Analysis and Design for Linear Singular Multirate Networked Control Systems¶

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Abstract—**This note proposes a multirate sampled-data system to model the discrete schedule feature of the networked singular control systems. Sampled-data system with time-delay in its discrete-time subsystem shows some nice features to admit many network-induced topics, for example, time-delay, packet dropout. Beside of the networked control systems with different combinations of the time-driven or event-driven mode of the devices, including the samplers, the controllers and the holders, this work concerns with the networks with multirate devices. By transforming the time-delay in the discrete-time subsystem into the continuous-time singular subsystem, a less conservative time-delay dependent stability result for the networked singular control systems is obtained. The stability condition guarantees that the networked singular system is regular, impulse free and stable. Furthermore, a stabilized control law for the networked singular systems is presented. Finally, a numerical example is provided to demonstrate the potential of the proposed techniques.**

Keywords: networked control system, singular system, stability, sampled-data system, multirate

1 Introduction

Nowadays, networks play an important role in many industrial control applications, especially for complex control systems and remote control systems. Comparing with the conventional point-to-point control systems, the networked control systems(NCSs) show many nice features, such as, flexibility of operation, ease of diagnosis and maintenance, small volume of wiring, low cost, and etc.. However, because of the limitation of the network resource shared by all devices of the control systems, time-delay caused by data transmission and/or packet drop will inevitably degrade control performance of the NCSs, or even cause the systems instable [7, 10, 12]. Recently, the regular systems combined with the network-induced time-delay, packet dropout, multi-packet transmission and the data transmission mechanism have received widely attentions, more details see ref.[5, 6, 7, 10, 12] and there in. What's more, certain additional problems appear when the communication is not always available. For example, the sampled data may be lost or damaged during the transmission such that the controller can not to generate and send the useful instruction at the given instant. Usually, a multirate solution is used to retransmit a new sampling or control data to the destination to improve the network reliability and safety. However, the analysis and synthesis problems for the networks with the devices working at different rates are still remained open.

On the other hand, singular systems, also referred to as descriptor systems, implicit systems, generalized state space systems, differential-algebraic systems, or semi-state systems [2], have an ability to better describe physical systems than regular ones. A great number of results based on the theory of regular systems (or state-space systems) have been extended to the area of singular systems [2, 11]. It should be pointed out that the stability problem for singular systems is much more complicated than that for regular systems because it involves some special issues such as regularity and absence of impulses (for continuous singular systems) and causality (for discrete-singular systems) at the same time [3].

Basically, the NCS is hybrid, which involves a continuous plant and event-driven or time-driven devices (digital controller, holder and sampler) and networks. Hybrid nature of NCSs makes the synthesis and analysis problems for NCSs difficult. This kind of systems is usually referred as sampled-data system which simultaneously contains continuous-time and discrete-time signals [1]. Conventionally, the lifting technique [1] is used as a tool to consider the synthesis and the analysis problems [10]. However, this technique only works for LTI systems. In [12], the authors considered the analysis problem for the NCSs under hybrid system framework. In the work [4], the authors proposed a robust control method by transforming the sampled-data system into a continuous-time system with control time-delay. However, this transformation is under a condition: the sampling period is infinite small. In our previous work [6], we considered the modeling and synthesis problems for the NCSs with the regular plants.

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However, the NCS considered are restrict to the one with single rate devices.

In this paper, a sampled-data system with time-delay in its discrete-time subsystem was presented for the networked singular control system with time-driven digital controller and event-driven holder device. All devices may work at multirate setting, but with a common frame period.The configuration shows that this model admits many network-induced issues, for example, time-delay, packet dropout, multi-packet transmission. Moreover, it allows different combinations of the time-driven or event-driven mode of the devices, including the samplers, the controllers and the holders. By transforming the time-delay in the discrete-time regular subsystem into the continuous-time singular subsystem, a time-delay dependent stability result is obtained for the networked singular control systems, which is the extension of the result in paper[6]. This stability result is less conservative by introducing a special equality, which involves the terms related to the states, the delayed states, the states derivative and an additive switched state weighted by four free matrices. Finally, a numerical example demonstrates the effectiveness of the proposed method.

Notations: Throughout this paper, for real symmetric matrix X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension, the superscript T represents the transpose, $\|x\|$ is the Euclidean norm of the vector x, while $\rho(M)$ denotes the spectral radius of the matrix M. $\{t_k, k = 1, 2 \ldots, \}$ are the controller switching instants satisfying $t_{k+1} - t_k = T_s$, where T_s denotes the controller switch period. $\frac{T_s}{n_s}$ denotes the sampling period with which outputs of a plant are synchronously measured by ideal samplers, that is, in the frame period T_s, the samplers measure the plant outputs n_s times.

2 Problem Formulation

Consider a NCS shown as in Fig.1, in which a continuous-

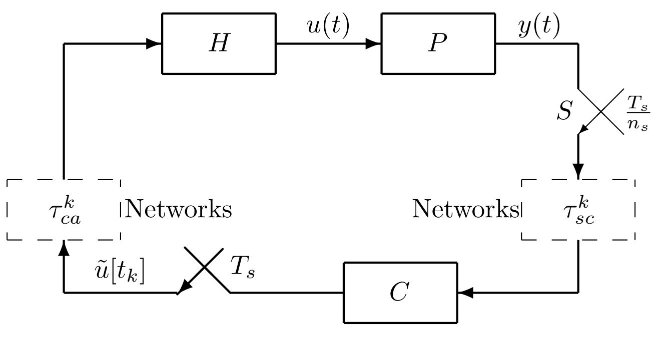

Figure 1: Schematic of Networked Control System

time regular or singular plant P is controlled by a digital controller C. The output signals $y(t)$ of the plant are synchronously measured with ideal samplers S at a sampling rate $\frac{n_s}{T_s}$. The digital controller uses the information of the plant transmitted through the networks to generate a digital control action $\tilde{u}[t_k]$. If the controller is failed to receive the measurements on time, it will request the consective measurements. The control action $\tilde{u}[t_k]$ is then transmitted via the networks again at the rate $\frac{1}{T_s}$, held by a zero-order holder H to drive the plant. In this formulation, the measurements $y(t)$ and the control action $\tilde{u}[t_k]$ may be delayed because of the network traffic. For the interval $(t_k, t_{k+1}]$, τ_{sc}^k and τ_{ca}^k denote the network-induced delays in the control input channels and the plant input channels, respectively. Here, the superscript k of τ_{sc} and τ_{ca} is used to describe time-varying nature of the time-delay. To well understand the net-

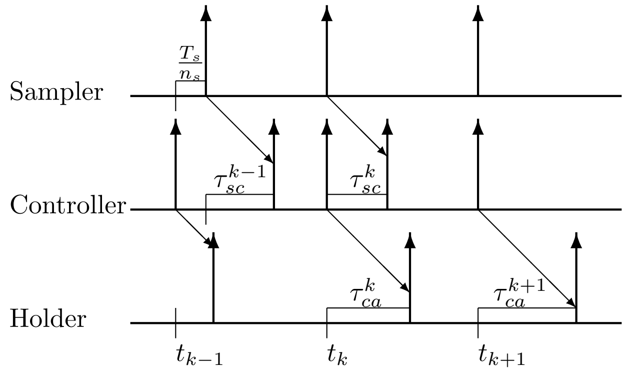

Figure 2: Timing Mechanism of Networked Control System

worked control system considered, its timing mechanism is shown in Fig. 2. In this paper, we assume that the digital controller and the sampler are working in time-driven mode, while the holder is working in event-driven modes. Moreover, each data transmitted can reach its destination within one sampling period T_s.

Consider the continuous-time singular plant described as

$$E\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

which is controlled by a digital state-feedback controller

$$\tilde{u}[t_k + \tau_{ca}^k] = Fx(t_{k-1+\frac{n_{k-1}}{n_s}}). \tag{2}$$

Recalling the fact, every control action $\tilde{u}$ is held by a zero-order holder and only valid over the interval $t \in (t_k + \tau_{ca}^k, t_{k+1} + \tau_{ca}^{k+1}]$, we have

$$u(t) = \tilde{u}[t_k + \tau_{ca}^k], \tag{3}$$

for $t \in (t_k + \tau_{ca}^k, t_{k+1} + \tau_{ca}^{k+1}]$, where $x(t) \in \mathbf{R}^n$ denotes the state vector of the plant, $u(t) \in \mathbf{R}^m$ denotes the control input vector. The matrix $E \in \mathbf{R}^{n \times n}$ may be singular, we assume that $\text{rank}(E) = r \leq n$. A, B and F are the known real matrices with appropriate dimensions.

Remark 1 *This sampled-data system model is adapted to describe the NCSs with the different network conditions, such as the variant time-delay τ_{sc}^k and τ_{ca}^k within*

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or longer than one sampling period, packet dropout and multi-rate sampling. Also, they may consider the NCSs with the time-driven or event-driven samplers, controllers and holders. Detailed discussion see [6] *for example.*

Definition 1 [2]

1. *The pair (E, A) is said to be regular if* $\det(sE - A)$ *is not identically zero.*
2. *The pair (E, A) is said to be impulse free if* $\deg(\det(sE - A)) = rankE$.

The continuous-time singular system (1) may have an impulsive solution, however, the regularity and the absence of impulses of the pair (E, A) ensure the existence and uniqueness of an impulse free solution to this system, which is shown in the following lemma.

Lemma 1 [11] *Suppose the pair (E, A) is regular and impulse free, then the solution to the continuous-time singular system (1) exists and is impulse free and unique on* $[0, \infty)$.

Definition 2 *The networked singular control system (1)-(3) is said to be*

1. *regular and impulse free if the pair (E, A) is regular and impulse free.*
2. *stable if, for any $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that, for any compatible initial conditions $\phi(t)$ satisfying* $\sup_{-\tau \le t \le 0} \|\phi(t)\| \le \delta(\varepsilon)$, *the solution $x(t)$ of system (1) satisfies $\|x(t)\| \le \varepsilon$ for $t \ge 0$. Furthermore $x(t) \to 0$, $t \to \infty$.*

Then the problems considered in this paper can be formulated as,

1. set up a stability result for the networked singular control system described by (1)-(3).
2. design a controller (2) such that the networked singular control system stable with possible maximum time-delay.

For self-completeness, some existing results are presented here for the following development.

Lemma 2 [8] *The singular system $E\dot{x}(t) = Ax(t)$ is regular, impulse free and stable if and only if there exists a matrix P such that $EP^T = PE^T \ge 0$, $AP^T + PA^T < 0$.*

3 Model Transformation

The system formulated above section is referred as a sampled-data one which contains continuous-time and discrete-time signals simultaneously with the network-induced delays τ_{sc}^k and τ_{ca}^k. Not like the conventional sampled-data systems where the controllers switch at the sampling instant t_k and the control action is held at the same time, the controller considered in this paper switches at the sampling instant t_k, but its control action starts to take effect after a network-induced time-delay τ_{ca}^k. Time-varying nature of the transmission delay τ_{sc} and τ_{ca} makes the lifting technique [1] for the sampled-data systems not working any more. In [4], a robust control method is proposed by transforming the sampled-data system into a continuous-time system with control time-delay. However, this transformation is under a condition: the sampling period is infinite small. In this paper, we proposed a new synthesis method by transforming the time-delay in the discrete-time subsystem into the continuous-time subsystem and keeping the sampled-data system nature unchanged. In this framework, no limitation on sampling period is introduced, which shows high application potential.

For simplification, we assume $\tau_{ca}^{k+1} \le \tau_{ca}^k + \frac{n_k - n_{k-1}}{n_s} T_s$. Letting $\bar{u}[t_k + \tau_{ca}^k] = Fx(t_k + \tau_{ca}^k)$ and $\hat{u}(t) = \bar{u}[t_k + \tau_{ca}^k]$, for $t \in (t_k + \tau_{ca}^k, t_{k+1} + \tau_{ca}^{k+1}]$, then the NCS (1)-(3) is equivalent to the following closed-loop system for $t \in (t_k + \tau_{ca}^k, t_{k+1} + \tau_{ca}^{k+1}]$:

$$E\dot{x}(t) = Ax(t) + B\hat{u}(t - \frac{n_s - n_{k-1}}{n_s} T_s - \tau_{ca}^k), \quad (4)$$

$$\hat{u}(t) = Fx(t_k + \tau_{ca}^k). \quad (5)$$

Actually, the above method defines a mapping $(u, \tilde{u}) \longrightarrow (\hat{u}, \bar{u})$ to transform the time-delay in the digital controller (2) to the continuous-time subsystem (4) and keep the dynamics of the closed-loop systems of (1)-(3) unchanged.

Letting $\tilde{x} = (x^T(t), \hat{u}^T(t))^T$ and $d = (1 - \frac{n_{k-1}}{n_s})T_s + \tau_{ca}^k$, the networked singular control system is rewritten as:

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{x}(t - d), \quad (6)$$

for $t \in (t_k + \tau_{ca}^k, t_{k+1} + \tau_{ca}^{k+1}]$, and

$$\tilde{x}(t_k + \tau_{ca}^k + 0) = (\bar{A} + \bar{B}F\bar{C})\tilde{x}(t_k + \tau_{ca}^k), \quad (7)$$

where $\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} 0 & B \\ 0 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $\bar{C} = \begin{bmatrix} I & 0 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$.

By $det(s\tilde{E} - \tilde{A}) = det(sE - A)det(sI)$, $deg(det(s\tilde{E} - \tilde{A})) = deg(det(sE - A)) + m$ and Definition 1, it is easy to prove that the augmentation (6)-(7) dose not change the finite modes or impulsive modes of the original singular system [8]. Hence, the augmented singular system (6)-(7) is equivalent to the original networked singular control system (4)-(5).

4 Stability Issue

We will present a delay-dependent Lyapunov function to ensure the stability of networked singular control system (4)-(5) with possible maximum time-delay d_0 satisfying $d \le d_0$.

Theorem 3 *The networked singular control system* (1)-(3) *is regular, impulse free and stable for delay* $d \le d_0$*, if there exist matrices* $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $\Theta > 0$, $T = [T_1, T_2, T_3, T_4]$, N *such that the following matrix inequalities hold:*

$$\tilde{E}P^T = P\tilde{E}^T \ge 0, \tag{8}$$

$$\begin{bmatrix} \Theta & N \\ N^T & Q_2 \end{bmatrix} \ge 0, \tag{9}$$

and

$$M + T^T\hat{A} + \hat{A}^T T + \Upsilon^T N^T + N\Upsilon + d_0\Theta < 0, \tag{10}$$

$$(\bar{A} + \bar{B}F\bar{C})^T P^T (\bar{A} + \bar{B}F\bar{C}) - P^T < 0, \tag{11}$$

where $M = \begin{bmatrix} Q_1 & 0 & P^T & 0 \\ 0 & -Q_1 & 0 & 0 \\ P & 0 & d_0Q_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\hat{A} = \begin{bmatrix} \tilde{A} & \tilde{B} & -\tilde{E} & 0 \end{bmatrix}$, $\Upsilon = \begin{bmatrix} I & -I & 0 & -I \end{bmatrix}$.

Proof. Suppose both (8) and (10) hold for $Q_1 > 0$, then by setting $T_1 = P$, $T_2 = 0$, $T_3 = 0$, $N = 0$, from (10) it is easy to see that

$$\tilde{A}P^T + P\tilde{A}^T < 0. \tag{12}$$

By Lemma 2, it follows from (8) and (12) that the pair $(\tilde{E}, \tilde{A})$ is regular and impulse free. Moreover, the augmented singular system (6)-(7) is regular and impulse free by [2].

Actually, by letting $P = diag(P_{11}, P_{22})$, it is easy to see from (8) and (10) that $EP_{11}^T = P_{11}E^T < 0$, $AP_{11}^T + P_{11}A^T < 0$. Hence, the origin system (4)-(5) is regular, impulse free.

Next, we shall show the stability of the singular delay system (4)-(5). To this end, we note that the regularity and the absence of impulses of the pair $(\tilde{E}, \tilde{A})$ implies that there exist two invertible matrices Φ and $\Gamma \in \mathbb{R}^{(n+m)\times(n+m)}$ such that [2]

$$\Phi\tilde{E}\Gamma = \begin{bmatrix} I_{r+m} & 0 \\ 0 & 0 \end{bmatrix}, \ \Phi\tilde{A}\Gamma = \begin{bmatrix} \tilde{A}_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}, \tag{13}$$

where $I_{r+m} \in \mathbb{R}^{(r+m)\times(r+m)}$ and $I_{n-r} \in \mathbb{R}^{(n-r)\times(n-r)}$ are identity matrices, $\tilde{A}_1 \in \mathbb{R}^{(r+m)\times(r+m)}$. According to (13), $\Phi\tilde{B} = \begin{bmatrix} \tilde{B}_{11} & \tilde{B}_{12} \\ \tilde{B}_{21} & \tilde{B}_{22} \end{bmatrix}$.

Let

$$\varsigma(t) = \begin{bmatrix} \varsigma_1(t) \\ \varsigma_2(t) \end{bmatrix} = \Gamma^{-1}\tilde{x}(t), \tag{14}$$

where $\varsigma_1(t) \in \mathbb{R}^(r+m)$, $\varsigma_2(t) \in \mathbb{R}^{n-r}$. Using the expressions in (13)-(14), the singular delay system (6) can be decomposed as

$$\begin{aligned} \dot{\varsigma}_1(t) &= \tilde{A}_1\varsigma_1(t) + \tilde{B}_{11}\varsigma_1(t-\tau) + \tilde{B}_{12}\varsigma_2(t-\tau), \\ 0 &= \varsigma_2(t) + \tilde{B}_{21}\varsigma_1(t-\tau) + \tilde{B}_{22}\varsigma_2(t-\tau). \end{aligned}$$

Since $\varsigma_1(t)$ is piece-wise continuously differentiable for $t \ge 0$, Leibniz-Newton formula gives

$$\begin{aligned} \varsigma_1(t-\tau) &= \varsigma_1(t) - w_{k,1} - \int_{t-d}^{t_{k-1}+\tau_{ca}^{k-1}} \dot{\varsigma}_1(s)ds \\ &\quad - \int_{t_{k-1}+\tau_{ca}^{k-1}}^{t_k+\tau_{ca}^k} \dot{\varsigma}_1(s)ds - \int_{t_k+\tau_{ca}^k}^{t} \dot{\varsigma}_1(s)ds, \end{aligned}$$

with

$$w_{k,1} = \begin{cases} \begin{bmatrix} 0 & \bar{u}^T[t_k+\tau_{ca}^k] - \bar{u}^T[t_{k-1}+\tau_{ca}^{k-1}] \end{bmatrix}^T \\ \quad \text{for } t-d > t_{k-1}+\tau_{ca}^{k-1}, \\ \begin{bmatrix} 0 & \bar{u}^T[t_k+\tau_{ca}^k] - \bar{u}^T[t_{k-2}+\tau_{ca}^{k-2}] \end{bmatrix}^T \\ \quad \text{for } t-d \le t_{k-1}+\tau_{ca}^{k-1}. \end{cases}$$

Moreover, for $t \ge d$, we define

$$\int_{t-d}^{t_{k-1}+\tau_{ca}^{k-1}} \dot{\varsigma}_2(s)ds + \int_{t_{k-1}+\tau_{ca}^{k-1}}^{t_k+\tau_{ca}^k} \dot{\varsigma}_2(s)ds + \int_{t_k+\tau_{ca}^k}^{t} \dot{\varsigma}_2(s)ds$$
$$:= \varsigma_2(t) - \varsigma_2(t-d).$$

Then we have $\dot{\tilde{x}}(t) = \Gamma \begin{bmatrix} \dot{\varsigma}_1^T(t) & \dot{\varsigma}_2^T(t) \end{bmatrix}^T$, and $\tilde{x}(t-d) = \tilde{x}(t) - \int_{t-d}^{t_{k-1}+\tau_{ca}^{k-1}} \dot{\tilde{x}}ds - \int_{t_{k-1}+\tau_{ca}^{k-1}}^{t_k+\tau_{ca}^k} \dot{\tilde{x}}ds - \int_{t_k+\tau_{ca}^k}^{t} \dot{\tilde{x}}ds - \tilde{w}_k$, where $\tilde{w}_k = \Gamma \begin{bmatrix} w_{k,1}^T & 0 \end{bmatrix}^T$.

Let $y_t(d) = \int_{t-d}^{t_{k-1}+\tau_{ca}^{k-1}} \dot{\tilde{x}}ds + \int_{t_{k-1}+\tau_{ca}^{k-1}}^{t_k+\tau_{ca}^k} \dot{\tilde{x}}ds + \int_{t_k+\tau_{ca}^k}^{t} \dot{\tilde{x}}ds$, the equation (6) can be rewritten as:

$$(\tilde{A} + \tilde{B})\tilde{x}(t) - \tilde{E}\dot{\tilde{x}}(t) - \tilde{B}y_t(d) - \tilde{B}\tilde{w}_k = 0,$$

Obviously, there exist arbitrary matrices T_1, T_2, T_3 and T_4 with compatible dimensions such that

$$\begin{aligned} &(T_1\tilde{x}(t) + T_2\tilde{x}_t(d) + T_3\dot{\tilde{x}}(t) + T_4\tilde{w}_k) \times \\ &((\tilde{A} + \tilde{B})\tilde{x}(t) - \tilde{B}y_t(d) - \tilde{E}\dot{\tilde{x}}(t) - \tilde{B}\tilde{w}_k) = 0. \end{aligned} \tag{15}$$

where $\tilde{x}_t(d) = \tilde{x}(t-d)$.

Choosing the following Lyapunov function candidate for $t \in (t_k + \tau_{ca}^k, t_{k+1} + \tau_{ca}^{k+1}]$,

$$V(\tilde{x}) = V_1(\tilde{x}) + V_2(\tilde{x}), \tag{16}$$

$$V_1(\tilde{x}) = \tilde{x}^T(t)P^T\tilde{x}(t) + \int_{t-d}^{t_{k-1}+\tau_{ca}^{k-1}} \tilde{x}^T(s)Q_1\tilde{x}(s)ds$$

$$+\int_{t_{k-1}+\tau_{ca}^{k-1}}^{t_k+\tau_{ca}^k} \tilde{x}^T(s)Q_1\tilde{x}(s)ds$$
$$+\int_{t_k+\tau_{ca}^k}^{t} x^T(s)Q_1\tilde{x}(s)ds,$$
$$V_2(\tilde{x}) = \int_{-d}^{-t+t_{k-1}+\tau_{ca}^{k-1}}\int_{t+\theta}^{t} \dot{\tilde{x}}(s)Q_2\dot{\tilde{x}}(s)dsd\theta$$
$$+\int_{-t+t_{k-1}+\tau_{ca}^{k-1}}^{-t+t_k+\tau_{ca}^k}\int_{t+\theta}^{t} \dot{\tilde{x}}(s)Q_2\dot{\tilde{x}}(s)dsd\theta$$
$$+\int_{-t+t_k+\tau_{ca}^k}^{0}\int_{t+\theta}^{t} \dot{\tilde{x}}(s)Q_2\dot{\tilde{x}}(s)dsd\theta,$$

where $P > 0$, $Q_1 > 0$, $Q_2 > 0$.

Abusing the notation of $\dot{V}_2(\tilde{x}) = d\dot{\tilde{x}}^T(t)Q_2\dot{\tilde{x}}(t) - \int_{t-d}^{t}\dot{\tilde{x}}^T(s)Q_2\dot{\tilde{x}}(s)ds$, and letting $T = [T_1, T_2, T_3, T_4]$ and $\xi(t) = (\tilde{x}^T(t), \tilde{x}_t^T(d), \dot{\tilde{x}}^T(t), \tilde{w}_k^T)^T$, then we have

$$\dot{V}(\tilde{x}) = \xi^T(t)M\xi(t)+2\xi^T(t)T^T((\tilde{A}+\tilde{B})\tilde{x}(t)-\dot{\tilde{x}}(t)-\tilde{B}w_k) -2\xi^T(t)T^T\tilde{B}y_t(d) - \int_{t-d}^{t}\dot{\tilde{x}}^T(s)Q_2\dot{\tilde{x}}(s)ds \quad (17)$$

By inequality in [9]and [6]:

$$\begin{aligned}&-2\xi^T(t)T^T\tilde{B}y_t(d)\\ \le\ & d\xi^T(t)\Theta\xi(t) + 2\xi^T(t)(N - T^T\tilde{B})\int_{t-d}^{t}\dot{\tilde{x}}ds\\ &+\int_{t-d}^{t}\dot{\tilde{x}}^T(s)Q_2\dot{\tilde{x}}(s)ds\\ =\ & \xi^T(t)(d\Theta + 2N)\xi(t) - 2\xi^T(t)T^T\tilde{B}(\tilde{x}(t) - \tilde{x}_t(d) - \tilde{w}_k)\\ &+\int_{t-d}^{t}\dot{\tilde{x}}^T(s)Q_2\dot{\tilde{x}}(s)ds, \end{aligned} \quad (18)$$

where $\Upsilon = \begin{bmatrix} I & -I & 0 & -I\end{bmatrix}$, and for any matrices Θ and N satisfying (9). Therefore, by equations (17)-(18), $\dot{V}(\tilde{x})$ can be described as

$$\dot{V}(\tilde{x}) \le \xi^T(t)(M + d\Theta + 2N\Upsilon + 2T^T\hat{A})\xi(t), \quad (19)$$

where $\hat{A} = \begin{bmatrix}\tilde{A} & \tilde{B} & -\tilde{E} & 0\end{bmatrix}$. It is easy to see that $\dot{V}(\tilde{x}) < 0$ if LMI (10) holds for $\forall\tilde{x} \neq 0$ and any $d \le d_0$.

On the other hand, from (11), we have $\Delta V < 0$.

Noting (19), we have

$$\dot{V}(\tilde{x}) \le -\beta_1\xi^T(t)\xi(t),\ \ t \in (t_k + \tau_{ca}^k, t_{k+1} + \tau_{ca}^{k+1}], \quad (20)$$

where $\beta_1 = \lambda_{min}(-(M+d\Theta+2N\Upsilon+2T^T\hat{A})) > 0$. From $\Delta V < 0$ and (20), We have with the initial state $\tilde{x}(\tau) = \phi(\tau)$, $\tau \in [-d, 0]$ that

$$V(\tilde{x}(t)) - V(\phi) \le \int_0^t \dot{V}(\tilde{x}(s))ds \le -\beta_1\int_0^t \tilde{x}^T(s)\tilde{x}(s)ds,$$

with the initial state $\tilde{x}(t) = \phi(t), t \in [-d, 0]$.

On the other hand, we can show that there exists a scalar $\beta_2 > 0$ such that

$$V(\tilde{x}(t)) = V_1(\tilde{x}(t)) + V_2(\tilde{x}(t)) \ge \beta_2\tilde{x}^T(t)\tilde{x}(t).$$

The above two inequalities imply that

$$\tilde{x}^T(t)\tilde{x}(t) \le -\lambda_1\int_0^t \tilde{x}^T(s)\tilde{x}(s)ds + \lambda_2V(\phi),$$

where $\lambda_1 = \beta_1\beta_2^{-1}$, $\lambda_2 = \beta_2^{-1}$. Then, we obtain $\tilde{x}^T(t)\tilde{x}(t) \le \lambda_2\exp(-\lambda_1 t)V(\phi)$. Therefore,

$$\int_0^t \tilde{x}^T(s)\tilde{x}(s)ds \le \lambda_1^{-1}\lambda_2[1 - \exp(-\lambda_1 t)]V(\phi),$$

Taking limit as $t \to \infty$, we have

$$\begin{aligned}\lim_{t\to\infty}\int_0^t x^T(s)x(s)ds &\le \lim_{t\to\infty}\int_0^t \tilde{x}^T(s)\tilde{x}(s)ds\\ &\le \lambda_1^{-1}\lambda_2V(\phi).\end{aligned}$$

Noting that there always exists a scalar $c > 0$, such that $\lambda_1^{-1}\lambda_2V(\phi) \le c\sup_{d_0\le s\le 0}\|\phi(s)\|^2$, it then follows that the continuous-time singular subsystem is stable.

The conditions (19)and $\Delta V < 0$ show the closed-loop system is asymptotically stable. We therefore have the obtained the desired result. ■

Remark 2 *Theorem 3 gives the sufficient conditions of regularity, impulse free and stability for the networked singular control system. In fact, by letting $E = I$, we could get the same conditions as Theorem 2 in paper [6]. Therefore, Theorem 3 is the extension of the result in [6].*

5 Control Design

In this section, we present a method to design a control (2) such that the networked singular control system stable with possible largest time-delay.

Theorem 4 *If there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $\Theta > 0$, $T = [T_1, T_2, T_3, T_4]$, N satisfying the following matrix inequalities:*

$$\begin{bmatrix} -P^T+\bar{A}^TP^T\bar{A} & * & * & * \\ \bar{B}^TP\bar{A} & -Y & * & * \\ \bar{F}\bar{C} & 0 & -Y_1-Y_1^T+Y & * \\ \bar{F}\bar{C} & 0 & 0 & -Y_1-Y_1^T+\bar{B}^TP^T\bar{B}\end{bmatrix} < 0 \quad (21)$$

and (8), (9), (10), then the networked singular control system (1)-(3) is stabilized via a state feedback controller, where the gain can be calculated by $F = Y_1^{-1}\bar{F}$ whenever the loop delay satisfies $d < d_0$.

Proof Noting the fact $\bar{C}^T F^T \bar{B}^T P^T \bar{A} + \bar{A}^T P^T \bar{B} F \bar{C} \leq \bar{C}^T F^T Y F \bar{C} + \bar{A}^T P^T \bar{B} Y^{-1} \bar{B}^T P \bar{A}$, then the equation (11) can be expressed as:

$$\begin{bmatrix} -P^T + \bar{A}^T P^T \bar{A} & * & * & * \\ \bar{B}^T P \bar{A} & -Y & * & * \\ Y_1 F \bar{C} & 0 & -Y_1 - Y_1^T + Y & * \\ Y_1 F \bar{C} & 0 & 0 & -Y_1 - Y_1^T + \bar{B}^T P^T \bar{B} \end{bmatrix} < 0$$

Letting $\bar{F} = Y_1 F$, we therefore have obtained the desired result.

6 Numerical Example

Consider the following state-space plant model

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.8 & -0.01 & 0 \\ 1 & 0 & 0.1 \\ 0 & -0.3 & 0.6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.4 \\ 0.1 \\ 0.9 \end{bmatrix} u.$$

By Theorem 3, we obtain a feedback controller $u = [1.2625, -1.2679, 1]x$ for the maximum time-delay $d_0 = 2s$. Fig. 3 shows the dynamic response of the singular system. Within the maximum allowable network-induced time-delay, the controller can still stabilize the plant during the variant control period.

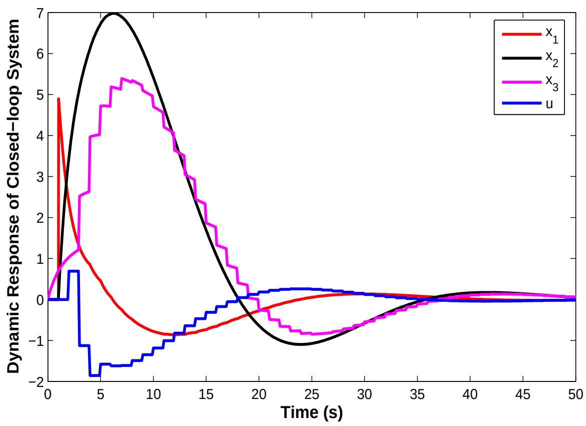

Figure 3: Simulation results

7 Conclusion

In this paper, a networked singular control system with a continuous-time singular plant, a multirate sampler, a time-driven digital controller and a event-driven holder device is considered. This system is modeled as a sampled-data one with discrete-time delay. The model shows its nice features to include many cases of the networked-control systems. By transferring time-delay in the discrete-time subsystem to its continuous counterpart, an equivalent sampled-data system with continuous-time delay is obtained with no limitation on the sampling period. Moreover, a stability result for the networked singular control system is also established. Based on the stability result, a state feedback control such that the closed-loop system with possible maximum time-delay caused by the networks is considered. The numerical example demonstrates the effectiveness of the proposed method for the networked-control systems.

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