

# Fuzzy ARTMAP Based Target Tracking Control System

Riyadh Kenaya, *Member, IEEE* and Ka C Cheok, *Member, IEEE*

**Abstract**—Fuzzy ARTMAP neural networks have been widely used in the areas of pattern and image recognition. They are well known for their ability to learn new patterns and efficiently identify previously learned patterns. However, they are rarely used in control applications. Our concept in this paper is to construct, train, and test a Fuzzy ARTMAP controller to do a certain control action. We would like to show that the Fuzzy ARTMAP neural network is capable of performing the control action that the human being is able to perform. This would help in training the robot to do tasks the human is in charge of. A human based moving target tracking experiment is employed. In this research we choose a fuzzy ARTMAP neural network to imitate the behavior of the human in such way to track a target the way the human does. This may completely replace the human and may result in transferring a lot of tasks the human does daily to an intelligent system if it is trained to do the job in the right way. Necessary data is collected from the said tracking experiment and is used to train the fuzzy ARTMAP network. The network is then used to replace the human in the testing and application phases.

**Index Terms**— Fuzzy ARTMAP, Adaptive Resonance Theory, Neural Networks, Intelligent Control.

## I. INTRODUCTION

The term ART stands for adaptive resonance theory. It is basically a clustering algorithm proposed by Carpenter and Grossberg in 1987 [1]. The algorithm was intended for unsupervised clustering of binary data and known as ART-1.[2]. Later on, they generalized the ART-1 algorithm to become applicable to both binary and analog clustering. This generalized algorithm is known as ART-2 [2]. Since then many researches have suggested different forms of ART algorithms and modifications [2].

Fuzzy ARTMAP is a supervised learning algorithm for analog data. It is a clustering technique with supervision that redirects training inputs which would be grouped in an incorrect category to a different cluster.

A fuzzy ARTMAP system consists of two fuzzy ART modules; ART<sub>a</sub> and ART<sub>b</sub>. Each Fuzzy ART module clusters vectors in an unsupervised fashion [3]. Both ART<sub>a</sub> and ART<sub>b</sub> are linked by the map field module, fig. 1. This linkage establishes a one to one correspondence between ART<sub>a</sub> categories (clusters) and their counterparts in ART<sub>b</sub>. Training the fuzzy ARTMAP system is done by introducing input patterns to ART<sub>a</sub> module. Those patterns represent an M-dimensional vectors. ART<sub>a</sub> groups those patterns into clusters based on a certain factor called vigilance factor  $\rho$  [4]. The bigger the vigilance factor, the bigger the number of clusters to result from training and the smaller the number of patterns to be grouped within each cluster. Every input pattern introduced to ART<sub>a</sub> is to be compared with all of its

categories. Based on  $\rho$  value, an input vector is to be combined with a current existing cluster or would be considered as a new category. High vigilance leads to narrow generalization and to prototypes that represent fewer input exemplars. Thus a single ART system may be used, say, to recognize abstract categories of faces and dogs, as well as individual faces and dogs [1]. The above mentioned description fully applies to ART<sub>b</sub> as well. ART<sub>b</sub> also accepts patterns in vector form and decide whether those patterns have seen before (cluster members) or they are new categories. The major difference between ART<sub>a</sub> and ART<sub>b</sub> is the nature of each module patterns. ART<sub>a</sub> patterns represent the overall system input to be recorded, whereas ART<sub>b</sub> patterns represent the overall system target output to be reached in the classification phase. Fuzzy ART requires input pattern to be normalized to prevent category proliferation [2]. Each input pattern is an M-dimensional vector  $(I_1, \dots, I_M)$ , where each component  $I_i$  is in the interval  $[0,1]$ . [1].

$$\mathbf{I} = \frac{\mathbf{a}}{\|\mathbf{a}\|} \quad (1)$$

where

$$\|\mathbf{a}\| = \sum_{i=1}^N |a_i|, N \text{ is the length of vector } \mathbf{a} \quad (2)$$

Input patterns are further complementary coded. Complement coding represents both the on-response and off-response to  $\mathbf{a}$ . [1]. So If  $\mathbf{a}$  represents the on-response, the off-response is represented by  $\mathbf{a}^c$  where

$$a_i^c = 1 - a_i \quad (3)$$

## II. ART STRUCTURE AND TRAINING

ART module learning process is based on checking the module's input vectors (patterns). Based on certain criteria, each new pattern is compared with all of the nodes (categories) saved in the ART memory. Before learning starts, the ART module memory is empty, which means no categories have been yet recorded. With the first pattern to arrive, the first category is created. The created category (which is in fact the pattern itself) is then saved in the ART module's memory. When the second pattern reaches the ART module, a comparison between the arrived pattern and the saved category(s) is established. Comparison results would specify the nature of the second pattern. If the pattern is close enough to the category saved, learning starts. The said learning modifies the saved category according to equation (6). Otherwise, they are not similar and the input pattern is considered to be a new category. This process continues for

the rest of the training patterns. Vigilance factor  $\rho$  determines the number of created categories by the end of the training phase. Vigilance factor ranges between 0 and 1. If the vigilance factor is set to 0, the resulting number of categories is zero, and if it is set to 1 the resulting number of categories would be exactly the same number of patterns used in the training process. In either case no learning occurs. Comparison is based on calculating the activation value [2] for each input pattern (4).

$$\mathbf{T}_j = \frac{\|\mathbf{I} \wedge \mathbf{w}_j\|}{\alpha + \|\mathbf{w}_j\|} \quad (4)$$

where  $\mathbf{I}$  is the input pattern currently applied to the ART module and  $\mathbf{w}_j$  is the  $j$ th category found in the ART module memory,  $\wedge$  is the fuzzy AND operation,  $(\mathbf{X} \wedge \mathbf{Y}) = \min(x_i, y_i)$  [5],[9].  $\alpha$  is a constant  $> 0$ .  $J$  is the index that represents the category of which the activation function is highest.  $\mathbf{w}_j$  undergoes another test called vigilance test [5] shown in (5).

$$\frac{\|\mathbf{I} \wedge \mathbf{w}_j\|}{\|\mathbf{I}\|} > \rho \quad (5)$$

Vigilance test is performed in order to measure the match of the category whose activation is highest with input pattern  $\mathbf{I}$ . If the match function exceeds the vigilance parameter  $\rho$ , we say that the  $J$ th category has won, the resonance is reached [6],[7], and learning is performed. Otherwise the said category is excluded from the search for this pattern. The search process resumes looking for another category maximizes the activation function and satisfies the vigilance test (5). If no category is found to satisfy both of the activation and vigilance conditions, a new category is formed and added to the ART module's memory. The new formed category would be the input pattern  $\mathbf{I}$  itself. In case of resonance occurrence, learning rule (6) is used to modify the category weights.

$$\mathbf{w}_j \text{ new} = \beta (\mathbf{I} \wedge \mathbf{w}_j \text{ old}) + (1 - \beta) \mathbf{w}_j \text{ old} \quad (6)$$

Where  $\beta \in (0,1]$  is known as learning rate. When  $\beta = 1$ , it is the case of fast learning where the category vector is directly equated to the input pattern. It is notable that not the input pattern but the attended portions of it are learned. This causes detection of relevant feature groupings of the categories and focusing attention on these portions while trying to match new input patterns [11].

In our research, fuzzy ARMAP algorithm is employed to do a control action in a similar way a regular control system does. We monitor the error signal and generate the necessary control signal to reduce that error.  $\text{ART}_a$  is responsible for recording the error vectors and cluster them into categories known as error categories.  $\text{ART}_b$  is responsible for recording the desired control actions needed to reduce this error and cluster them into categories known as target categories. Recording those error and target patterns and clustering them into categories is the outcome of the whole fuzzy ARTMAP system training. Fig. 1 shows the fuzzy ARTMAP system.

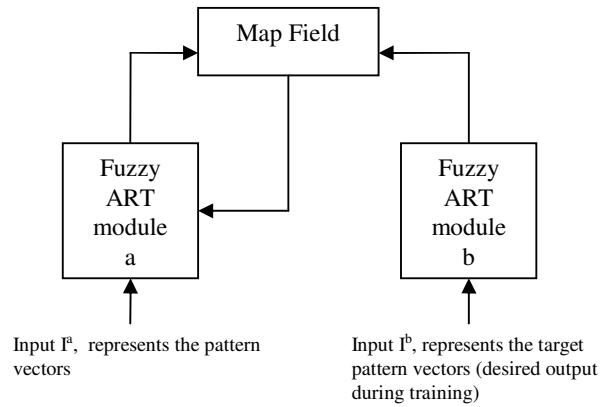


Fig. 1: Fuzzy ARTMAP system showing the two fuzzy ART modules and the map field link between them.

Each pattern arrives to  $\text{ART}_a$  or  $\text{ART}_b$  would undergo the activation and vigilance tests. Error patterns are compared with  $\text{ART}_a$  categories whereas target patterns are compared with  $\text{ART}_b$  categories. Both of the activation and vigilance tests would ensure that the number of categories in both  $\text{ART}_a$  and  $\text{ART}_b$  are not going to increase without bound. If the number of categories grows without bound, the system efficiency and response would be highly degraded during classification phase. It is the case that is referred to as category proliferation. Category proliferation would result in flooding the ART module memory with huge number of adjacent category vectors that are alike.  $\text{ART}_a$  vigilance parameter is  $\rho$  and  $\text{ART}_b$  vigilance parameter is  $\rho_{ab}$ .  $\text{ART}_a$  categories are mapped to  $\text{ART}_b$  categories via a map field containing a matrix of weights  $\mathbf{w}^{ab}$ . The winning category in  $\text{ART}_a$  would be given the index (row number)  $J$ . The said index would also refer to the corresponding output category node in  $\text{ART}_b$ . We can say that a one-to-one correspondence between a category in  $\text{ART}_a$  and its counterpart in  $\text{ART}_b$  is established by the index  $J$ . The map field is essentially a look-up table [6], retrieving an analog-valued weight  $w_J^{ab}$  when module "a" category  $J$  and module "b" category  $J$  are active. It is necessary to say that only one category node in each module is active at a time. During the training phase, the map field performs a vigilance test similarly to  $\text{ART}_a$  vigilance test. In the said test if the match function exceeds the map field vigilance parameter (7)

$$\frac{\|\mathbf{I}^b \wedge \mathbf{w}_J^{ab}\|}{\|\mathbf{I}^b\|} \geq \rho_{ab}, \rho_{ab} \in (0,1] \quad (7)$$

then both the resonance and learning occurs [5]. Else a match tracking procedure [2],[3],[4] is initiated to find a better candidate in  $\text{ART}_a$ . The procedure is represented by incrementing  $\text{ART}_a$ 's vigilance parameter  $\rho$  by  $\delta$ , where  $\delta$  is a positive number much less than 1. This would result in exempting category  $J$  which has failed the competition and open the door for a new search for other candidates so as to fulfill the vigilance tests (5) and (7). This search for an  $\text{ART}_a$  category that predicts the correct  $\text{ART}_b$  category proceeds until it is found, otherwise a new category is created. The association between an  $\text{ART}_a$  category and an  $\text{ART}_b$  category is gained by the following learning rule.

$$\mathbf{w}_j^{ab \text{ new}} = \beta_{ab} (\mathbf{I}^b \wedge \mathbf{w}_j^{ab \text{ old}}) + (1 - \beta_{ab}) \mathbf{w}_j^{ab \text{ old}} \quad (8)$$

It is the case of map field resonance.

Once the training is pursued, the fuzzy ARTMAP system becomes ready for testing. This phase is called; classification phase [8],[10],[12]. In this phase, each input vector applied to  $ART_a$  is compared with all of the categories stored in  $ART_a$  memory (9).

$$\mathbf{T}_j = \frac{\|\mathbf{I}_{a(\text{testing})} \wedge \mathbf{w}_j\|}{\alpha + \|\mathbf{w}_j\|} \quad (9)$$

A search for the category of  $J_{th}$  index whose activation is highest is run (10)

$$\mathbf{T}_j = \max(\mathbf{T}_j) \quad (10)$$

The category of the highest activation  $\mathbf{T}_j$  is considered to be the winner,  $\mathbf{w}_j$ . The map field unit returns the corresponding  $ART_b$ 's winner category  $\mathbf{w}_j^{ab}$  and would be considered as the fuzzy ARTMAP system's corresponding output.

### III. METHODOLOGY AND EXPERIMENT SETUP

In this research we use an experiment based data representing the performance of a certain system as training data for a fuzzy ARTMAP neural network. The trained network is then used to substitute the above mentioned system so that it gives the same performance that the system itself gives. The system we have used in this research is simply a man's eye-brain-hand trying to track a moving target in the plane. The target is simply a moving trapezium in the plane with certain frequency and displacement.

Proportional and proportional-differential trained fuzzy ARTMAP controllers are to be used for result comparisons. Fig. 2, 3, and 4 show signals collected from the best run of the experiment. To get best training data, it is important to run the experiment more than once. It might be hard for the man to do the best tracking from the first trial. Fig. 2 shows the man tracking error signals resulting from missing the target.  $P_x$  is the x-component of the error vector and  $P_y$  is its y-component. The faster the target motion, the larger the error obtained.

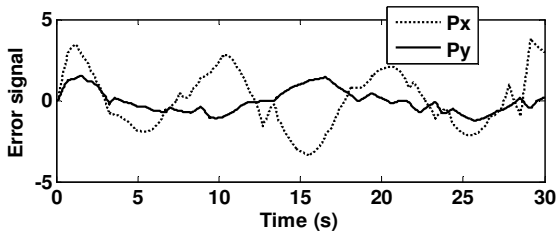


Fig 2: Error signal represents the mismatch between target position and tracking line.

Fig. 3 shows the man control command signals  $U_x$  and  $U_y$ . Those are the brain signals sent to the hand muscles so that

tracking is pursued and less error is obtained. Fig. 4 shows the function generators' excitation signals (set values) versus feedback signals. The difference between excitation and feedback results in error signal shown in fig. 2.

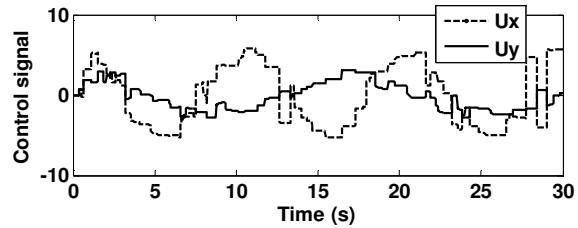


Fig 3: Man output signal represents the brain commands to the hand muscles

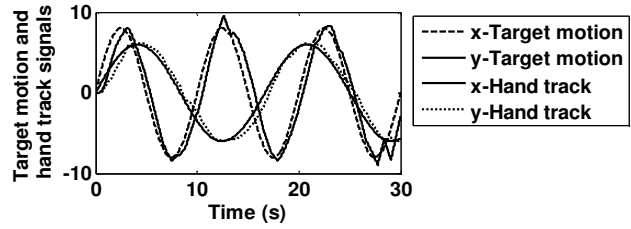


Fig. 4: Function generator signal versus feedback signal

### IV. FUZZY ARTMAP SCHEME

We construct a fuzzy ARTMAP based control system to replace the man's eye-brain-hand controller. The fuzzy ARTMAP controller monitors the error signal and generates the necessary output to reduce the said error. In the training phase,  $ART_a$  module accepts error signal vector patterns and cluster them into categories whereas  $ART_b$  module accepts the controller's output signals collected from the tracking experiment and clusters them into categories known as target vector categories.

Vector patterns applied to  $ART_a$  module during training should be normalized and complementary coded. In our case those patterns are the error vectors collected from the eye-hand experiment and saved on a data file. They are two dimensional error vectors representing the position mismatch between the target and the hand track in the plane. Fig. 5 shows the error data after normalization.

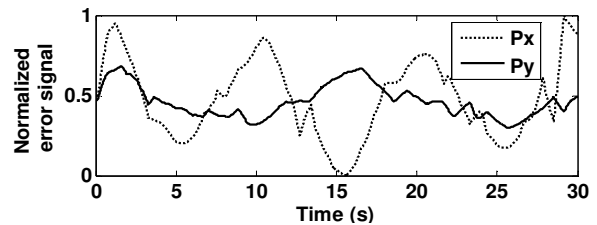


Fig. 5: Normalized error data in eye-hand tracking experiment

Normalization is also applied on the output tracking signals. Fig. 6 shows the controller's output signals after normalization. Once the normalized patterns are generated, they are complementally coded (3). Complement coding doubles the vector size. In all of our experiments, fuzzy ARTMAP parameter were set to  $\beta = 0.5$ ,  $\beta_{ab} = 1$ ,  $\rho_{ab} = 1$ ,  $\alpha = 1$ , and  $\rho$  was given different values to see the effect of

this parameter on the number of categories generated and classification accuracy.

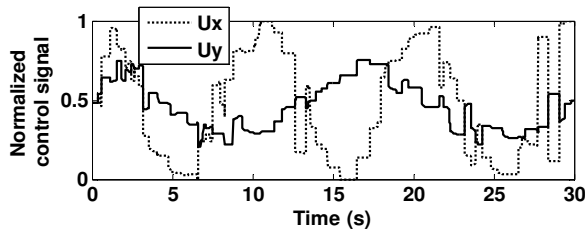


Fig. 6: Man output signal represents brain commands to hand muscles

For training, we have used 15000 input patterns and got different numbers of categories in each training process depending on  $\rho$ 's value.

### V. FUZZY ARTMAP NEURAL NETWORK TRAINING AND CLASSIFICATION RESULTS

Training the fuzzy ARTMAP neural network with  $\rho = 0.8$  and 15000 input patterns results in only 155 categories. Each category in  $ART_a$  module is a 4-dimensional vector (row vector with four columns). The first column represents the x axis error category, the second column represents the y-axis error category, the third and fourth columns represent the complement codes of the first and the second columns.  $ART_b$  categories are also four dimensional row vectors. First and second columns represent x and y control outputs, the third and fourth columns represent their corresponding complement codes. The trained fuzzy ARTMAP neural network is then involved in a testing process to evaluate its performance. The trained system consists of two ART modules "a", "b", and map field.  $ART_a$  stores in its memory error categories whereas  $ART_b$  stores in its memory the corresponding target categories. In classification phase an  $ART_a$  input vector which in our case is an error vector would undergo an activation test to find the category it belongs to. The said activation test would return the cluster J which would be the category index of  $ART_b$  module and would be considered as the network output. Once the output category is selected, it is demoralized to give the original output level. Demoralization processes is exactly the opposite of the normalization process.

#### A. Fuzzy ARTMAP Proportional Target Tracking Controller Simulation Results

Fig. 7 shows fuzzy ARTMAP based proportional control system where the human eye-brain-hand controller is replaced with the trained fuzzy ARTMAP neural network. In this simulation we test the generality of the trained network by calculating the tracking mean square error for both x and y axes. In the testing process, we try a mixture of sinusoidal and step target motions. Fig. 8, 9, and 10 shows the simulation results for  $\rho = 0.95$  and sinusoidal target motion. Another set of results is obtained for a mixture of sinusoidal and step target motion.  $\rho = 0.95$ . See fig. 11, 12, and 13.

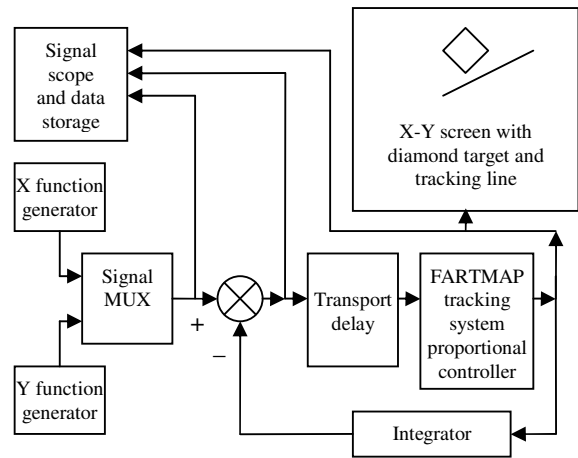


Fig. 7: Fuzzy ARTMAP target tracking control system.

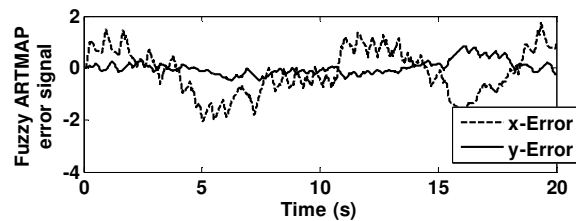


Fig. 8: Fuzzy ARTMAP controller error

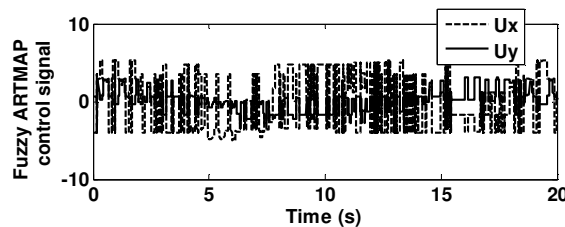


Fig. 9: Fuzzy ARTMAP controller's output

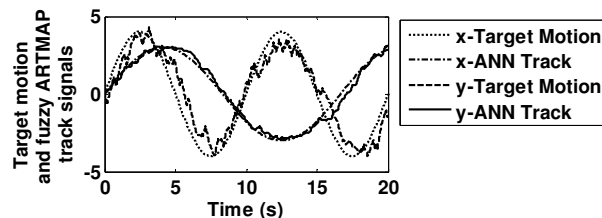


Fig. 10: Fuzzy ARTMAP controller tracking

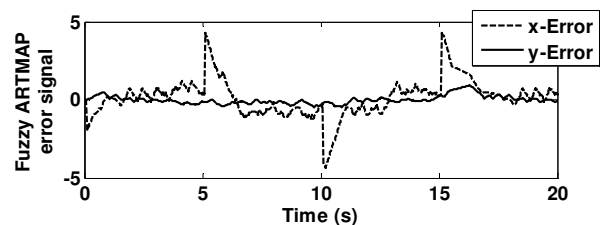


Fig. 11: Fuzzy ARTMAP controller error

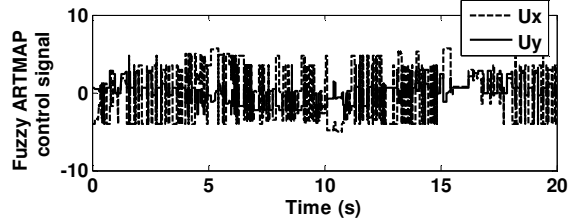


Fig. 12: Fuzzy ARTMAP controller's output

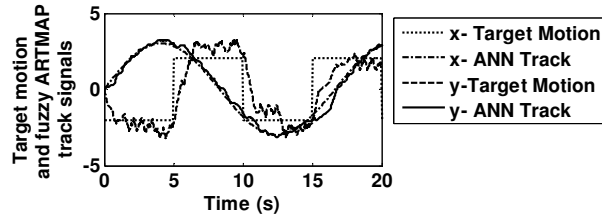


Fig. 13: Fuzzy ARTMAP controller tracking

**B. Fuzzy ARTMAP Proportional Differential Target Tracking Controller Simulation Results**

In this experiment we train the fuzzy ARTMAP network to cluster patterns of error and differential error vectors. Instead of making the training based on error vectors only, we introduce two additional patterns; x and y differential errors. ART<sub>a</sub> module categories that result from the training process are eight-dimensional row vectors. First and second columns represent x and y error categories. The third and fourth columns represent x and y differential error categories and the rest of the columns represent the error and differential error corresponding complement codes. ART<sub>b</sub> module categories are still four dimensional row vectors same as those found in ART<sub>b</sub> module of the fuzzy ARTMAP proportional controller. Fig. 14 shows the PD fuzzy ARTMAP based control system. Fig. 15 shows the normalized error and differential error data collected from eye-brain-hand experiment.

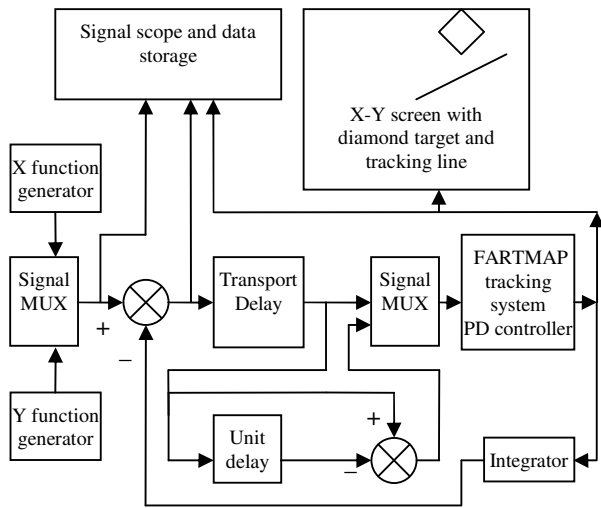


Fig. 14: Fuzzy ARTMAP target tracking system, PD control

An error differential is constructed by subtracting the current error vector from the previous one and feeding the difference

to the fuzzy ARTMAP controller as an additional pattern appending the original error vector.

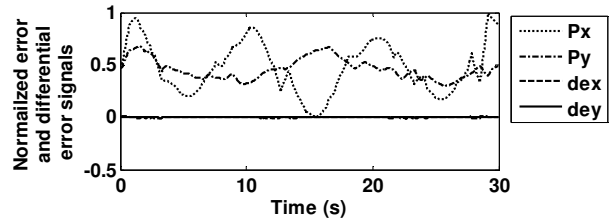


Fig. 15: Normalized error and differential error data obtained from hand eye-hand tracking experiment

Fig. 16, 17, and 18 shows the system response for sinusoidal target motion and fig. 19, 20, and 21 shows the system response for a mixture of sinusoidal and step target motions.

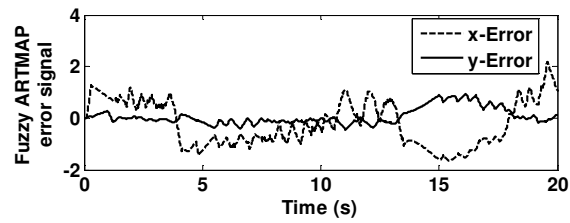


Fig. 16: Fuzzy ARTMAP controller error, PD control,  $\rho=0.9$

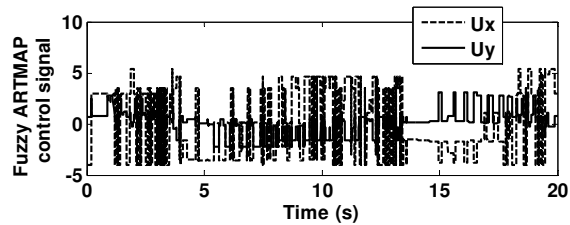


Fig. 17: Fuzzy ARTMAP controller's output, PD control,  $\rho=0.9$

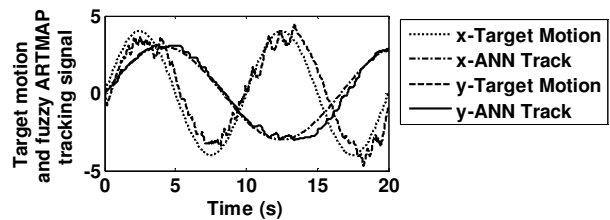


Fig. 18: Fuzzy ARTMAP controller tracking, PD control,  $\rho=0.9$

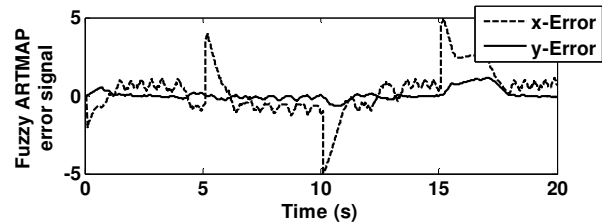


Fig. 19: Fuzzy ARTMAP controller error, PD control,  $\rho=0.9$

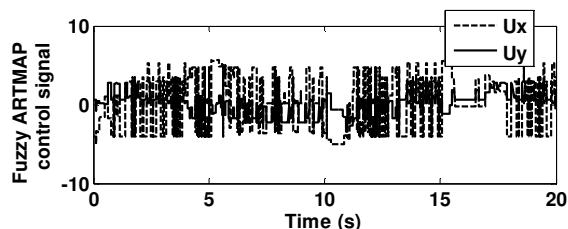


Fig. 20: Fuzzy ARTMAP controller's output, PD control,  $\rho=0.9$

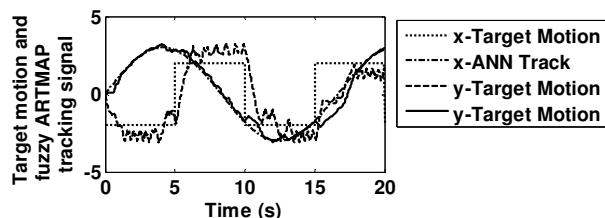


Fig. 21: Fuzzy ARTMAP controller tracking, PD control,  $\rho=0.9$

### C. Error Calculation for the fuzzy ARTMAP Controller

We evaluate the performance of the trained networks by calculating the mean square value of the error signal.

Let the error mean square value be

$$MSE = \frac{\sum_{i=1}^N (e_i)^2}{N} \quad (11)$$

where  $N$  is the vector length and  $e_i$  is an individual element within the error vector. Different  $\rho$  values result in different training outcomes. Table 1 shows that the number of categories increases as  $\rho$  increases with gradual drop in MSE value. We can see that there are two major factors that contribute to the popularity of neural networks. The first factor is the ability of neural networks to approximate arbitrary nonlinear functions. The second factor is the capability of neural networks to adapt [13],[14].

Table 1: Fuzzy ARTMAP proportional and proportional differential control system simulation results

$\rho$	Proportional Control			PD Control		
	Number of Clusters	x-axis MSE	y-axis MSE	Number of Clusters	x-axis MSE	y-axis MSE
0.800	153	0.6535	1.0167	153	1.0022	0.0486
0.850	158	0.8563	0.0218	158	1.0245	0.0332
0.900	255	0.5206	0.0344	255	0.8403	0.1035
0.920	255	0.6896	0.0637	255	0.8403	0.1035
0.940	257	1.8109	0.0624	257	0.8228	0.0961
0.960	264	0.6964	0.0800	264	0.6770	0.0691
0.980	290	1.8072	0.1111	290	0.6268	0.0886
0.990	372	1.3656	0.1602	372	1.3699	0.1607
0.999	2113	1.3706	0.1720	2113	1.4229	0.1712

## VI. CONCLUSION

It is obvious from table 1 that MSE value decreases as  $\rho$  value increases in the training phase. This is an important indication which means that the number of categories (clusters) governs the behavior similarity between the trained fuzzy ARTMAP network and the original system we are

trying to imitate. We can see that the higher the number of categories obtained during training, the closer to the original system we will be. This means that the trained fuzzy ARTMAP network will respond to the error input it receives on ART<sub>a</sub> module and generate a tracking action by ART<sub>b</sub> module the same way the imitated system does.

Increasing the number of categories however does not guarantee an improved system performance. This is apparent from table 1. As we increase  $\rho$  and we get more categories, MSE value of the controller starts to increase as well. This happens because the trained fuzzy ARTMAP network becomes closer and closer to the original eye-brain-hand system we want to imitate. If we consider all of the input patterns as categories –which is the case of  $\rho=1$ , training would result in a fuzzy ARTMAP network whose behavior is exactly the same as the original system's behavior. This means that original system that is not perfect would also result in a fuzzy ARTMAP controller that is not perfect too.

## REFERENCES

- [1] G. A. Carpenter, S. Grossberg, and D. B. Rosen. "Fuzzy ART: Fast stable learning and categorization of analog patterns by an adaptive resonance system." *IEEE transactions on Neural Networks*, V4: 759-771, 1991.
- [2] H. Xu. *Mahalanobis Distance-Based ARTMAP Networks*, Master's thesis of Computer Science, San Diego State Univ., 2003.
- [3] S. Rajasekaran and G. A. Vijayalakshmi pai. "Image Recognition Using Simplified fuzzy ARTMAP Augmented with a Moment Based Feature Extractor." *International Journal of Pattern Recognition and Artificial Intelligence*, Vol. 14, No. 8 (2000) 1081-1095.
- [4] G. A. Carpenter, Stephen Grossberg, and John H. Reynolds, "A Fuzzy ARTMAP Nonparametric Probability Estimator for Nonstationary Pattern Recognition Problems." *IEEE Transactions on Neural Networks*, Vol 6, No.6, November 1995
- [5] B. Vigdor and B. Lerner. "Accurate and Fast Off and Online Fuzzy ARTMAP-Based Image Classification With Application to Genetic Abnormality Diagnosis." *IEEE Transactions on Neural Networks*, VOL. 17, NO. 5, SEPTEMBER 2006
- [6] M. Blume and S. C. Esener. "Optoelectronic Fuzzy ARTMAP Processor", *Optical Computing*, Vol 10, 1995 OSA Technical Digest Series (Optical Society of America, Washington DC, 1995), p.213-215, March 1995.
- [7] A Pérez-Urbe. *Structure-Adaptable Digital Neural Networks*, PhD thesis, Federal polytechnic school of Luazzane, Switzerland, 1999.
- [8] F. Alilal, S. Loumi, and B. Sansal. "Modified Fuzzy ARTMAP and Supervised Fuzzy ART: Comparative Study with Multispectral Classification." *International Journal of Computer Science*, Volume 1 Number 4 2006 ISSN 1306-4428.
- [9] J.S.R. Jang, C.T. Sun, E. Mizutani. *Neuro-Fuzzy and Soft Computing*, Prentice Hall, Upper Saddle River, NJ 1997.
- [10] S. Rajasekaran and G. A. Vijayalakshmi Pai. "Image Recognition Using Simplified Fuzzy Artmap Augmented with a Moment Based Feature Extractor." *International Journal of Pattern Recognition and Artificial Intelligence*, Vol. 14, No. 8 (2000) 1081-1095
- [11] O.C. Ozcanli – Ozbay, *An Image Retrieval System Based on Region Classification*, Master's thesis of Computer Science, Middle East Technical University, Turkey, 2004.
- [12] G. A. Carpenter, Stephen Grossberg, and John H. Reynolds. "ARTMAP: Supervised Real-Time Learning and Classification of Nonstationary Data by a Self-Organizing Neural Network." *Nural Networks*, Vol. 4, pp. 565-588. 1991
- [13] J. B. D. Cabrera and K. S. Narendra, "Issues in the application of neural networks for tracking based on inverse control", *IEEE Transactions on Automatic Control*, Vol. 44, pp 2007-2027, 1999.
- [14] D. S. Chen and R. C. Jain, "A robust rack propagation learning algorithm for function approximation", *IEEE Transactions on Neural Networks*, Vol. 5, pp 467-479, 1994