Estimation of Scour Downstream of Spillways Using SVM Modeling

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***Abstract*— The problem of accurate prediction of the maximum depth, width and length of scour hole downstream of spillways has been based on the experimental studies and the equations developed are mainly empirical in nature. In this paper, prediction of the scour hole parameters like maximum depth, width and length downstream of a spillway using ski jump bucket type arrangement with linear regression and RBF & polynomial kernel based SVM technique have been attempted. The performance of different schemes was compared using two error criteria such as correlation coefficient and root mean square error. The study shows that RBF kernel based SVM scheme has emerged as the most satisfactory on the present data set as compared to the polynomial kernel based SVM model and the linear regression modeling.**

***Index Terms*— Spillways, Scour, Support vector machines, linear regression.**

I. Introduction

Prediction of maximum depth, width and length of scour with a reasonable accuracy is of immense importance for proper planning, design and management of hydraulic structures. Spillways like over-fall, ogee etc are provided for disposal of surplus water and to the control of water flow at the downstream channel. Scouring is a complex and dynamic phenomenon affected by many parameters those are often interrelated and difficult to understand because flow in open channel is turbulent; geometry is irregular and varies with time [1], [2]. There are various hydraulic, morphologic and geotechnical factors governing the depth, width and length of scour namely discharge intensity q, height of fall H_1, bucket radius R, bucket lip angle, phi Ø, type of rock, degree of rock homogeneity, run time and mode of operation of spillway etc. The literature review indicates that a regression mathematical model for predicting maximum depth, width and length of scour under all circumstances is not readily available using different flow, material and fluid parameters. However, deterministic models of varying degree of complexity have been employed in the past for modeling the scouring process, with varying degree of accuracy. The researchers have mainly relied on the conventional experimental approach to study the scouring by using physical modeling. Recently references [3], [4],[5] have applied soft computing modeling (ANN) for the prediction of scour parameters downstream of ski jump type spillway successfully. So far a few studies have reported the use of support vector machines (SVMs) for the scour prediction. Reference [6] examined the potential of support vector machines in the long-term prediction of lake water levels. Investigators [7] have successfully explored the usefulness of SVMs based modeling technique for predicting of real time flood stage forecasting on Lan-Yang river in Taiwan one to six hour ahead. However, literature review indicates that that no one has attempted SVM based modeling on the same data set. The present study aims to explore utility of the support vector machines for the scour hole parameters modeling on laboratory and field data and comparing its performance with linear regression for ski-jump type of spillways [3], [4], [5].

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II. Support Vector Machines (SVMs)

Support vector machines (SVMs) are classification and regression methods, which have been derived from statistical learning theory [8], [9]. The SVMs classification methods are based on the principle of optimal separation of classes. If the classes are separable - this method selects, from among the infinite number of linear classifiers, the one that minimize the generalization error, or at least an upper bound on this error, derived from structural risk minimization. Thus, the selected hyper plane will be one that leaves the maximum margin between the two classes, where margin is defined as the sum of the distances of the hyper plane from the closest point of the two classes [8]. The modeling techniques like support vector machines have the capability to reproduce the unknown relationship present between a set of input variables and the output of the system. Support vector machines performance was found to be better due to its use of the structural risk minimization principle in formulating cost functions and of quadratic programming during model optimization. These advantages lead to a unique optimal and global solution as compared to conventional neural network models.

If the two classes are non-separable, the SVMs tries to find the hyper plane that maximizes the margin and that, at the same time, minimizes a quantity proportional to the number of misclassification errors. The trade off between margin and misclassification error is controlled by a positive constant that has to be chosen beforehand. This technique of designing SVMs can be extended to allow for non-linear decision surfaces. This can be achieved by projecting the original set of variables into a higher dimensional feature space and formulating a linear classification problem in the feature

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space [9]. Support vector machines can be applied to regression problems and can be formulated as below:

Reference [8] has proposed ε-support vector regression (SVR) by introducing an alternative ε-insensitive loss function. The purpose of the SVR is to find a function having at most ε deviation from the actual target vectors (y_i) for all given training data and have to be as flat as possible [10]. This can be put in other words as the error on any training data has to less than ε. For a given training data with k number of samples, represented by $(\mathrm{x}_1, y_1), \ldots\ldots\ldots\ldots, (\mathrm{x}_k, y_k)$ and a linear function

$$f(\mathrm{x}) = \langle \mathrm{w}, \mathrm{x} \rangle + d \tag{1}$$

where $w \in \mathrm{R}^N$ and $d \in \mathrm{R}$. Set $\langle \mathrm{w}, \mathrm{x} \rangle$ represents the dot product in space R^N and N is the dimension of input space. A smaller value of w indicates the flatness of (1), which can be achieved by minimizing the Euclidean norm as defined by $\|\mathrm{w}\|^2$ [10].

Thus, an optimization problem for this can be written as: minimize $\frac{1}{2}\|\mathrm{w}\|^2$

subject to

$$\begin{cases} y_i - \langle \mathrm{w}, \mathrm{x}_i \rangle - d \leq \varepsilon \\ \langle \mathrm{w}, \mathrm{x}_i \rangle + d - y_i \leq \varepsilon \end{cases} \tag{2}$$

The optimization problem in (2) is based on the assumption that there exists a function that provides an error on all training pairs which is less than ε. In real life problems, there may be a situation like one defined for classification by [11]. So, to allow some more error slack variables ξ, ξ' can be introduced and the optimization problem defined in Equation 1 can be written as:

$$\text{Minimize } \frac{1}{2}\|\mathrm{w}\|^2 + C\sum_{i=1}^{k}\left(\xi_i + \xi_i'\right) \tag{3}$$

Subject to

$$y_i - \langle \mathrm{w}, \mathrm{x}_i \rangle - d \leq \varepsilon + \xi_i \tag{4}$$

$$\langle \mathrm{w}, \mathrm{x}_i \rangle + d - y_i \leq \varepsilon + \xi_i' \tag{5}$$

and ξ_i , $\xi_i' \geq 0$ for all i = 1, 2,……, k.

The parameter C is determined by the user and it determines the trade-off between the flatness of the function and the amount by which the deviations to the error more than ε can be tolerated. The optimization problem in (3) can be solved by replacing the inequalities with a simpler form determined by transforming the problem to a dual space representation using Lagrange multipliers

λ_i , λ_i' , η_i , η_i' i = 1,….,k [12].

The prediction problem can finally be written as

$$f(\mathrm{x}, \alpha) = \sum_{i=1}^{k}\left(\lambda_i' - \lambda_i\right)\langle \mathrm{x}_i, \mathrm{x} \rangle + d \tag{6}$$

This technique can be extended to allow for non-linear support vector regression by introducing the concept of the kernel function [8]. This is achieved by mapping the data into a higher dimensional feature space, thus performing linear regression in feature space. The regression problem in feature space can be written by replacing $\mathrm{x}_i \cdot \mathrm{x}_j$ with $\Phi(\mathrm{x}_i) \cdot \Phi(\mathrm{x}_j)$.

where

$$K(\mathrm{x}_i, \mathrm{x}_j) \equiv \Phi(\mathrm{x}_i) \cdot \Phi(\mathrm{x}_j) \tag{7}$$

The regression function for this can be written as:

$$f(\mathrm{x}, \alpha) = \sum_{i=1}^{k}\left(\lambda_i' - \lambda_i\right) K(\mathrm{x}_i , \mathrm{x}) + d \tag{8}$$

III. Data Set Used

The data set [5] were used in the present study from the laboratory and the field for an ogee spillway having ski jump bucket type energy dissipation arrangement at the toe. This data set has been given in Appendix. The data set comprises of a total of 95 runs with a discharge intensity per unit with q, upstream Head H_1, bucket radius R, angle of bucket phi, downstream tail water depth d_w , length of scour hole l_s, width of scour hole w_s and depth of scour hole d_s (where m means meter) on different types of spillway models in lab as well as in the field.

IV. Performance Evaluation

Much success has already been achieved using neural network algorithms in other applications, such as rainfall-runoff modeling, stage-discharge analysis and prediction of dimensions of scour hole downstream of a ski jump bucket[3],[4],[5], [13],[14], [15]. The Neural networks are now being applied to several other problems related to the hydraulics and hydrologic modeling, while the use of support vector machines is comparatively new to the field of hydraulics and water resource engineering [6], [7]. One of the important factors in using support vector machines for prediction of scour is that it requires setting up of the few user-defined parameters. The SVMs, in addition to the choice of kernel require setting up of kernel specific parameters. The optimum values of the regularization parameter C and the size of the error-insensitive zone ε need to be determined. To select user-defined parameters i.e. (C, γ and d*) a large number of trials were carried out by using different combination of these parameters on each of the data sets. Similarly, a number of trials were also carried out to find a suitable value of ε (error-insensitive zone) with a fixed value of C and kernel specific parameters. To reach at a suitable choice of these parameters, the correlation coefficients (CC) and Root Mean Square Error (RMSE) were compared and a combination of parameters providing smallest value of RMSE and the highest value of correlation coefficient was selected for the final results. A number of trials were carried out with different data set to select a suitable value of regularization parameter C. Variation in the error-insensitive zone ε have no effect on the predicted scour so a value of 0.0010 was chosen for all experiments.

The data sets from references [3], [4], [5] was used in the present study for model building and validation to assess the potential linear regression based modeling and RBF & Poly based kernels support vector machines in predicting the scour parameters downstream of spillway. Further, measured scour values were plotted against the computed values obtained with linear and SVMs algorithms. To study the scatter around the line of perfect agreement (i.e. a line at 45 degrees) was also plotted for the data set. Due to the availability of small data sets, a cross validation was used to train and test the performance of the SVMs. The cross-validation is a method of estimating the accuracy of a classification or regression model. The input data set is divided into several parts (a number defined by the user), with each part in turn used to test a model fitted to the remaining parts. In this study, the data sets of the laboratory and field data were used for both creating and testing the models. The choice of input parameters used in modeling the scour may influence the predicting capabilities of support vector machines. Graphs have been plotted for difference in actual and predicted values of the scour results as shown in Figure 1, 2 and 3.

V. PREDICTION OF SCOUR HOLE PARAMETERS

The first set of analysis was carried out by using SVMs and linear regression with data [6] for predicting the scour on downstream of spillway. The SVMs determine a relationship (i.e. create a model) between the input and the output of the available data set of any system. These models are than used to predict the output from the known input values of the same system, thus requiring sufficient number of data to create and test the models. Six parameters namely upstream head H1(m), radius of ski jump bucket R(m), angle of ski jump bucket phi, tail water depth dw (m), length of scour hole ls (m), width of scour hole ws(m) and depth of scour hole ds(m) from the data sets provided by the studies carried out by [3],[4],[5] were used to predict the scour. A 10 fold cross validation was used to create and test the models. A number of trials were carried out to reach at the various user-defined parameters required for The SVMs using both polynomial and RBF kernels and linear regression based algorithms using WEKA software. Measured versus calculated values of the maximum scour depth, width and length are plotted as given in Figures 1, 2 and 3 respectively.

The Table I, II provides the value different user defined parameters and correlation coefficients, RMSE for the data set. For maximum depth of scour, a correlation coefficient of 0.9502 and 0.7163 (RMSE = 0.0306 & 0.070) are obtained by using RBF and polynomial kernels respectively in comparison to a value of 0.6610 (RMSE = 0.0746) achieved by using linear regression based modeling (Table II). Further, it is evident from Figure 1 that more number of points are lying on the 45^{o} line when rbf kernel was used to predict the scour in comparison to polynomial kernel and linear regression based algorithm.

The values of correlation coefficient of 0.9201 and 0.7515 (RMSE = 0.1654 & 0.2871) are achieved with RBF and polynomial kernels respectively for maximum width of scour hole. In comparison to this Table II provides the value of correlation coefficient (i.e. 0.7459) and RMSE (i.e.0.2803) with this data using linear regression approach. A perusal of the Figure 2 indicates a better performance by rbf kernel based SVMs for this data set.

A higher value of correlation coefficient of 0.9803 and 0.9401 (RMSE = 0.0998 & 0.1729) are achieved with RBF and polynomial kernels respectively for maximum length of scour hole. In comparison to this Table II provides the value of correlation coefficient (i.e. 0.9369) and RMSE (i.e.0.1770) with this data using linear regression approach. A perusal of the Figure 3 indicates a better performance by rbf kernel based SVMs for this data set. The rbf and polynomial based SVM and linear regression is performing best when length of scour hole is to be predicted. Thus critical examination of three figures indicates that the performance of the SVM modeling is data dependent.

TABLE I. VALUES OF KERNEL SPECIFIC PARAMETERS OF SVM MODELING

S.No	Type of parameter	SVM(RBF)		SVM(POLY)	
		C	γ / d*	C	γ / d*
1	Scour depth d_s	5	0.2	0.1	0.1
2	Scour width w_s	5	5	0.1	2
3	Scour length l_s	5	1	5	2

TABLE II. COMPARISON OF RESULTS

S No	Scour hole parameter	Type of technique	Correlation coefficient (r)	Root mean squared error (RMSE)
1	d_s	SVM(RBF)	0.9502	0.0306
2		SVM(POLY)	0.7163	0.0700
3		Linear	0.6610	0.0746
4	w_s	SVM(RBF)	0.9201	0.1654
5		SVM(POLY)	0.7515	0.2871
6		Linear	0.7459	0.2803
7	l_s	SVM(RBF)	0.9803	0.0998
8		SVM(POLY)	0.9401	0.1729
9		Linear	0.9369	0.1770

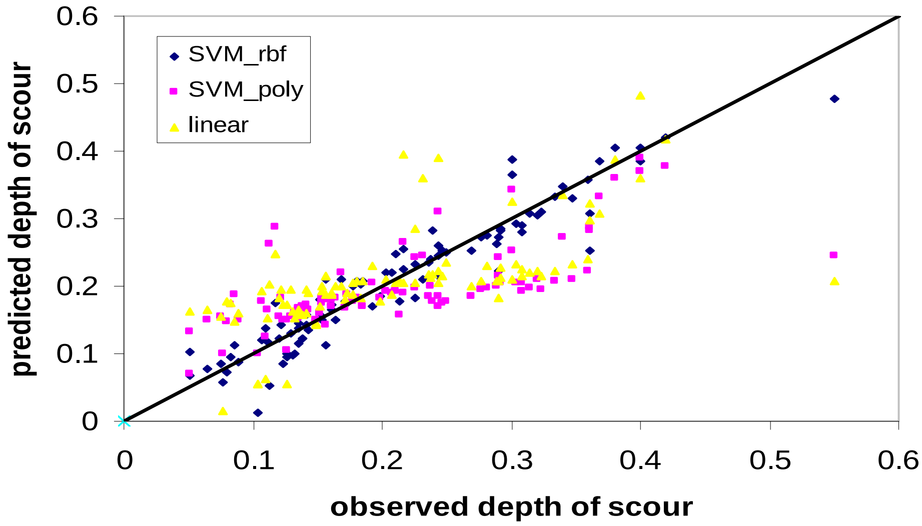

Fig.1Observed scour versus predicted scour depth

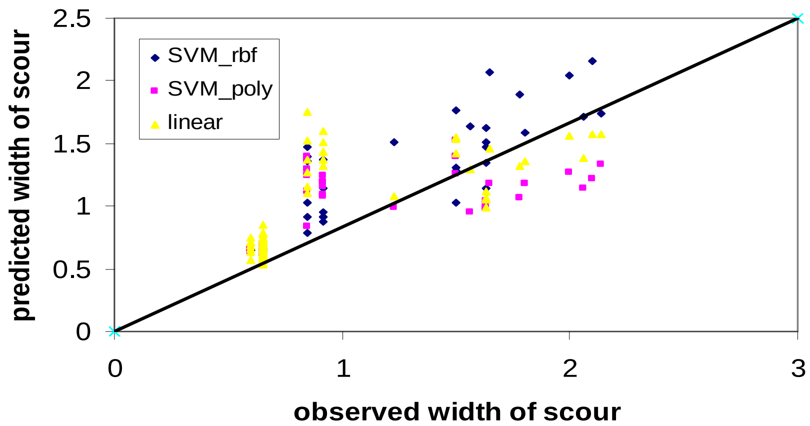

Fig.2 Observed scour versus predicted scour width

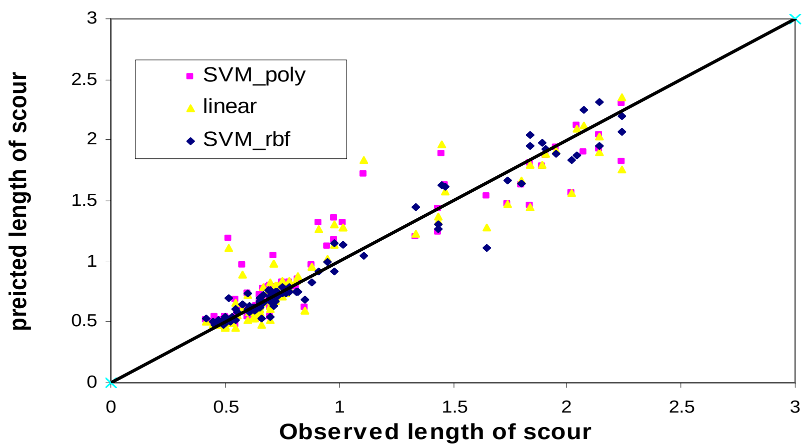

Fig.3 Observed scour versus predicted scour length

VI. Conclusions

This study was carried out to judge the potential and suitability of SVMs and the linear regression based modeling the scour on downstream of spillway using laboratory as well as field data sets. A conclusion from this study is that the RBF based SVM technique is yielding better results as compared to polynomial kernel based SVM and the linear regression with this data as indicated by the higher values of correlation coefficients and smaller values of root mean square values. The findings of this study, encourages the use of RBF kernel based SVM approach in predicting the scour on spillways downstream of a ski-jump bucket used in water resources projects, although the results are data dependent.

Appendix

Spillway Scour Data Set

S.No	q	H_1(m)	R(m)	phi	dw (m)	ls(m)	ws(m)	ds(m)
1	0.1703	0.5083	0.4	0.472	0.1667	1.1116	0.85	0.55
2	0.1792	1.4268	0.406	0.612	0.23	1.9512	0.85	0.2439
3	0.0842	1.4268	0.609	0.698	0.15	2.02	0.92	0.2246
4	0.0634	1.1328	0.406	0.612	0.03	0.9807	1.63	0.1128
5	0.0266	1.3659	0.61	0.698	0.17	0.9756	0.92	0.1259
6	0.1616	1.7962	0.254	0.349	0.2337	1.9055	1.5	0.3608
7	0.0709	1.4146	0.61	0.698	0.16	1.7378	0.92	0.1922
8	0.0204	0.3505	0.18	0.524	0.0286	0.697	0.6	0.1218
9	0.0374	0.3328	0.14	0.524	0.0687	0.72	0.6	0.236
10	0.0093	1.0718	0.406	0.612	0.234	0.5742	1.63	0.0762
11	0.1239	1.3659	0.406	0.612	0.18	1.4634	0.85	0.1677
12	0.1446	1.3902	0.406	0.126	0.265	1.6463	0.85	0.2165
13	0.0399	1.3902	0.61	0.698	0.18	1.4329	0.92	0.1485
14	0.0471	0.3827	0.14	0.524	0.0286	0.75	0.6	0.347
15	0.0204	0.3104	0.18	0.524	0.0687	0.5	0.6	0.0889
16	0.0204	0.2991	0.14	0.524	0.1	0.53	0.65	0.1235
17	0.0186	1.0822	0.406	0.612	0.215	0.7165	1.23	0.1037
18	0.0285	0.3188	0.14	0.524	0.0687	0.63	0.6	0.1609
19	0.1616	1.7962	0.254	0.78	0.2337	2.0709	1.5	0.3608
20	0.0471	0.3676	0.14	0.524	0.0437	0.7	0.6	0.3238
21	0.0089	1.3415	0.61	0.698	0.178	0.5183	0.92	0.0512
22	0.0725	1.3415	0.406	0.612	0.09	0.9146	0.85	0.0854
23	0.025	1.0922	0.406	0.612	0.25	0.8781	1.63	0.1098
24	0.1616	1.7962	0.254	0.174	0.2337	1.4482	1.5	0.2998
25	0.1626	1.4146	0.406	0.612	0.248	1.8902	0.85	0.2317
26	0.087	1.1532	0.406	0.612	0.033	1.0162	1.63	0.1169
27	0.1616	1.7962	0.254	0.523	0.2337	2.1439	1.5	0.2998
28	0.0204	0.3354	0.1	0.524	0.0437	0.495	0.65	0.136
29	0.0398	1.3902	0.61	0.698	0.18	1.4329	0.92	0.1485
30	0.0285	0.3589	0.25	0.567	0.0286	0.65	0.65	0.1642
31	0.0435	1.1125	0.3	0.612	0.248	0.9502	1.63	0.1113
32	0.0374	0.3328	0.25	0.567	0.0687	0.7	0.65	0.1772
33	0.0374	0.3015	0.25	0.567	0.1	0.67	0.65	0.1516
34	0.0374	0.3015	0.25	0.567	0.1	0.65	0.6	0.2135
35	0.0471	0.3827	0.25	0.567	0.0286	0.82	0.65	0.3085
36	0.0285	0.3188	0.25	0.567	0.0687	0.64	0.65	0.1432
37	0.0204	0.2991	0.25	0.567	0.1	0.455	0.65	0.0512
38	0.0285	0.2875	0.3	0.612	0.1	0.55	0.65	0.157
39	0.1532	1.075	0.56	0.611	0.146	1.84	2.06	0.38
40	0.0511	0.965	0.56	0.611	0.146	1.34	1.56	0.29
41	0.2042	1.13	0.56	0.611	0.146	2.04	1.65	0.4
42	0.1021	1.03	0.56	0.611	0.146	1.8	1.78	0.34
43	0.2042	1.474	0.56	0.611	0.146	2.24	2.14	0.42
44	0.1532	1.485	0.56	0.611	0.146	2.144	2.1	0.4
45	0.0511	1.505	0.56	0.611	0.146	1.84	1.8	0.29
46	0.1021	1.5	0.56	0.611	0.146	2.24	2	0.368
47	0.0285	0.3589	0.18	0.524	0.0286	0.65	0.65	0.1725
48	0.0374	0.3578	0.14	0.524	0.0437	0.71	0.65	0.2112
49	0.0471	0.3113	0.14	0.524	0.1	0.6	0.65	0.2459
50	0.0285	0.2875	0.18	0.524	0.1	0.63	0.65	0.1297
51	0.0374	0.3578	0.2	0.524	0.0437	0.725	0.65	0.2032
52	0.0471	0.3827	0.18	0.524	0.0286	0.78	0.65	0.3199
53	0.0471	0.3676	0.18	0.524	0.0437	0.775	0.65	0.3036
54	0.0204	0.3354	0.1	0.524	0.0437	0.495	0.65	0.136
55	0.0285	0.2875	0.2	0.524	0.1	0.62	0.65	0.1207
56	0.0285	0.3438	0.18	0.524	0.0437	0.65	0.65	0.1607
57	0.0471	0.3426	0.18	0.524	0.0687	0.78	0.65	0.2808
58	0.0374	0.3328	0.18	0.524	0.0687	0.7	0.65	0.181
59	0.0374	0.3578	0.18	0.524	0.0437	0.71	0.65	0.2172

60	0.0471	0.3113	0.1	0.524	0.1	0.7	0.65	0.2394
61	0.0204	0.3505	0.2	0.524	0.0286	0.525	0.65	0.0816
62	0.0471	0.3426	0.1	0.524	0.0687	0.72	0.65	0.3153
63	0.0374	0.3015	0.14	0.524	0.1	0.7	0.65	0.1848
64	0.0285	0.3438	0.2	0.524	0.0437	0.65	0.65	0.1542
65	0.0285	0.3589	0.14	0.524	0.0286	0.58	0.65	0.1986
66	0.0204	0.3354	0.2	0.524	0.0437	0.47	0.65	0.0752
67	0.0204	0.3104	0.1	0.524	0.0687	0.45	0.65	0.135
68	0.0204	0.3505	0.14	0.524	0.0286	0.5	0.65	0.139
69	0.0285	0.2875	0.14	0.524	0.1	0.6	0.65	0.1405
70	0.0471	0.3827	0.1	0.524	0.0286	0.815	0.65	0.3587
71	0.0374	0.3729	0.2	0.524	0.0286	0.75	0.65	0.2263
72	0.0285	0.3589	0.1	0.524	0.0286	0.61	0.65	0.2065
73	0.0471	0.3426	0.2	0.524	0.0687	0.72	0.65	0.2693
74	0.0471	0.3676	0.2	0.524	0.0437	0.76	0.65	0.292
75	0.0204	0.3304	0.14	0.524	0.0687	0.5	0.65	0.1309
76	0.0204	0.3354	0.18	0.524	0.0437	0.66	0.65	0.1068
77	0.0285	0.3438	0.1	0.524	0.0437	0.605	0.65	0.1839
78	0.0471	0.3426	0.14	0.524	0.0687	0.67	0.65	0.3091
79	0.0471	0.3113	0.25	0.524	0.1	0.69	0.65	0.243
80	0.0204	0.3505	0.1	0.524	0.0286	0.49	0.65	0.1424
81	0.0374	0.3328	0.1	0.524	0.0687	0.66	0.65	0.2426
82	0.0471	0.3676	0.1	0.524	0.0437	0.73	0.65	0.3343
83	0.0204	0.3104	0.2	0.524	0.0687	0.5	0.65	0.0643
84	0.0285	0.3438	0.14	0.524	0.0437	0.65	0.65	0.1765
85	0.0285	0.3188	0.18	0.524	0.0687	0.65	0.65	0.1526
86	0.0204	0.2791	0.1	0.524	0.1	0.5	0.65	0.1255
87	0.0374	0.3729	0.14	0.524	0.0286	0.74	0.65	0.2885
88	0.0471	0.3113	0.18	0.524	0.1	0.765	0.65	0.2497
89	0.0285	0.3188	0.1	0.524	0.068	0.555	0.65	0.1706
90	0.0204	0.3354	0.14	0.524	0.0437	0.42	0.65	0.1325
91	0.0374	0.3015	0.18	0.524	0.1	0.85	0.65	0.156
92	0.0374	0.3578	0.1	0.524	0.0437	0.715	0.65	0.2755
93	0.0374	0.3729	0.18	0.524	0.0286	0.72	0.65	0.2382
94	0.0374	0.3729	0.1	0.524	0.0286	0.72	0.65	0.2915
95	0.0204	0.2791	0.18	0.524	0.1	0.55	0.65	0.0785

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