to sense this signal using an on-board range sensor. We present a non-linear control law that enables this mode of signaling. We also present simulations and experimental prototyping of this idea, along with a discussion of potential applications of this concept.**

***Index Terms*—gesture, communication, non-linear control, robotics**

I. Introduction

Gesturing is a prevalently used mode for communication. We use it in our daily lives, be it signaling by traffic police, construction workers, workers on run-ways, bees dancing, birds in formation flight, dumb-charades, planes in formation flight, field sports like soccer, football - the list goes on. This mode of signaling adds one more degree of freedom in the design space for robotic interaction, be it between robots or between robots and humans. We anticipate significant utility of this idea in a variety of contexts - from fundamental questions, to important engineering applications.

At a fundamental level, the questions that can be addressed include the determination of the fundamental limit to the amount of information that can be communicated using gestures. Such a limit will depend on a measure of the space of all possible gestures by a gesturing agent, the precision with which the agent receiving the gesture signals is able to decode the transmitted gesture, and a code book that maps the received signal to the corresponding meaning. One may also think in terms of augmenting secure communication using gestures. This can add to the existing bio-based authentication schemes including iris scanning, fingerprint identification and voice recongnition. One other application lies in the case of signaling in formation. Such signaling can serve as a natural means for signaling between formations of robots, reserving the commonly used wireless data exchange methods for other transactions. Indeed, avoiding the use of conventional wireless messaging may offer a stealthy mode for communication.

In this article, we describe a way by which we have achieved this kind of signaling between two non-holonomic robots moving in a plane, and present the control strategies that we have used to achieve this.

Recognizing pre-defined gestures (such as hand-gesture recognition) accurately using vision based image processing is an active area of research. The goal is to reverse engineer the joint angles, motion sequence and other kinematic details of the motion, and map this information to a pre-determined knowledge-base of gestures to estimate the message. The focus is mainly on the vision task (see [4] for instance for a survey.) There is also research into the notion of using relative motion as a means for camouflage. The goal has been to understand the dynamics and control of prey-predator motion of insects. Predators like dragonflies achieve camouflage with respect to a prey by maintaining a constant orientation in space with respect to the prey. By doing this, slower predators are able to capture faster prey. This works because for most insects, detection of motion transverse to its orientation is very well developed. However, longitudinal perception (ie motion towards or away from the insect) is not that well developed[2][3]. Another interesting related problem is that of optimal traffic management, wherein, one attempts to control the flow of traffic by controlling the relative separation between vehicles in order to form one-dimensional vehicular chains[6].

This article is organized as follows. We present a mathematical formulation of the problem of signaling using relative motion in the plane between two non-holonomic robots. We then present some properties that the signals being transmitted must possess given the limitations on the dynamics of the robots. Following that, we present control

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paradigms for this kind of transmission. Beyond that, we present simulations that illustrate this concept, followed by a discussion of directions in which we intend to extend and apply this research.

II. Problem Formulation

Figure 1 illustrates the idea of signaling between two robots using their relative motion. Robot R_1 is moving along the positive x-axis. R_2 is attempting to signal to R_1 the space curve that is shown in the figure, all the time trying to keep up with R_1's x-coordinate. Thus R_2 has two goals - one, to keep up with R_1 (tracking goal), and two, overlay the space curve signal to R_1. R_1 has a range sensor that is able to estimate the distance of R_2 from R_1 at each point in R_1's trajectory and build an estimate of the curve that R_2 is attempting to transmit to it. It could further *cooperate* with R_2 by regulating its trajectory and velocity to a state favorable to R_2 to track.

We constrain ourselves to the case of R_1 and R_2 being two wheeled non-holonomic robots in the plane that are often referred to as Dubins' vehicles in the literature [5](Figure 2.) The orientation of such a vehicle is described relative to the positive x-axis, while the coordinates of the vehicles' center is expressed relative to an inertial frame. We represent the velocity of the midpoint between the wheels of the robot as u and the angular velocity of the robot as ω, to get

$$\dot{x} = u\cos\theta \quad (1)$$
$$\dot{y} = u\sin\theta \quad (2)$$
$$\dot{\theta} = \omega \quad (3)$$

For the case that we are interested in, the goal is for R_2 to

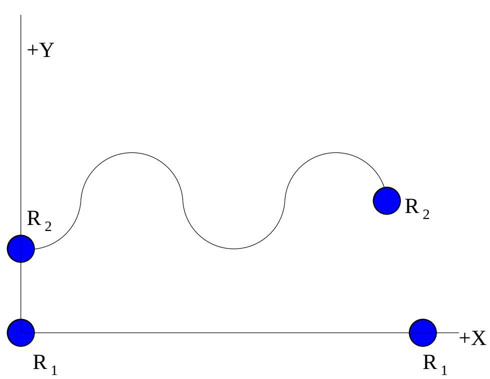

Fig. 1. Robot R_2 signaling to robot R_1 using relative motion.

track R_1's velocity. We assume, without loss in generality, that R_1 is moving along the positive x-axis with a velocity profile of $V(t)$. We now write out the equations of motion of the two robots $R_1(x_1, y_1, \theta_1)^T$ and $R_2(x_2, y_2, \theta_2)^T$ as

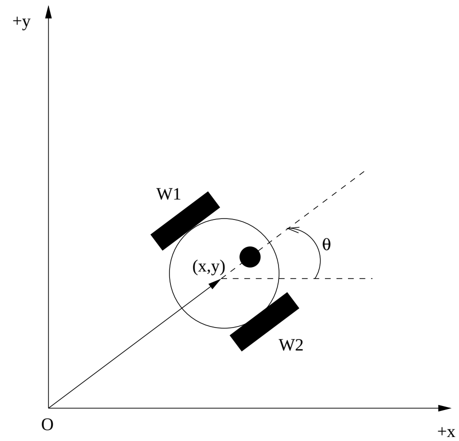

Fig. 2. A Dubins' vehicle with its coordinates and orientation shown. The distance between wheels W1 and W2 is d.

follows. Note that $\mathbf{x} = (x_1\ y_1\ \theta_1\ x_2\ y_2\ \theta_2)^T$.

$$\dot{x}_1 = V(t) + u_1(\mathbf{x}, t) \quad (4)$$
$$\dot{y}_1 = (V(t) + u_1(\mathbf{x}, t))\tan\theta_1 \quad (5)$$
$$\dot{\theta}_1 = \omega_1(\mathbf{x}, t) \quad (6)$$
$$\dot{x}_2 = u_2(\mathbf{x}, t) \quad (7)$$
$$\dot{y}_2 = u_2(\mathbf{x}, t)\tan\theta_2 \quad (8)$$
$$\dot{\theta}_2 = \omega_2(\mathbf{x}, t) \quad (9)$$

These equations are a little different in form from (1)-(2). As will be seen in the sequel, the goal is for R_2 to be able to track R_1, which we shall, for the convenience of analysis, assume is moving along the positive x-direction. In this case, we would like to control the x-components of the velocity directly with control inputs and hence, the form shown.

Problem statement *Find* $(u_1, \omega_1, u_2, \omega_2)$ *and the family of curves* $\mathcal{C} \in \Re^2$ *for the system* (4)-(9) *satisfying the following constraints and requirements. We use* $\kappa(\mathcal{C})$ *to represent the instantaneous curvature of the planar curve* $\mathcal{C}$.

Robot motion constraints

$$\dot{x}_i \sin\theta_i - \dot{y}\cos\theta_i = 0, i = 1, 2 \quad (10)$$
$$|u_i| \leq V_{max}, i = 1, 2 \quad (11)$$
$$|u_i \sec\theta_i| \leq V_{max}, i = 1, 2 \quad (12)$$
$$|\kappa(\mathcal{C})| \leq 2/d, i = 1, 2 \quad (13)$$

Protocol Requirements

$$\mathcal{C} = \mathcal{C}(s, \beta(s)) \quad (14)$$
$$\beta(s + L) = \beta(s), \textit{for a fixed } L \in (0, \infty) \quad (15)$$
$$\lim_{t \to +\infty} |y_2(t)| < +\infty \quad (16)$$

Constraint (14) forces the curve $\mathcal{C}$ to be parametrized in terms of the path traversed by R_1. This is important as R_2 is attempting to signal to R_1 while maintaining the component of its (R_2's) velocity parallel to that of R_1 the same as R_1. The family of curves $\mathcal{C}$ represents the set of messages

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that can be transmitted by this means of signaling *assuming perfect sensing by* R_1. Constraint (15) requires the signal to be spatially periodic, so that once a signal is transmitted, a re-transmission of the same signal adds no new information unless there is noisy observation at the receiver. Also, a transmission cannot be of infinite duration. We assume that R_2 is able to traverse the entire spatial signal in finite time. Finally, Constraint (16) requires that the signal that is being transmitted ensures that the separation between R_2 and R_1 does not diverge, but is bounded.

We can parametrize a continuously differentiable space curve (so that the required velocity profile of R_2 is feasible) in terms of the path length s of the curve in the Euclidean plane as $\mathcal{C}(\alpha(s), \beta(s))$. The tangent at any point to $\mathcal{C}$ is

$$\begin{aligned} \tan\theta &= \frac{d\beta(s)}{d\alpha(s)} = \frac{d\beta/ds}{d\alpha/ds} \\ \Lambda(s) &= \frac{\alpha'(s)\beta''(s) - \alpha''(s)\beta'(s)}{(\alpha'^2(s) + \beta'^2(s))^2} \\ \frac{d\theta}{dt} &= \frac{ds}{dt}\Lambda(s) \end{aligned} \tag{17}$$

We thus have a relation between $\mathcal{C}$ and the rate of change of the tangent (orientation) of a robot that is attempting to track the curved trajectory.

A. *Properties of Transmittable Functions*

Based on the constraints listed in the previous section, one can determine some conditions that $\beta(s)$ must satisfy (we note that requirement (14) make $\alpha(s) = s$.) We denote by $BV(\Omega)$ functions that are of bounded variation on the set Ω. In what follows, we further specialize Ω to represent the closed interval $[0, L]$ based on Requirement (15).

Lemma II.1. $\beta'(s) \in BV(\Omega)$.

Proof: From (13), $\kappa(\mathcal{C}(s, \beta(s)) = \frac{|\beta''|}{(1+\beta'^2)^{3/2}} \leq |\beta''| \leq 2/d$. Thus $\beta'(s)$ is *Lipschitz* which in turn means that $\beta'(s) \in BV(\Omega)$. ∎

Lemma II.2. $\beta(s) \in BV(\Omega)$.

Proof: $\beta'(s) \in BV(\Omega) \implies \beta'(s)$ is uniformly continuous. This in turn implies that $|\beta'(s)|$ is bounded $\implies \beta(s) \in BV(\Omega)$. ∎

Corollary II.3. $|\Lambda(s)| \leq 2/d$.

Proof: $|\kappa(\mathcal{C})| = \frac{|\beta''|}{(1+\beta'^2)^{3/2}} \geq \frac{|\beta''|}{(1+\beta'^2)^2} = |\Lambda(s)| \implies |\Lambda| \leq 2/d$ (from Lemma II.1). ∎

Theorem II.4. $\mathcal{C} \in C^2$.

Proof: This follows from Lemmas II.2 and II.1. ∎

Constraint (15) *along with Theorem II.4 implies that* $\beta(s)$ *can be expanded out in terms of a Fourier series.*

Theorem II.5. *Constraint* (16) *holds for any* $\mathcal{C}(s, \beta(s))$ *satisfying Lemma II.2, Lemma II.1 and Constraint* (15).

Proof: One can write $y_2(t)$ as

$$\begin{aligned} y_2(t) &= y_2(0) + \int_0^t \tan\theta(s(\tau))u_2(\tau)d\tau \\ &= y_2(0) + \int_{s(0)}^{s(t)} \tan\theta(s)ds \text{ [since } u_2(\tau)d\tau = ds] \\ &= y_2(0) + \int_{s(0)}^{s(t)} \beta'(s)ds \\ &= \beta(s(t)) - \beta(s(0)) \end{aligned}$$

Over a period L, we thus have $\int_\gamma^{\gamma+L} \tan\theta(s)ds = \beta(\gamma + L) - \beta(\gamma) = 0$ (from (15).) Hence, for $\beta(s)$ periodic and $\mathcal{C} \in C^2$, we have $\lim_{t\to\infty} |y_2(t)| < \infty$. ∎

Lemma II.6. $\theta_2(s)$ *is periodic.*

Proof: As $\beta(s)$ can be represented as a Fourier series, $\beta'(s)$ and $\beta''(s)$ can also be represented by Fourier series. $\Lambda(s) = \Lambda(\beta(s), \beta'(s), \beta''(s))$ is a function of other periodic functions and so, is periodic itself.

$$d\theta = \Lambda(s)ds \implies \theta(s) = \theta(0) + \int_0^s \Lambda(r)dr$$

Since $\Lambda(s)$ is periodic, one can write

$$\theta(s) = g(s) + \bar{\Lambda}\cdot s + \theta(0)$$

where $g(s)$ is a periodic function of s and $\bar{\Lambda} = (1/L)\int_\gamma^{\gamma+L} \Lambda(s)ds$. Since $\beta'(s)$ is bounded for all $s \geq 0$, the slope angle $\theta(s)$ is bounded. The only way this is possible for all possible $s \geq 0$ is if $\bar{\Lambda} = 0$. We know that $\beta(s)$ can be represented as a Fourier series.

$$\beta(s) = a_0 + \sum_{n=0}^{\infty}(a_n \cos(\frac{2\pi}{L}ns) + b_n \sin(\frac{2\pi}{L}ns)) \tag{18}$$

$$\beta'(s) = \frac{2\pi}{L}\sum_{n=0}^{\infty}(-a_n n \sin(\frac{2\pi}{L}ns) + b_n n \cos(\frac{2\pi}{L}ns)) \tag{19}$$

$$\beta''(s) = -\frac{4\pi^2}{L^2}\sum_{n=0}^{\infty}(a_n n^2 \cos(\frac{2\pi}{L}ns) + b_n n^2 \sin(\frac{2\pi}{L}ns))$$

Hence we have

$$\begin{aligned} \left|\frac{1}{L}\int_\gamma^{\gamma+L} \Lambda(s)ds\right| &= \frac{1}{L}\left|\int_\gamma^{\gamma+L} \frac{\beta''}{(1+\beta'^2)^{3/2}}ds\right| \\ &\leq \frac{1}{L}\left|\int_\gamma^{\gamma+L} \beta'' ds\right| = 0 \end{aligned}$$

The last inequality follows from the Fourier expansion of $\beta''(s)$, which is purely a function of $2\pi/L$ periodic sines and cosines the corresponding higher integral harmonics. Hence, $\beta(s)$ being periodic implies $\bar{\Lambda} = 0$, and hence, the result. ∎

III. Control Strategies to Enable Simultaneous Tracking and Overlay

In this section, we present a control strategies that can achieve simultaneous tracking and overlay. We require two control laws. First, we need a control law that enables the trajectories of the transmitter and receiver to be *coupled*. For instance, we might require R_1 and R_2 to travel parallel to each other for optimal detection of R_2 by the sensors onboard R_1. Second, we need to overlay on this control law a strategy that enables the transmitter to trace the desired space curve for the receivers' benefit. This responsibility can be shared in three different ways:

1) **The transmitter performs both controls.** In this case, the transmitter both couples the trajectories *and* overlays the desired curve on this trajectory. There is no explicit *cooperation* between the transmitter and receiver. It is possible that the receiver tacitly follows a trajectory that is easy to track for the transmitter. For the specific scenario we have described in the preceding section, one can choose a standard PID control strategy for the tracking mode, and overlay the desired space curve as shown by the laws (20)-(23) below (with an initial constraint of $\theta_1(0) = 0$):

$$\begin{aligned} u_1 &= V && (20)\\ \omega_1 &= 0 && (21)\\ u_2 &= K_{p2}(x_1 - x_2) + && (22)\\ & \quad K_{i2}\int_0^t (x_1(\tau) - x_2(\tau))d\tau + K_{d2}\dot{x}_2 \\ \omega_2 &= u_2\Lambda(x_2) && (23) \end{aligned}$$

2) **The transmitter and receiver cooperate in coupling the trajectories; in addition, the receiver handles the signal overlay.** This strategy presents a scheme with *explicit cooperation* between the transmitter and the receiver. The following is an example control strategy in this spirit:

$$\begin{aligned} u_1 &= V + K_{p1}(x_1 - x_2) && (24)\\ \omega_1 &= 0 && (25)\\ u_2 &= K_{p2}(x_1 - x_2) && (26)\\ \omega_2 &= u_2\Lambda(x_2) && (27) \end{aligned}$$

In this second strategy, R_1 is willing to sacrifice its speed V in order to accommodate a possibly slower robot R_2, allowing for a steady state error.

3) **The receiver handles the receiver-transmitter trajectory coupling, while the transmitter handles only the signal overlay.** The following control laws are an instance of this idea:

$$\begin{aligned} u_1 &= K_{p1}(x_1 - x_2) + && (28)\\ & \quad K_{i1}\int_0^t (x_1(\tau) - x_2(\tau))d\tau + K_{d1}\dot{x}_1 \\ \omega_1 &= 0 && (29)\\ u_2 &= V && (30)\\ \omega_2 &= u_2\Lambda(x_2) && (31) \end{aligned}$$

All three of these strategies are feasible for achieving our desired goals of tracking and overlay. However, we prefer to use the Among the three, strategy 1 is easily extended to duplex transmission - both R_1 and R_2 simultaneously transmitting data between each other by signaling with respect to a pre-determined baseline curve.

The pair of equations (20) and (22) represent, respectively, a mode that is being tracked, and, an agent that is tracking it with zero steady state error given some limitations on V. Equation (22) actually is a standard PID control law that can track the x_1 coordinate if $V(t) = constant$ for instance.

In what follows, we present simulations illustrating this idea.

A. Simulations

Figure 3 illustrates R_2 signaling to R_1 while simultaneously tracking the trajectory of R_1. The same two robots simulated for a longer duration of time is shown in Figure 4. Finally, Figure 5 shows how one can super-impose two sinusoidal signals to get a more complex signal that can be transmitted using the scheme described in the previous section. In all cases, the trajectory tracking error is being driven to 0 by the use of a PID control law for the tracking mode. The received signal closely follows the transmitted signal as can be seen from the simulations. We note that the steep slopes shown in the figure are due to the scaling of the plot; the actual slopes on the space curves are much smaller.

IV. Conclusions

We have shown a method for signaling between robots that attempts to emulate the idea of gestures that we commonly use and encounter. We have decomposed the notion of gesturing into two parts - a motion generation part, and a motion perception part. We have presented a formal approach to formulating and solving this problem in 2-D as a non-linear control problem and have presented our results. We have also determined properties of signals that can be transmitted using this mode of gesturing given the limitations of the dynamics of the non-holonomic robots that we have considered.

This mode of signaling augments several existing modes for communication - wireless, optical to name just a few. This mode of signaling is of relatively low bandwidth compared to other technologies such as 802.11 wireless. Nevertheless it does give us benefits such as stealthy communication.

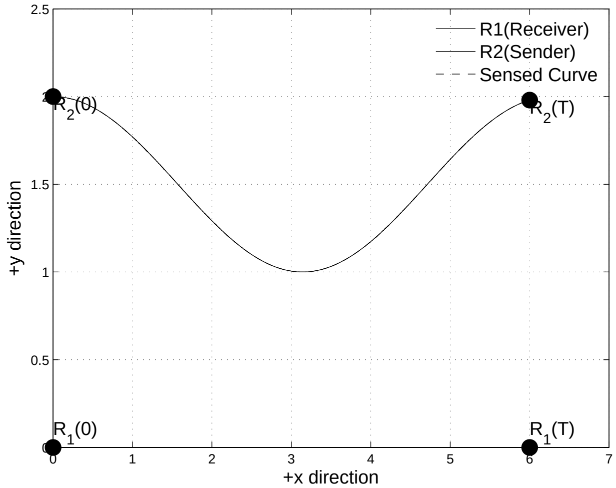

Fig. 3. R_1 receiving a signal with frequency and amplitude $\omega = 1.0\ rad/s,\ amplitude\ = 0.5m$ from R_2. R_1 moves with 0.1m/s, control laws are (20)-(23). Simulation run for 60s.

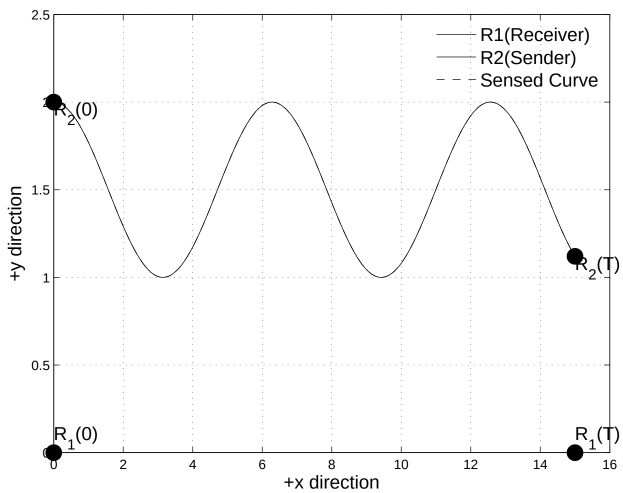

Fig. 4. Same as Figure 3, but run for 150s.

There are several research questions that this line of inquiry opens up. One open question is that of finding a measure on the set of signals that can be transmitted in this fashion given limitations of sensors (noisy readings for instance.) In a sense, this is an information theoretic perspective of this notion of signaling. Local coordinates rather than the inertial reference frames that have been used throughout this analysis will be more useful to achieve this task in a distributed manner[1]. Duplex signaling, where two agents simultaneously signal to each other relative to a constant baseline curve should be possible, and control laws need to be developed to enable this. Finally, such a mode of signaling should be extensible to larger formations of robots, where a leader can signal to the rest of the group of robots. These questions form an active part of the authors' current research interests.

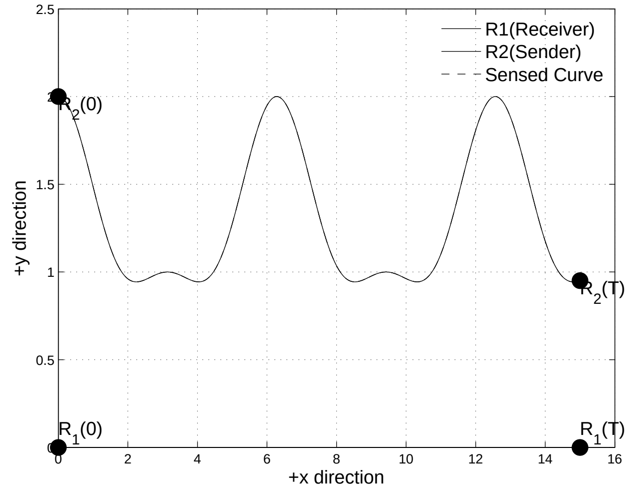

Fig. 5. R_1 receiving a signal with frequencies and amplitudes $(\omega = 1.0\ rad/s,\ amplitude\ = 0.5m), (\omega = 2.0\ rad/s,\ amplitude\ = 0.2m)$ from R_2. R_1 moves with 0.1m/s, control laws are (20)-(23). Simulation run for 150s.

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