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Cycle-to-Cycle Control of the Thermoforming Reheat Process

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***Abstract*—In this paper, we present a cycle-to-cycle repetitive control scheme for plastic sheet reheat in a thermoforming oven. Based on previous research on terminal iterative learning control, the notion of mode is introduced to the system to implement cyclic control of the sheet temperature. A hybrid dual-mode cascade-loop system is then designed where the inner loop monitors the real-time temperatures to ensure the quality of control in Mode 0 with the original PID control system of an AAA thermoforming machine, while the outer loop reads temperatures and commands at the end of each cycle to decide the necessity of switching to the mode of adjustment (MA, Mode 1), or vice versa. The simulation results present satisfactory performance with only a small sheet temperature error at the end of the cycle.**

***Index Terms*—Cycle-to-cycle control, industrial automation, repetitive control, thermoforming.**

I. Introduction

A. Thermoforming

Thermoforming is an industrial process in which plastic sheets are heated and then formed into useful parts. It consists of three consecutive phases, namely heating, forming and cooling. The sheet is heated, usually in some kind of oven, until it becomes soft or reaches the desired temperature. Then, the softened sheet is formed over the mold and cooled until it can retain its shape. Fig. 1 demonstrates the procedure of drape forming, a typical example of thermoforming [1].

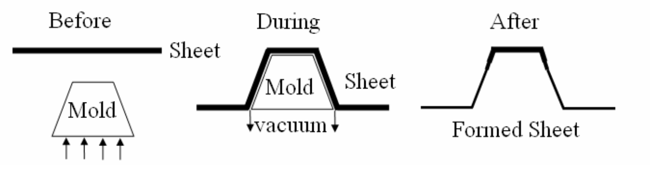

Fig. 1: The Process of Drape Forming

Today, thermoforming has become one of the fastest, if not the fastest growing methods of processing plastics. With the development of thermoforming technique, more and more relevant theoretical research on thermoforming control is carried out. Since the repetitive production of sheets meeting the standards is very demanding in the industry, the implementation of cyclic control on the machine has naturally become an important objective for researchers. A representative achievement is the Iterative Learning Control approach, which is surveyed by K. L Moore and then introduced to areas of thermal processes [2], [3]. Based on this approach, the Terminal Iterative Learning Control (TILC) is invented, further analyzed [4]–[6], and redesigned for various applications [7], [8], which has paved the way for this research.

B. AAA Thermoforming Machine

The AAA thermoforming machine at the National Research Council of Canada's Industrial Materials Institute (IMI) is the main test facility for our thermoforming research. All the critical experiments of this project, mainly heating process prototype work, actuator prototype work, and design of a converter for cyclic control, are carried out with the AAA thermoforming machine.

During the heating process, the sheet enters the oven between the trays of heating elements (heaters), which are located above and below the entry space. Fig. 2 provides the distribution of heating elements inside the AAA thermoforming machine, along with the exact sizes [1]. 36 ceramic heating elements are distributed equally on top and at the bottom of the oven (18 elements on each side), and divided up in 12 groups (heating zones) so that each zone encloses 3 heating elements connected in parallel. Each heating element can supply up to 650W of power. The basic design of a ceramic infrared heater is evenly distributing a resistance coil made of nickel-chromium alloy into the ceramic during the casting process. The coil is embedded as close as possible to the element's surface in order to deliver maximum efficiency [1]. The infrared wavelengths vary from 2 to 10 μm [9].

Fig. 2 (a): Heating Elements in the Bottom Tray

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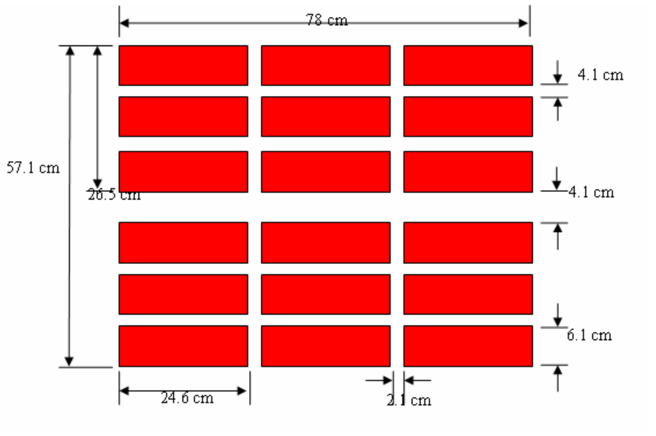

Fig. 2 (b): Sizes and Distribution of Heating Elements

Mark Ajersch established a sensor-based plant model and developed a controller successful in producing sheets discontinuously [1]. Through the sensors' projections to the sheet, the temperatures of five layers (points) across the thickness of the sheet are adopted as state variables. The temperatures on the two surfaces, which reflect the heating of the sheet, are also treated as outputs.

Fig. 3 (a) shows the structure and the flows of data inside this system with a controller connected, whereas Fig. 3 (b) simplifies the structure to an ordinary one-loop feedback system without loss of accuracy. Both of them are applied to the MATLAB controlled system designed by Mark Ajersch and also the control mode (Mode 0) of our improved system.

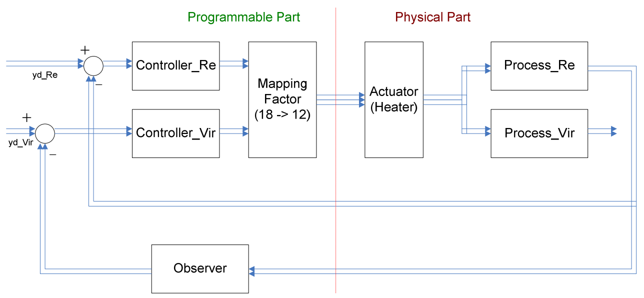

Fig. 3 (a): Structure of Original Computer-Controlled System

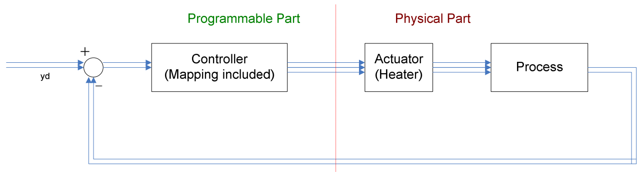

Fig. 3 (b): Simplified Structure of Original Computer-Controlled System

C. *Problem Statement*

Despite the fact that thermodynamic analysis and research have been conducted on the AAA machine itself, the control design and implementation has stayed at a relatively primitive stage before the current research topic, which is addressing the mass production of satisfactory plastic parts through consistent heating. Two main problems prevent the previous research from being applied:

1) Simulations with traditional PID control strategy showed that a certain delay always exists in the response of control signals (heater temperatures) to their commands. This delay causes excursions of heater temperatures, which is the source of wasted heating energy. It also causes difficulties in designing/improving controllers, and produces some inaccuracies in display panels of the apparatus.

2) The TILC model established by G. Gauthier and B. Boulet provides a blueprint of model-based control [10], [11]. It needs constant control signals within each cycle, however; the delay of control signals stops them from keeping steady upon different setpoints.

The first problem of actuator prototyping for the AAA machine is solved in [12] by changing input signals, so that heater temperature excursions are reduced. The second problem will be discussed in the remaining sections with the design of cycle modes and switching in producing sheets.

II. A Dual Mode Controller for Production with Fixed Cycle Time

A. *Two Modes*

The cycle-to-cycle production of sheets in a unified format calls for the self-tracking ability of heater temperatures; that is, the final value of each heater's temperature in each cycle should correspond to its initial value. We select 600 seconds as the default cycle time.

Define $y_t(k)$ as the real-time sheet temperature at the t^{th} second of the k^{th} cycle, $u_t(k)$ as the real-time actuator temperature at time t of the k^{th} cycle, and $y_d(k)$ and $u_d(k)$ as the desired values. Then, the ideal case and the goal of cyclic control is $u_0(k) = u_{\text{end}}(k) = u_d(y_d(k)) = u_d(k)$ ($u_0(k)$ and $u_{\text{end}}(k)$ respectively denote the heater temperatures at the beginning and at the end of the k^{th} cycle) for every cycle number k. Note that the heater temperatures must be continuous in the time axis due to the limit of their response speeds, that is, $u_0(k+1) = u_{\text{end}}(k)$; thus, in the ideal case there would be $u_0(k+1) = u_0(k)$, creating a consistently identical condition of production for every sheet.

From the previous analysis and experiments, it is easy to discover that $u_{\text{end}}(k)$ depends on both $y_d(k)$ and $u_0(k)$, in the case where no change occurs in the system properties. That means $u_0(k+1) = u_{\text{end}}(k) = u_0(k)$ could theoretically happen, if the initial heater temperatures $u_0(k)$ are tuned to appropriate values according to the current y_d. Especially, the subtopic "Design of the Converter Module" within this paper provides a way to select $u_0(k) = u_{\text{stable}}(y_d(k))$ according to Table I or the linear relationship described in (8). For example, a proportional controller with $K_p = 14$ shows the self-tracking ability of u_0 when parameters are selected as in Table I ($y_d = 150$°C, $u_{0\_\text{edge}}(k) = 184.2$, $u_{0\_\text{center}}(k) = 169.464$). It can lead to not only fairly short rising time and small overshoot, but also small deviations of heater temperatures from their initial values. Thus we use $K_p = 14$ as the default K_p of the classic PID control mode (Mode 0).

However, such a kind of self-tracking ability of the oven itself is vulnerable. If u_{0_edge} and u_{0_center} happens to be far from their desired values (according to Table I), there will be no guarantee that u_{0_edge} and u_{0_center} will return, which would cause consecutive sheets to be out of specification. Besides, the difference between the final values of heater temperatures and the initial values always exists, although the controller and the setting of u_{0_edge} and u_{0_center} help to reduce it. The error would be cumulative if caused by imprecise modeling; in this case, it would grow noticeable after a couple of cycles, and the sheets would not meet the standards any more.

Now, we introduce the concept of a **dual-mode controller** to solve the problems. The most characteristic improvement is to associate the **Mode of Adjustment (MA)** with the original mode of PID control, in order to constrain $u_0(k)$ at every cycle k to an acceptable range. MA is expected to directly control the heater temperatures and only acts in necessary cases (cycles). Since the main purpose of MA is not controlling the sheet temperature, no sheet should be placed in the oven during MA cycles to prevent unnecessary waste of materials.

An auto-switching mechanism should be designed with MA to form a complete set. Let $u_0(k)$ be consecutively examined at the beginning of each cycle. If the results show any element of $u_0(k)$ (any heater's initial temperature) goes out of the desired range, the system will automatically switch to the MA. The MA will last until all the u_0 components are driven back to the desired range; it is designed to accomplish the goal in one cycle, which is the general case (two or more cycles are also possible because of the limitations of heaters' hardware properties).

In MATLAB implementation, a suitable approach is to define a Boolean variable *mode*, which could be assigned a value cyclically and depends on $u_0(k)$ and $y_d(k)$ as shown in (1).

$$mode(k) = mode\left(u_0(k), y_d(k)\right) = \begin{cases} 1, \text{ (Mode of Adjustment)} \\ 0, \text{ (Mode of Classic PID Control)} \end{cases}. \tag{1}$$

Nevertheless, the control signal when $mode = 1$ (MA) is still unknown, the analysis of which is the subject of the next section.

B. Prototype of Actuator for the Mode of Adjustment

The research on actuator prototyping shows the actual command signals (control signals) are composed of the rising and falling curves, which are described by their corresponding different equations. According to the previous analysis, the rising and descending curves, which obey the upper and the lower bound of the saturation block, are given by (2):

$$u(t) = \begin{cases} \left(u(0)-600\right)\exp(-0.005t)+600 & \text{(rising)} \\ \left(u(0)-100/3\right)\exp(-0.0018t)+100/3 & \text{(descending)} \end{cases}. \tag{2}$$

Here u denotes heater temperatures (in Celsius), and $u(0)$ denotes the initial value of the cycle. $u < 600$ is a necessary condition of the rising curve equation, while $u > 100/3$ is a necessary condition of the descending curve. k denotes the k^{th} second within any unspecified cycle, since the saturation bounds are computed once per second.

In order to mass-produce heated sheets, the shape of the cyclic control signal is carefully considered. Generally, the variation of heater temperatures within one cycle should cause short rising time of sheet temperatures as well as reduce their overshoots (inertia), which requests combinations of both kinds of the rising and descending curves mentioned above. A typical model of rising and descending exponential curve is introduced to simplify the computation:

$$u(t) = \begin{cases} \left(u(0)-600\right)\exp(-0.005t)+600, \text{ when } t \le \tau \text{ (rising)} \\ \left(u(\tau)-100/3\right)\exp\left(-0.0018(t-\tau)\right)+100/3, \text{ when } \tau \le t \le T. \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(descending)} \end{cases} \tag{3}$$

τ is the switching time within the cycle and should satisfy $0 \le \tau \le T$, where T is the period of each cycle. It is easy to show that $u_{\text{end}}(k)$ is uniquely determined when $u_0(k)$ and τ are known.

As a result, the MA controller (when the variable $mode = 1$) could be written in the following function style:

$$\tau(k) = f\left(u_0(k), y_d(k)\right) \text{ with equivalent form}$$
$$u_0(k+1) = u_{\text{end}}(k) = f_u\left(u_0(k), y_d(k)\right). \tag{4}$$

The index k means the k^{th} cycle. Here, u_{stable} corresponds to y_d, as it denotes desired heater temperatures causing sheet temperatures to be equal to y_d (the relationship between u_{stable} and y_d is in Table I). Then (4) can be transformed as:

$$\tau(k) = g\left(u_0(k), u_{\text{stable}}\left(y_d(k)\right)\right) \text{ with equivalent form}$$
$$u_0(k+1) = u_{\text{end}}(k) = g_u\left(u_0(k), u_{\text{stable}}\left(y_d(k)\right)\right). \tag{5}$$

All of the function expressions do not contain a feedback of the sheet temperature $y(k)$, showing that the MA controller is an open-loop controller.

Now, selecting the switching time τ for each cycle becomes the core of the newly introduced controller. Since $u_0(k)$ and $u_{\text{stable}}(k)$ are known at the beginning of the k^{th} cycle, and the expressions of the two available kinds of curves are determined, a unique $u_{\text{end}}(k)$ will come out based on each determined $\tau(k)$. In order to reach the best control results, $\tau(k)$ is selected so that the difference between $u_{\text{end}}(k)$ and $u_{\text{stable}}(k)$ is reduced to a minimum (theoretically, zero). MATLAB optimal toolbox command *fsolve.m* can be applied to solve the equivalent optimization nonlinear problem:

$$\begin{cases} u_\tau(k) - u_{\text{stable}}(k) = 0 \\ u_\tau(k) = g_\tau^{-1}\left(u_0(k), \tau(k)\right) \end{cases}. \tag{6}$$

The usage of fsolve.m has the form:

$$x = fsolve\left(fun(x), x_0\right). \tag{7}$$

x_0 is the initial value of x before the iteration process; when the error is less than the tolerance, x solved for the equation $fun(x) = 0$. Besides, the two possible cases of MA (heater temperatures are not able to rise or descend to the desired range by the end of the current cycle period) are discussed and examined before the usage of fsolve.m.

Note that computed values of τ could not be directly applied as the control command signal; it is necessary to add a step to transfer τ into desired heater temperature data at every second of the corresponding cycle. Due to the saturation block within each heater element and the direct control property of MA, the given control signal source for MA could be generated as in Fig. 4, which depends upon τ.

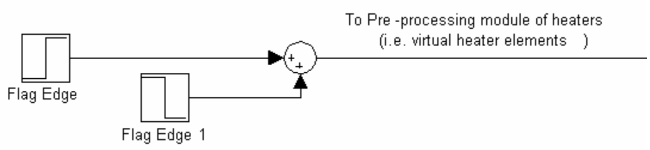

Fig. 4: Decoupled Forced Control Signal Source for MA

Both the positive "Flag Edge" and the negative one give step signals with high absolute values, and the negative step signal happens at the τ^{th} second. Such a compound signal will satisfy the saturation block—upper bound before τ and lower bound after τ, and it will be transferred by the pre-processing module into the curves we desire, which are described by (3).

C. Design of the Converter Module

In the previous parts, we have defined two modes, the switching conditions, and the MA controller computing $\tau(k)$. However, as the industrial production requires, the input variable given by the operator should be sheet temperatures rather than heater temperatures. Therefore, it is necessary to set up a converter before the mode switching module. The function of the converter should be to transfer sheet temperatures to corresponding desired heater temperatures. Since the principle of cyclic production is "one sheet per cycle", the converter operates at the beginning of each cycle, the input of which is thus treated as the desired temperature of the current sheet.

Now we need to establish the converting rules, and two methods are adoptable. The first method is based on the definition of modeling: treating the output (corresponding desired heater temperature) as a constant heater temperature able to keep the sheet stable at given converter input (desired sheet temperature) when the time goes to the infinity. We define $u_{\text{stable}}(k)$ as $u_{d\_0}(k)$, which is computed following the instructions above, in order to distinguish it from the counterpart computed in the second method. Then, a general modeling experiment can be introduced to set up the relation between $y_d(k)$ and $u_{\text{stable}}(k)$, and the result is as follows:

Table I: Data of desired sheet temperatures y_d and the corresponding heater temperatures u_{stable} which makes actual sheet temperatures stable at y_d

y_d	u_{stable}	
	Edge	Center
120	151.4124	139.2994
130	162.4644	149.4672
140	173.5164	159.6351
150	184.2000	169.4640
160	194.8836	179.2929
170	205.4935	189.0540
180	215.8824	198.6118

The linear relationship can be computed with the MATLAB command "polyfit.m":

$$\begin{bmatrix} u_{\text{stable\_edge}} \\ u_{\text{stable\_center}} \end{bmatrix} = \begin{bmatrix} 1.075 \\ 0.989 \end{bmatrix} y_d + \begin{bmatrix} 22.3510 \\ 20.5629 \end{bmatrix}. \tag{8}$$

With such a linear converter, the input signal (desired sheet temperature y_d) can be transformed into the desired value of heater u_{d_end} regarding the end of each cycle. Every u_{d_end} is valid throughout the whole cycle, during which the subsequent PID control loop produces satisfactory sheets, or the MA controller adjusts the heater to the desired temperature. However, this linear converter has flaws in practice described in the conclusion part, and the second method is introduced as a solution, which will be presented in a future publication.

D. Structure of the Dual Mode Controller

Now the Dual-Mode Controller is completely formed:

1) At (before) the beginning of each cycle (specified as the k^{th} cycle), the values of $u_{\text{stable}}(k) = u_{\text{stable}}(y_d(k))$, containing both $u_{\text{stable\_edge}}(k)$ and $u_{\text{stable\_center}}(k)$, are computed according to the linear relationship in (8) and the desired sheet temperature constant $y_d(k)$ of the k^{th} cycle.

2) Then, the difference between $u_{\text{stable}}(k)$ and the current heater temperature $u_0(k)$ at the beginning of the k^{th} cycle, that is, $u_0(k) - u_{\text{stable}}(k)$, is computed for every heater. If $u_0(k) - u_{\text{stable}}(k)$ of all the heating elements is smaller than the desired tolerance, then the classical PID control mode (Mode 0) will be carried out for the k^{th} cycle. Otherwise, if $u_0(k) - u_{\text{stable}}(k)$ of some heater(s) exceeds the value of the desired tolerance, then the mode of adjustment (MA, Mode 1) will be selected for the k^{th} cycle.

3) The MA controller shown in (4) and (5) will be activated when MA is selected. The computed output $\tau(k)$ for each heater will be applied to the external signal source in Fig. 4, in order to compute ideal control signal $u_t(k)$ for every moment t within the cycle.

The complete structure of the dual-mode control system is finally designed as in Fig. 5. This figure shows that the dual-mode control system is structured as a **hybrid cascade-loop system** under the PID control mode, or a **hybrid open-loop system** under MA, since the inner loop of PID mode and the open loop of MA pass real-time signals, while selection of modes and the switching point τ happens at most once per cycle. The legend for Fig. 5 is as follows:

1) Colors: light green flows denote actual input signals, red flows denote actual output signals with their feedback routes, and purple diamonds denote mode selectors.

2) Flow patterns: Wide, hollow arrows denote values computed only once per cycle (at/before the beginning of each cycle), and 4-line arrows denote real-time values.

According to the order of each function, the necessary judgments, and the boundary conditions, we can sketch the flow charts of the MATLAB program as Fig. 6, including the main script file and the MA controller function.

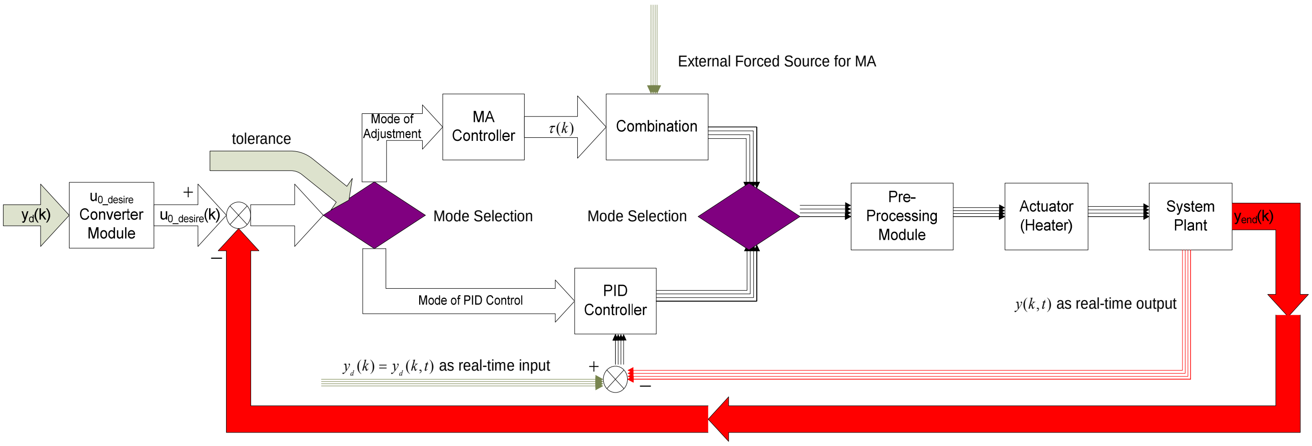

Fig. 5: Block Diagram of the Dual-Mode Controlled AAA Thermoforming Machine

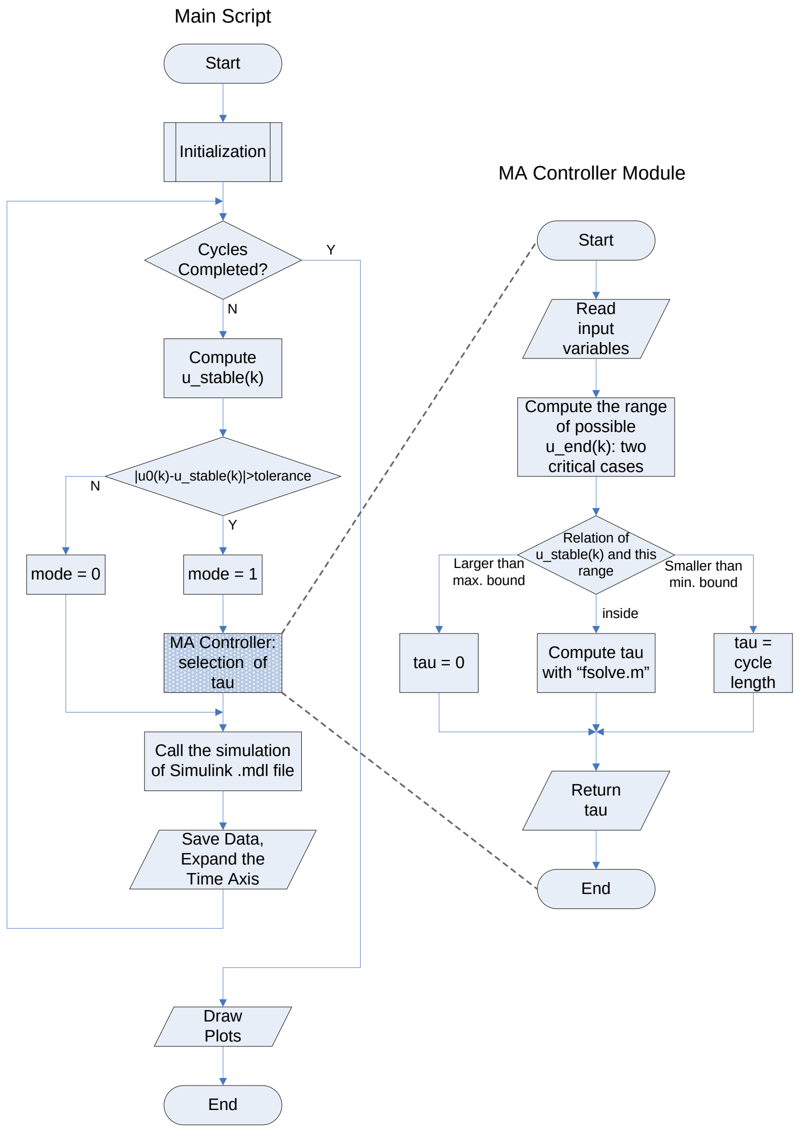

Fig. 6: Flow Charts of Script and MA Controller Function

III. Simulation Results

Now, we carry out a finite-time simulation using a script file designed according to Fig. 6. We fix the proportional parameter for the PID mode at K_p = 14, and use a varying desired sheet temperature at

$$y_d(k) = \begin{cases} 150, & k \le 5 \\ 170, & k \ge 6 \end{cases} \tag{9}$$

The temperatures are in degrees Celsius, and k denotes the corresponding cycle number.

Fig. 7 shows the results of the dual-mode control, especially the ability of switching between the two modes according to the given tolerance. The 3rd plot shows the mode switches with time. The read marks denote the switching point of cycles, at which the commands of modes are generated and become valid for the entire following cycle.

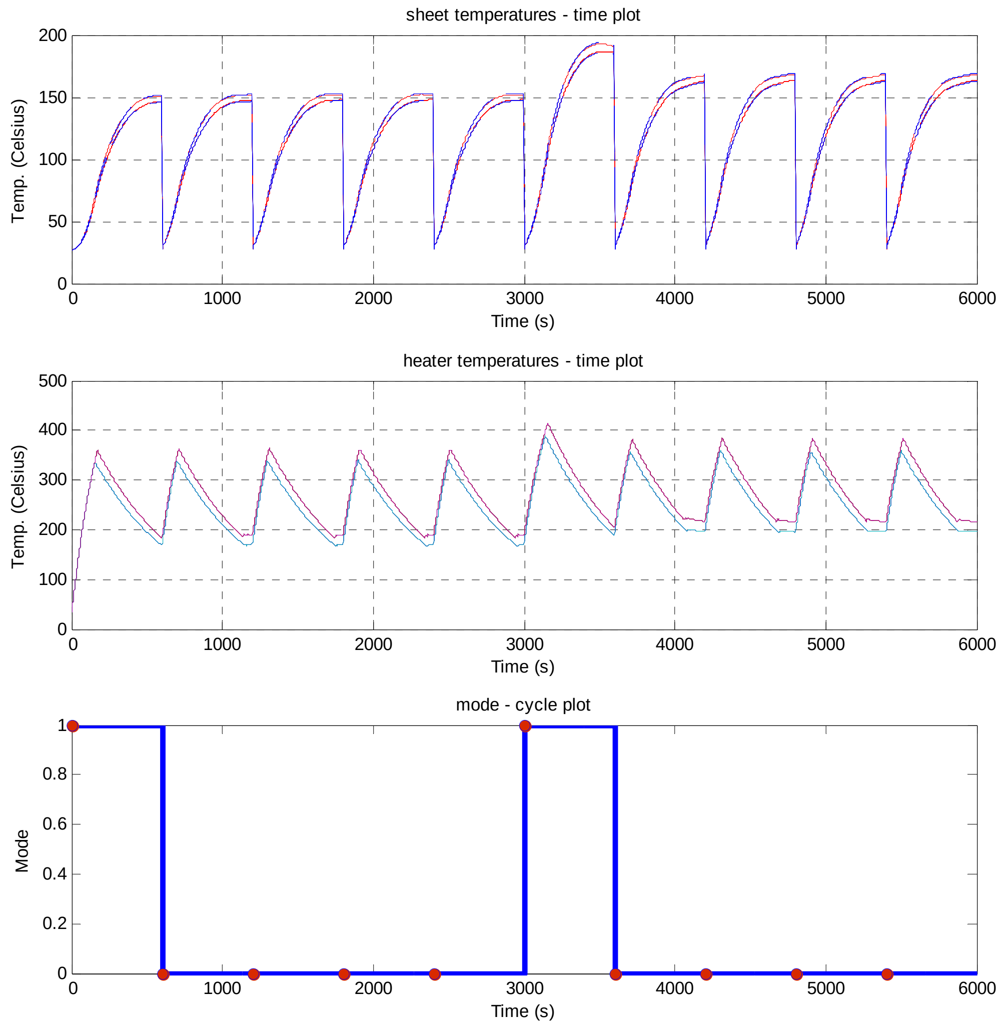

Fig. 7: Data Collected from a Dual-Mode Control Simulation with Fixed K_p = 14

It can be noticed that the examining module causes the controller to switch to MA at the beginning of the 6th cycle (when y_d changes from 150 to 170) and comes back to the PID control mode at the beginning of the next cycle. When an obvious change appears on desired sheet temperature y_d, the examining part of the program script will catch it, and switch to the MA (mode = 1) if any of the 12 heating elements satisfy

$$\left\| u_{\text{end}}(k) - u\left(y_d(k+1)\right) \right\| > \textit{tolerance}. \tag{10}$$

After a few cycle(s), the system will switch back to the original PID control mode (mode = 0). Since the batch production of sheets request the heaters generally to stay between 100ºC ~ 250 ºC, MA lasts only one cycle in most cases.

From another perspective, a MA cycle lasts at least 600 seconds if there should be one, however, which means a waste of sheets and time. Therefore, an improvement consisting of a flexible cycle length and a variable K_p is developed in order to reduce the opportunities of unnecessary mode switching caused by the actual nonlinear model and produce smaller errors of control results (sheet temperatures) under Mode 0.

IV. Conclusion

In summary, we presented for thermoforming reheat control a model of the heating elements, the notion of cyclic control, and a relevant dual-mode controller for heating plastic sheets. The simulation of the dual-mode controller with a fixed cycle length and K_p presents satisfactory performance. The constraints of industrial production require a reduction of heater temperature excursions and successful limitation of end temperature deviations (both sheets and heaters). The auto-correction succeeds in tracking operator's commands with errors in a small range by introducing MA cycles when it is necessary.

However, the present controller, as well as other modules within the cascade feedback loops, still need to be improved. The current plan needs at least an entire cycle, which is equal to 600s, to complete the heater adjustments; in other words, a long time of potential machine operation is wasted in this case. Besides, the present module converter does not perform very well, since there exists measurable difference between the temperature (of heater) at 600s and its final value (infinite time), and fixed proportional coefficient K_p of the PID controller can result in higher or lower sheet temperatures when the desired value is far from 150°C. An improved control system, containing variable cycle length, instant response, a different converter, and flexible PID controller, has been designed in order to overcome the problems above. It will be presented in a future publication.

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