Proceedings of the World Congress on Engineering and Computer Science 2008
WCECS 2008, October 22 - 24, 2008, San Francisco, USA

Modeling A PV-Diesel-Battery Power System: An Optimal Control Approach

Siew Fang Woon *, Volker Rehbock †, Ahmad Agus Setiawan ‡

Abstract—**The optimal design and operation of hybrid power systems used in remote area electrification are difficult tasks due to a large variety of location specific factors. Several mathematical models have been proposed in literature, aiming to capture the behavior of hybrid power system and optimize its overall operating cost. In this paper, we review an existing optimal control model of a PV-diesel-battery hybrid power system. We then compare the characteristics of the model with several generic computer simulation tools. Finally, we identify the limitations of the model and propose several improvements for future development.**

Keywords: Hybrid power systems, optimal control, modeling, optimization.

1 Introduction

A hybrid power system describes a stand-alone electrical power system which incorporates conventional (i.e. hydrocarbon powered) generators, renewable energy supplies, and energy storage devices. Such systems are vital for electrification in remote areas, where grid-connected infrastructure is not available. A combination of one or more diesel generators with additional renewable energy resources and battery storage helps to significantly reduce fuel consumption when compared to traditional stand-alone diesel generators.

There are various computer programs available to simulate the hybrid power systems, such as HOMER, HYBRID2, INSEL, and RESAD [10], as well as an optimization model developed in [12]. However, most of these simulation tools, with the exception of [12], involve no or only very basic optimization algorithms. Also, the simulations are usually performed via a time series with fairly large time steps (typically in the order of one hour or more). On the contrary, Tiryono [15] formulated a continuous time dynamic model for the operation of a hybrid power system with the objective of minimizing a weighted sum of several terms reflecting various operating costs. He also developed a specialized optimal control algorithm to generate optimal operating schedules for this model. A critical review of the formulation in [15] will be given in the next section.

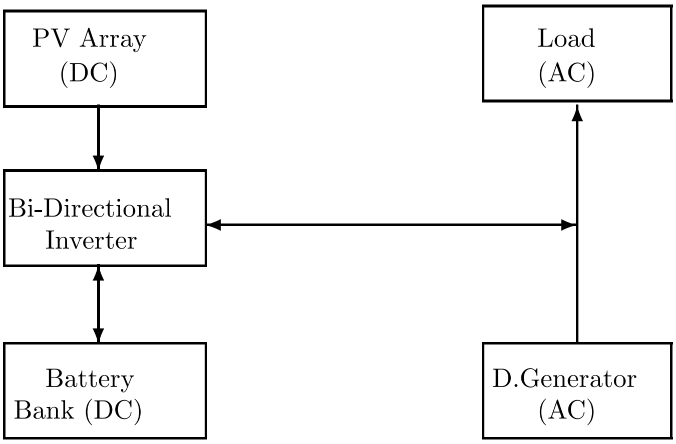

Figure 1: Schematic diagram of a hybrid power system

2 Hybrid Power System Formulation

Figure 1 illustrates the configuration of a hybrid power system modeled in [15], which uses a parallel topology. An alternating current (AC) diesel generator is coupled directly to the load to avoid converter losses and thus increase the efficiency of the system. Since PV array and battery bank are operated on direct current (DC), a bi-directional inverter, a device which converts DC to AC and vice versa, is used as an interface between the DC and AC parts of the system. The inverter also acts as a battery charger. Typically, a fraction of power is lost when passing through an inverter. For a 5 kW inverter, a conversion efficiency of 90% is assumed [1]. The diesel generator is assumed to be sufficiently large to handle the peak load demand. The assumed load demand profile is based on data provided by the Centre for Renewable Energy & Sustainable Technologies Australia (CRESTA), where the estimated power consumption at 15 minute intervals was collected for a day. Then, cubic spline interpolation

---

[*] Department of Mathematics and Statistics, Curtin University of Technology, Australia Tel/Fax: +61-8-92662942/3197 Email: woonsiewfang@yahoo.com

[†] Western Australia Centre of Excellence in Industrial Optimization, Curtin University of Technology, Australia Tel/Fax: +61-8-92663486/3197 Email: rehbock@maths.curtin.edu.au;

[‡] Department of Electrical & Computer Engineering, Curtin University of Technology, Australia Tel/Fax: +61-8-92661784/2584 Email: setiawan@ieee.org

is adopted to generate a continuous load demand profile. The estimated load demand is 340 kWh per day.

The hybrid power system model discussed in [15] assumes that the majority of the load demand is met by the diesel generator, with a PV array to supplement energy to the load during certain periods of the day. Diesel generators are popular for rural electrification due to their reliability, low initial cost, and ease of installation [15]. The operating cost of a diesel generator depends on its specific fuel consumption and maintenance cost. Frequent starts of the diesel generator from cold and running the generator for long hours at low load increase the engine wear. In addition, operating the diesel engine at low load significantly reduces the fuel efficiency. Typically, the minimum load of a diesel generator is set at 40% of its rated capacity in order to prevent glazing on the cylinder walls [16]. In other words, unless it is turned off, a diesel generator is assumed not to operate at less than 40% of its rated capacity to avoid low fuel efficiency.

A battery bank, which consists of lead acid batteries, is used to store excess energy and to supplement the load demand when needed. A battery bank of 100 kWh capacity was assumed in [15]. The charge of the battery bank is governed by

$$\dot{C}(t) = R(t) + D(t), \tag{1}$$

where the recharge rate is presented by

$$R(t) = \begin{cases} \dfrac{K_1 P_B(t)}{K_1 + C(t)}, & \text{if } P_B(t) \geq 0, \\ 0, & \text{if } P_B(t) < 0, \end{cases} \tag{2}$$

while the discharge rate is given by

$$D(t) = \begin{cases} K_2 P_B(t), & \text{if } P_B(t) < 0, \\ 0, & \text{if } P_B(t) \geq 0. \end{cases} \tag{3}$$

$C(t)$ refers to the capacity of the battery bank and $P_B(t)$ is the net power available at the battery bank. $P_B(t) > 0$ indicates that the battery bank is undergoing charging while $P_B(t) < 0$ implies that the battery bank is being discharged. The parameters K_1 and K_2 assume the use of lead acid batteries with the values of 250 and 1.4, respectively. The parameters assume that the charging efficiency at a near full battery state is just over 70% of the corresponding charging efficiency at a near empty battery state and 70% of power stored in the battery can be converted for load use, respectively.

The dynamics of the battery bank (4) is represented by one of the following modes:

- If the diesel generator produces excess power, the inverter directs the excess power from both the diesel generator and the PV arrays to the battery bank;
- If the power from the generator alone is insufficient to meet the load demand, a combination outputs of diesel generator and PV array are used to supply the load. In this case, the inverter directs a fraction of the output from PV array to meet the load demand while the excess is used to charge the battery bank;
- If the combined outputs of diesel generator and PV array fail to meet the load demand, the battery bank is discharged to make up the load demand.

Mathematically, these modes can be described as

$$\dot{C}(t) = \begin{cases} \dfrac{K_1 K_3 [P_R(t) + P_G(t) - P_L(t)]}{K_1 + C(t)}, \\ \quad \text{if } P_G(t) \geq P_L(t), \\[2ex] \dfrac{K_1 [K_3 P_R(t) + P_G(t) - P_L(t)]}{K_1 + C(t)}, \\ \quad \text{if } P_G(t) + K_3 P_R(t) \geq P_L(t), \\[2ex] K_2 \left[P_R(t) - \dfrac{P_L(t) - P_G(t)}{K_3} \right], \\ \quad \text{if } P_G(t) + K_3 P_R(t) < P_L(t), \end{cases} \tag{4}$$

$$C_{min} \leq C(t) \leq C_{max}, \quad \forall t \in [0, t_f], \tag{5}$$

and

$$C(t_f) = C_f, \tag{6}$$

where $P_R(t)$ is the power generated by PV array, $P_G(t)$ is the power produced by diesel generator, and $P_L(t)$ is the load demand.

The battery operating cost is associated with

$$\int_0^{t_f} (C(t) - K_4)^2 dt. \tag{7}$$

Term (7) measures the deviation of the charge state from a desired set point. Deviation from this set point is considered undesirable for two reasons. Firstly, extended deep cycle discharges are harmful to the battery. Secondly, a too high charge state may limit the battery's ability to store excess power when it becomes available. K_4 is set at 80 in [15].

In [15], the control function, $P_G(t)$, is restricted to a set of discrete values, because the generator is assumed to operate at only 0%, 40%, 60%, 80%, or 100% of capacity for ease of implementation of the optimal control policy. Thus, for a diesel generator with a 20 kW maximum capacity, we require $P_G(t) \in \{0, 8, 12, 16, 20\}$ and we are

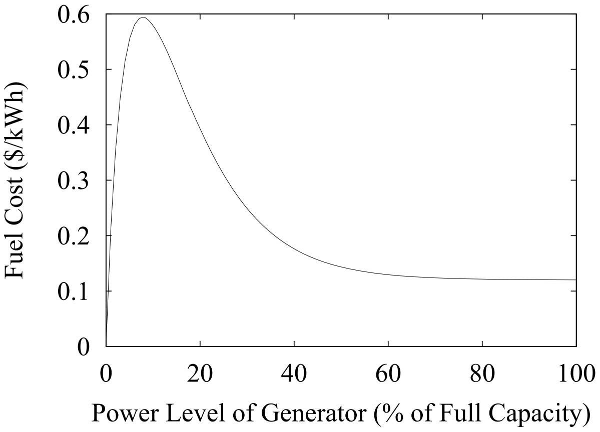

Figure 2: Approximation of the diesel generator fuel efficiency function.

dealing with a discrete valued control function. The operating cost of a hybrid power system is governed by

$$\int_0^{t_f} P_G(t) g_1(5P_G(t))dt, \tag{8}$$

where

$$g_1(x) = 2((0.2x+0.5)^{0.4} - 0.5^{0.4})e^{-0.1x} + 0.15(1 - e^{-0.1x}). \tag{9}$$

Term (9) is derived from Figure 2, which describes a typical relationship between the fuel consumption cost of a diesel generator and its operating level, based on information in [1].

Note that the objective terms (7) & (8) are formulated in such a way as to satisfy the differentiability requirements demanded by the optimal control software, MISER3.3 [5]. Both terms are to be minimized, subjected to the dynamics described by (4), (5) & (6). At this stage, we are dealing with a discrete valued optimal control and optimal parameter selection problem. An additional cost term which models frequent switches in the operating mode of the generator will be discussed in the next section, where a time scaling transformation is applied in order to transform the problem into a standard optimal control problem.

3 Control Parameterization Enhancing Technique

The control parameterization enhancing technique (CPET) initially developed in [7] transforms an optimal discrete-valued control problem into an equivalent optimal parameter selection problem. The transformed problem can then be solved by a standard optimal control software, such as MISER3.3 [5].

A new time horizon $[0, N)$ is defined with the partition $\{0, 1, 2, \cdots, N\}$. Assume $\tau \in [0, N)$. Consider the fixed control function $U_G(\tau)$ is defined by

$$U_G(\tau) = \begin{cases} 0, & \text{if } \tau \in [i, i+1), \text{ where } i \bmod 5{=}0, \\ 8, & \text{if } \tau \in [i, i+1), \text{ where } i \bmod 5{=}1, \\ 12, & \text{if } \tau \in [i, i+1), \text{ where } i \bmod 5{=}2, \\ 16, & \text{if } \tau \in [i, i+1), \text{ where } i \bmod 5{=}3, \\ 20, & \text{if } \tau \in [i, i+1), \text{ where } i \bmod 5{=}4. \end{cases} \tag{10}$$

Hence, $U_G(\tau)$ represents $P_G(t)$ in the transformed problem. Next, $u_{enh}(\tau)$, the enhancing control, is defined as a piecewise constant function consistent with the above partition and subjected to

$$0 \le u_{enh}(\tau) \le t_f. \tag{11}$$

$u_{enh}(\tau)$ transforms the original time horizon $[0, t_f)$ to the new time horizon $[0, N)$ through

$$\frac{dt}{d\tau} = u_{enh}(\tau), \quad \tau \in [0, N), \quad t(0) = 0, \tag{12}$$

with an additional constraint

$$t(N) = t_f. \tag{13}$$

Therefore, the original dynamics are transformed to

$$\dot{\bar{C}}(\tau) = \begin{cases} \dfrac{K_1 K_3 [P_R(t(\tau)) + U_G(\tau) - P_L(t(\tau))]}{K_1 + \bar{C}(\tau)} u_{enh}(\tau), \\ \quad \text{if } U_G(\tau) \ge P_L(t(\tau)), \\ \\ \dfrac{K_1 [K_3 P_R(t(\tau)) + U_G(\tau) - P_L(t(\tau))]}{K_1 + \bar{C}(\tau)} u_{enh}(\tau), \\ \quad \text{if } U_G(\tau) + K_3 P_R(t(\tau)) \ge P_L(t(\tau)) > U_G(\tau), \\ \\ K_2 \left[P_R(t(\tau)) - \dfrac{P_L(t(\tau)) - U_G(\tau)}{K_3} \right] u_{enh}(\tau), \\ \quad \text{if } U_G(\tau) + K_3 P_R(t(\tau)) < P_L(t(\tau)), \end{cases} \tag{14}$$

and

$$\dot{t}(\tau) = u_{enh}(\tau), \quad \bar{C}(0) = C_0, \quad t(0) = 0. \tag{15}$$

Similarly, the constraints (5) & (6) are also transformed to constraints (16) & (17), respectively,

$$C_{min} \le \bar{C}(\tau) \le C_{max}, \quad \forall \tau \in [0, N), \tag{16}$$

$$\bar{C}(N) = C_f. \tag{17}$$

After the transformation, the terms measuring the fuel cost and the battery operating cost are given by

$$\int_0^N U_G(\tau) g_1(5U_G(\tau)) u_{enh}(\tau) d\tau, \tag{18}$$

and

$$\int_0^N (\bar{C}(\tau) - K_4)^2 u_{enh}(\tau) d\tau. \tag{19}$$

As a third objective term, frequent switching of the diesel generator mode can be discouraged by minimizing

$$\int_0^N g_2(u_{enh}(\tau)) d\tau, \tag{20}$$

where $g_2(x) = ((x + 0.01)^{0.25} - 0.01^{0.25})e^{-5x})$ is a function that satisfies $g_2(0) = 0$ and $\lim_{x\to\infty} g_2(x) = 0$. It has sharp peak at about $x = 10$. Under these circumstances, short time intervals (less than 15 minutes) are severely penalized while only a small cost penalty should be imposed for longer time intervals (more than an hour).

The cost functional of the hybrid power system model can thus be summarized as

$$\begin{aligned} g_0 \;=\; \int_0^N & \{\alpha U_G(\tau) g_1(5U_G(\tau)) u_{enh}(\tau) \\ & + \beta(\bar{C}(\tau) - K_4)^2 u_{enh}(\tau) \\ & + \gamma g_2(u_{enh}(\tau))\} \, d\tau. \end{aligned} \tag{21}$$

We refer to this problem as Problem (P). Problem (P) is minimized, subjected to dynamics (14) & (15) and constraints (13), (16) & (17). Note that α, β, γ are the weighting coefficients of the fuel cost, variation of battery charge, and switching cost, respectively. Problem (P) can be readily solved by using MISER3.3. The optimal diesel generator profile of this model is demonstrated in [15]. However, these solutions are only local optima and efforts are being made to determine globally optimal solutions.

4 Comparison with Alternative Hybrid Power System Simulation and Optimization Tools

HOMER [6] is a popular tool for preliminary design of hybrid power systems. It uses simple strategies with strong emphasis on economic factors by selecting the most appropriate system components to obtain an optimal design of a hybrid system. On the other hand, HYBRID2 [9] concentrates more on the technical characteristics of hybrid power systems and is able to optimize the operating strategies as well [6]. Barley et al. [2] suggests the use of HOMER for a quick search to find the lowest life-cost of a hybrid power system from a range of possible operating strategies, whereas HYBRID2 is used to verify HOMER models for more accurate results. A third tool, implemented in the Matlab environment [4], was developed in [12]. This includes considerable details on various power flows, interaction of components, and applying genetic algorithms to optimize the choice of system components as well as broad aspects of the operating strategies. Several actual systems were simulated and optimized to demonstrate the applicability of their tool [12]. However, no further development or application of the tool has appeared in literature since [13].

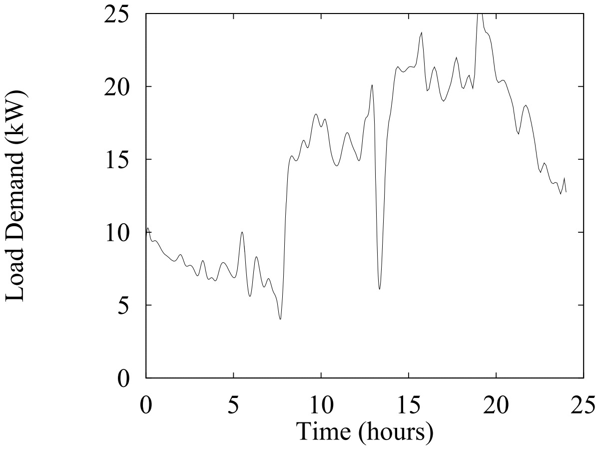

Figure 3: Load demand profile in [15].

A common feature of the above algorithms is that simulation is performed over relatively large time steps, typically at least 1 hour. This is done in order to simulate the system over at least several days to capture a variety of daily power demands and renewable power availability profiles. Smaller time steps would lead to excessively complex models under these circumstances. However, as can be seen from a typical load demand profile in Figure 3, there is significant variation in the model inputs over one hour period, and one would expect similar variation in the optimal operating schedules within this period.

While only a crude cost function was proposed in [15], the operating cost for the model was shown to vary significantly with respect to the switching times for the diesel generator and other time dependent parameters [11]. Such sensitivities would not have been captured in the above models with large time steps. In contrast to these models, [15] simulates a hybrid power system as a continuous time model that can capture the above mentioned variabilities. Fuel efficiency cost is represented by a nonlinear function in [15], while a linear relationship is used in both HOMER and HYBRID2. Next, both HOMER and HYBRID2 use the kinetic battery model to describe the charge and discharge rate, while [15] suggests a more basic formulation to represent these rates.

Calculating the total operating cost of a hybrid power system in HOMER and HYBRID2 is more comprehensive compared to [15], where only surrogate terms for real costs were suggested. HOMER calculates the total net present cost (NPC) of a system by incorporating the initial capital cost of the system components, replacement costs, maintenance costs, fuel costs, and costs of purchasing power from the grid. Likewise, HYBRID2 considers the fixed and marginal costs of the system components as well as the economic parameters, such as interest and inflation rates.

5 Limitations Of the Existing Model & Future Work

While the model in [15] is for a specific example of a hybrid power system, the structure of the model is relatively simple and can be adapted to other system configurations.

The first limitation of the model in [15] is that the battery dynamics in terms of recharge and discharge rates, represented by equations (1), (2) & (3), are too simple and backed by intuition only. We intend to adopt a more realistic kinetic battery model of [8] to measure the recharge and discharge behavior. This model has also been adopted in HOMER and HYBRID2 [6, 9]. According to the kinetic battery concept, a battery is divided into a two-tank system: an available energy tank and a bound energy tank. The available energy tank provides immediate energy for charging or discharging, while the rest is chemically bound in the latter. The conversion rate between these two tanks depends on the difference in 'height' between these tanks. The mathematical formulations that describe the kinetic battery model are

$$\frac{dq_1}{dt} = -I - k'\Big(\frac{q_1}{c} - \frac{q_2}{1-c}\Big) \tag{22}$$

and

$$\frac{dq_2}{dt} = k'\Big(\frac{q_1}{c} - \frac{q_2}{1-c}\Big), \tag{23}$$

where q_1 = available charge, q_2 = bound charge, k' is a fixed conductance, c is the width of the available energy tank, and I is the current.

We also intend to model the battery cost more realistically by relating the daily usage to the total lifetime. There are two common lifetime models for lead acid batteries, the post-processing models and the performance degradation models [3]. For the purpose of this paper, we discuss how to integrate the post-processing model into our optimal control problem only. We apply the Ah-throughput counting method to evaluate the lifetime consumption of the battery as the data of the total throughput is readily available in our optimal control formulation.

Ah-throughput assumes that there is a fixed amount of energy that can be cycled through a battery before it requires replacement. The estimated throughput is derived from [3],

$$throughput = Average\{E_{nom} D_i C_{F,i}\}_Y^X, \tag{24}$$

where E_{nom} is the nominal battery capacity, D_i refers to the specific depth of discharge being considered, $C_{F,i}$ is the number of cycles to failure to the specific depth of discharge, i represents each depth of discharge measurement, and X to Y is the range over which the measurements of depth of discharge are taken. Note that the relationship between the depth of discharge and the number of cycles to the failure curve is provided by the manufacturer. Based on [3], the total throughput over a variety of discharge depth is approximately constant for most lead acid batteries. To adopt the Ah-throughput into the control optimal formulation, note that

$$x(t) = \int_0^T \frac{|\dot{C}(t)|}{2} dt \tag{25}$$

captures the total throughput of the battery bank over a daily time horizon. The battery bank operating cost over this time is then modeled by

$$C_{BB} = \frac{x(t)}{T_{TP}} C_B, \tag{26}$$

where T_{TP} is the total throughput over a battery bank lifetime and C_B is the cost of a battery bank.

The second limitation is forecasting of load demand and renewable power profiles is not carried out in [15]. It would be interesting to include the predictions of the future load demand and the forecasts of solar resource or other renewable resources as part of the control strategy of a hybrid power system. Some of the forecasting issues related to the solar/wind resources are size of the PV/wind systems, daily temperature fluctuations, radiation forecasts, wind speed, humidity, ambient temperatures, observations of cloud cover and cloud movement, barometric pressure, and irradiation [17]. As for remote area electrification, size of the population, changes of consumer behavior, special community events, seasonal/short-term variation of environmental condition are among the factors which can bring significant changes to short-term and long-term load demand, as observed in [17]. Different load profiles, such as daily, weekly, or seasonal demand profiles on individual usage patterns should be considered when constructing a robust hybrid power system. Many of these issues can be built into our model if we extend the time horizon from a day to a week or more. While this will result in a more complex problem, we expect that solutions can still be obtained in a reasonable computational time.

Thirdly, the model in [15] only focuses on the operating strategy of a discrete value diesel generator. Further study on a wide range of generators, such as variable speed generators or continuous type generators, should be considered, where the output is not limited to discrete values.

Fourthly, the power from renewable energy, i.e. PV array (2.5 kW), is considered small compared to the diesel generator (20 kW), where the latter is the backbone of the energy supply. Nowadays, a system that is based primarily on renewable resources, with the diesel generator as a backup supply, is more realistic for long term usage due to increasing fuel costs and continually cheaper renewable power generation.

Next, formulation in [15] has neglected the initial setup cost of each component of the hybrid power system. It is vital to incorporate the initial capital cost of the components into the total cost of the hybrid power system to increase the efficiency of the system. This introduces discrete variables into the problem which complicate the optimization process considerably. Several algorithms in this regard have been proposed in the literature [13, 14].

6 Conclusions

An optimal control approach has been used in [15] to evaluate the differences in operating strategies and configurations during the design of a PV-diesel-battery model. However, [15] did not capture all realistic aspects of the hybrid power system. In this paper, the optimal control model is analyzed and compared with three different simulation and optimization programs. We propose several improvements to the current model to make it more representative of real systems. The revised model will be optimized by the existing optimal control software.

References

[1] M. Ashari. Optimisation of photovoltaic/diesel/battery hybrid power systems for remote area electrificatiom. Master's thesis, Department of Electrical and Computer Engineering, 1997.

[2] C. D. Barley, L. T. Flowers, P. J. Benavidez, R. L. Abergas, and R. B. Barruela. Feasibility of hybrid retrofits to off-grid diesel power plants in the Philippines. *Prepared for Windpower'99, Burlington, Vermont*, June 20-23, 1999.

[3] H. Binder, T. Cronin, P. Lundsager, J. F. Manwell, U. Abdulwahid, and I. Baring-Gould. Lifetime modelling of lead acid batteries. Technical report, Riso National Laboratory, Roskilde, Denmark, 2005.

[4] M. R. Guide. The MathWorks. *Inc., Natick, MA, http://www.mathworks.com/*, 1998.

[5] L. S. Jennings, M. E. Fisher, K. L. Teo, and C. J. Goh. *Miser 3.3–optimal control software: theory and user manual.* University of Western Australia, http://www.maths.uwa.edu.au/ les/MISER3.3/ch1.pdf, 2004.

[6] T. Lambert, P. Gilman, and P. Lilienthal. *Integration of alternative sources of energy, Chapter 15 Micropower system modeling with HOMER.* John Wiley & Sons, Inc., 2006.

[7] H. W. J. Lee, K. L. Teo, V. Rehbock, and L. S. Jennings. Control parametrization enhancing technique for optimal discrete-valued control problems. *Automatica*, 35(8):1401–1407, 1999.

[8] J. F. Manwell and J. G. McGowan. Lead acid battery storage model for hybrid energy systems. *Solar Energy*, 50(5):399–405, 1993.

[9] J. F. Manwell, A. Rogers, G. Hayman, C. T. Avelar, J. G. McGowan, U. Abdulwahid, and K. Wu. Hybrid2 - a hybrid system simulation model theory manual. Technical report, Renewable Energy Research Laboratory, Department of Mechanical Engineering, University of Massachusetts, June 30, 2006.

[10] R. K. Maskey and F. Nestmann. Hydro based renewable hybrid power system for rural electrification: A concept paper. Retrieved June 12,2008, from http://www.mtnforum.org/apmn/hybridconcept.htm.

[11] V. Rehbock and P. Souksomvang. Optimal reset time for hybrid power systems. In *Proceedings of the 18th National ASOR Conference & 11th Australian Optimisation Day, Perth, Australia*, 229–236, 2005.

[12] G. Seeling-Hochmuth. *Optimisation of hybrid energy systems sizing and operation control.* PhD thesis, University of Kassel, 1998.

[13] G. C. Seeling-Hochmuth. A combined optimisation concept for the design and operation strategy of hybrid-PV energy systems. *Solar Energy*, 61:77–87, 1997.

[14] R. Tiryono, W. R. Lee, and V. Rehbock. Optimal design and operation of hybrid power systems. In *Proceedings of Industrial Optimization Symposium, Perth, Australia*, 324–335, 2004.

[15] R. Tiryono, V. Rehbock, and W. B. Lawrence. Optimal control of hybrid power systems. *Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications & Algorithm*, 10:429–439, 2003.

[16] B. Wichert. *Control of photovoltaic-diesel hybrid energy systems.* PhD thesis, School of Electrical and Computer Engineering, Curtin University of Technology, 2000.

[17] B. Wichert and W. B. Lawrance. Photovoltaic resource and load demand forecasting in stand-alone renewable energy systems. In *2nd World Conference on Photovoltaic Solar Energy conversion, Vienna*, 1998.