# Bin Packing Model Implementation for Material Election in Read Switches Production Planning

Rainier Romero Parra, Larisa Burtseva

*Abstract* – **This paper presents an important real problem in Read Switch manufacturing where an efficient use of materials is required. The carrying out of the consumer demands depends on the stochastic results of a classification process where each lot of switches is distributed into bins according to an electric measure value. Various glass types are employed for the switch manufacturing. The effect caused by the glass type variation on the switch classification results was investigated. Based on real data statistic analysis, the problem is reduced to the lots number minimizing taking into consideration the glass type, and interpreted as a bin packing problem generalization. In contrast to the classic problem, in the considered case, an item represents a set of pieces; a container is divided into a number of bins (sub-containers); the bins capacity is variable; there are the assignment restrictions between bins and sets of items; the item sets are allowed to be fragmented into bins and containers. The problem has a high complicity. A heuristic offline algorithm is proposed to find: the number, types and packing sequence of containers, the set fragments coupled with containers and bins. The bin capacities do not affect the algorithm.**

*Index Terms*—**Bin packing, Fragmentation, Offline algorithm, Sub-containers, Variable capacity**.

## I. Introduction

An electrical switch (Read Switch) production plant for sensors and relays is examined. The company must carry out the consumer demands according to the delivery dates. The demands execution date depends on the results of the classification process, which is an intermediate operation in technological route. The results are stochastic and related to the material type. The efficient use of the material has direct influence to the production results.

The classification process has a key role in the switch production planning for the following reasons:

1. The stochastic results of the operation may cause a nonfulfilment of demands according to their priorities;
2. It feeds back the first stage with the information about demands execution.

Due to the information complexity, the production planning in the company is realized empirically, based on previous experience, and therefore it suffers from unexpected results when production is finalized.

The effect caused by the glass type on the switches distribution according to the operation value, which is an electrical measure, is investigated. The mathematical model is created using the glass type distribution tendencies. The goal is to minimize the number of lots.

The problem is interpreted in terms of the bin packing problem and a heuristic algorithm is proposed.

## II. Production Model

The switch components are a glass tube and two metallic knives (Fig. 1).

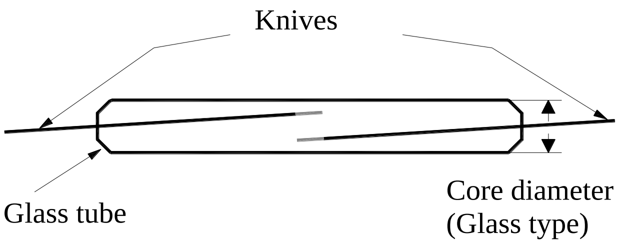


Fig. 1. Switch scheme

The switch manufacturing is carried out according to the consumer demands. A demand includes a part number, amount and delivery date. A part number corresponds to an acceptable value of operation range and a metallic knives form of switch.

All switches pass technological route which is composed of several successive operations. The investigation focuses on the classification process, which is the penultimate technologic operation in the route. On each switch the operation value is measured and then it is assigned to a bin associated with a deterministic interval of the operation range value. The procedure is performed per lots, in an identical machines group. Each lot contains 5500 pieces with tubes of the same glass type. A lot does not divide between machines. Several glass types are utilized to obtain different operation ranges. When a machine begins lot processing does not stop until finishing.

Each machine has 25 bins (Fig. 2, a). The lot concept disappears when the lot content is distributed between the bins, and then the product is handled in pieces.

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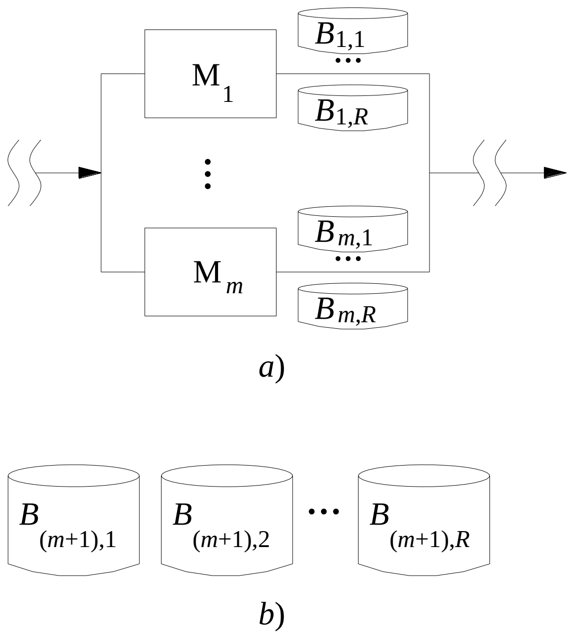


Fig. 2. Classification process: a) *m* identical machines with *R* bins; b) unlimited buffer with *R* bins.

To be complete, a demand uses one or more lots. The pieces requested by the demands are processed on next operation. The rest of pieces called “work in process” (WIP) is accumulated in an unlimited buffer according to its operation range waiting for a demand that will require it (Fig. 2, b).

## III. STATISTICAL ANALYSIS OF PARAMETERS

The experience shows that the glass type affects the distribution of the switches between the bins. The statistical analysis is focused on establishing such dependence.

Ten lots for every glass type were recollected as real samples of the classification process. The information used for analysis varies in size of lot due to scrap. To uniform the size, 1000 pieces are extracted from each lot using the MATLAB discrete uniform random numbers generator.

The means and standard deviations of the pieces distribution into 25 bins are evaluated, taking 10 lots for each glass type (G1, G2, G3, G4) (Table 1).

As the table shows, the means reveal an influence of the glass type on distribution results, so that a larger mean value corresponds to a highest glass type. The mean of each glass type fluctuates within a range and does not intersect with other ranges.

The mode value for the first two glass types shows a left bias respect to mean. The modes for the remaining two glass types, whose means are closed to the distribution center (bins 12-13), vary on both sides of the mean. The standard deviations for all glass types vary within the range [3.04, 4.75].

The samples are grouped according to glass type to reveal the distribution trends for each group.

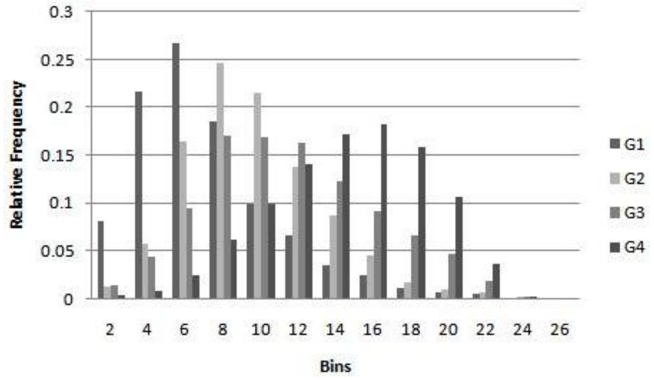


Fig. 3. Distribution histogram

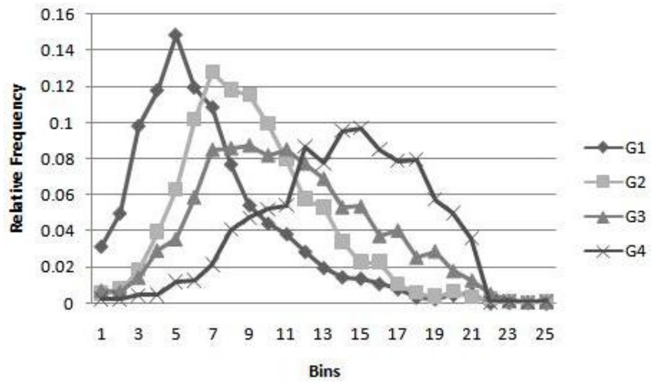


Fig. 4. Ditribution trends

Fig. 3 shows the pieces distribution histogram for glass types G1, G2, G3, G4; Fig. 4 illustrates the switches distribution trends for each glass type.

## IV. LOTS NUMBER MINIMIZING PROBLEM

The completion of demand according to their delivery dates depends on correct production planning, which implies to define: how much lots are necessary for demands execution; which kind of lot should be used to take the maximum number of pieces in required switch operation range; which order of lots processing should be selected. Actually, the company does not have a procedure that helps to solve the production planning in a trusty way.

A part number does not indicate the glass type, but specifies the operation range which permits demand completion per parts, from several lots regardless the glass type. The selection of specific glass type of lot implies the selection of bins in which is deposited the majority of the pieces (Fig. 5). As a result, the same quantity of pieces in the required operation range can be received from different number of lots. The problem is to satisfy the demands using the minimum number of lots taking advantage from differences in distribution that generates each glass type.

Table I. Statistics Sample

| Sample | GLASS TYPE | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | G1 | | | G2 | | | G3 | | | G4 | | |
| | **Mode** | **Mean** | **SD** | **Mode** | **Mean** | **SD** | **Mode** | **Mean** | **SD** | **Mode** | **Mean** | **SD** |
| 1 | 6 | 6.31 | 3.04 | 8 | 8.83 | 3.13 | 10 | 10.6 | 4.56 | 12 | 13.75 | 4.28 |
| 2 | 5 | 6.58 | 3.54 | 7 | 8.88 | 4.03 | 7 | 10.79 | 4.53 | 15 | 13.94 | 4.33 |
| 3 | 5 | 6.62 | 3.7 | 7 | 8.91 | 3.75 | 9 | 10.8 | 4.48 | 12 | 13.96 | 4.27 |
| 4 | 5 | 6.63 | 3.49 | 7 | 8.94 | 3.83 | 11 | 10.86 | 4.44 | 14 | 14.01 | 3.99 |
| 5 | 5 | 6.67 | 4.42 | 8 | 8.96 | 3.65 | 9 | 10.87 | 4.74 | 14 | 14.01 | 4.19 |
| 6 | 5 | 6.78 | 3.82 | 9 | 9.18 | 3.37 | 11 | 11.01 | 4.45 | 14 | 14.05 | 4.03 |
| 7 | 5 | 6.82 | 4.19 | 9 | 9.27 | 3.55 | 9 | 11.02 | 4.45 | 15 | 14.12 | 3.95 |
| 8 | 5 | 6.92 | 4.04 | 7 | 9.28 | 3.51 | 10 | 11.25 | 4.4 | 14 | 14.14 | 4.1 |
| 9 | 5 | 6.96 | 3.92 | 7 | 9.34 | 3.84 | 9 | 11.28 | 4.24 | 14 | 14.23 | 4.01 |
| 10 | 5 | 7.18 | 4 | 9 | 9.59 | 3.52 | 11 | 11.46 | 4.14 | 16 | 14.59 | 3.74 |
| **Mean** | 5 | 6.75 | 3.81 | 7 | 9.12 | 3.62 | 9 | 10.99 | 4.44 | 15 | 14.08 | 4.09 |

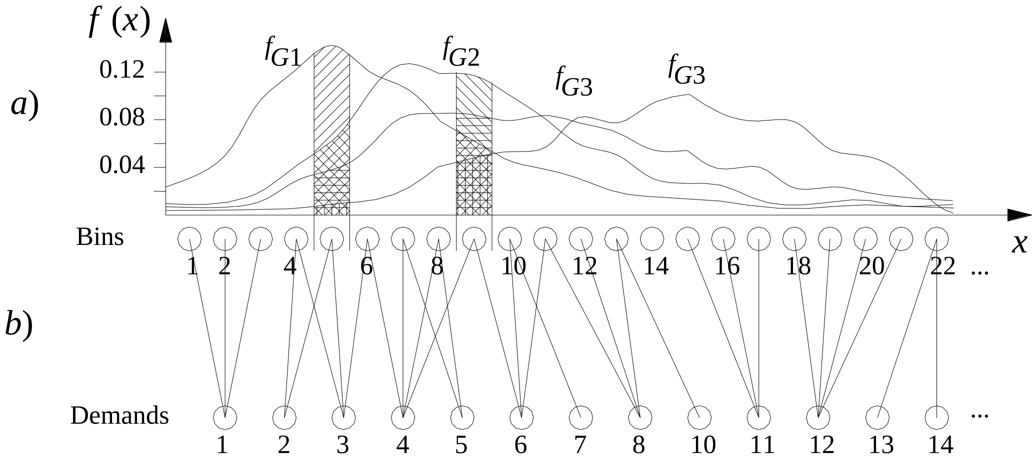


Fig. 5. Glass type selection: a) Probability density function $f(x)$ for the operation value *x*.
b) Bipartite graph of relationship between bins and demands.

**Notations**

| | |
|---|---|
| $j$ | Demand index, $j = \overline{1,N}$. |
| $Z_j$ | Number of items on demand $j$. |
| $g$ | Glass type index, $g = \overline{1,G}$. |
| $r$ | Bin index, $r = \overline{1,R}$. |
| $z_{jr}$ | Items quantity on demand *j* received from bin *r* |
| $t$ | Part number index, $t = \overline{1,T}$. |
| $p_j$ | Part number associated with the demand *j*, $p_j \in \{1,...,T\}$. |
| $o_{tr}$ | Utilization of bin *r* in the part number *t* (*T* chains of *R* bits). |
| $k_{gr}$ | Number of items distributed on bin *r* from one lot with the glass type *g*. |
| $l_g$ | Number of lots with glass type *g*. |

**Model**

A part number *t* through the element $p_j$ of vector *P* is associated with the demand *j*, $j = 1,...,N$. The row *t* of binary matrix $O = o_{tr}$ corresponds to the element $p_j$. An element of the matrix $o_{p_j r} = 1$ if the bin *r* is used for the part number $p_j$. In other cases $o_{p_j r} = 0$.

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The problem of lots number minimizing is:

$$L = \sum_{g=1}^{G} l_g \rightarrow \min \qquad (1)$$

s.t. $$\sum_{g \in \{1,\ldots,G\}} l_g k_{gr} \geq \sum_{j=1}^{N} z_{jr}, \quad r = 1,\ldots,R, \qquad (2)$$

$$\sum_{r=1}^{R} o_{p_j r} z_{jr} = Z_j, \; j = 1,\ldots,N, \qquad (3)$$

$$z_{jr} = 0 \text{ if } o_{p_j r} = 0, \; j \in \{1,\ldots,N\}, \qquad (4)$$

$$k_{vr}, z_{jr}, n_g \in Z^0, \; \forall j, r, g. \qquad (5)$$

The restriction (2), describes the relationship between production and demand for all bins. Due to (3) and (4), the expression (2) represents a system with *R* inequations and $G + \sum_{j=1}^{N} \sum_{r=1}^{R} o_{p_j r}$ unknowns.

The problem (1) is a generalization of bin packing. In the classical NP-Hard bin packing problem one is required to pack a given list of items into the smallest possible number of unit-sized bins. Bin packing applies in various areas, e.g.: stock cutting, television programming, transportation, computer storage allocation, bandwidth allocation, scheduling, etc. [1].

A difference from classical problem, in the considered case, one lot is associated with a container formed by *R* sub-containers (bins of machine) (Fig. 6). The bin capacities in a container vary depending on the parameter *g* (glass type of lot), and are defined according to the pieces distribution given by row *g* of the matrix *K*, so that $k_{g1} + k_{g2} + \ldots + k_{gR}$ represents the capacity of the container *g* (lot size). A demand is associated with an item that corresponds to a set of identical pieces. For each set is indicated the item quantity and bin numbers that are allowed for packing.

Detailed survey of the research on the bin packing problem is given by [1].

Shachnai and Tamir [2] define the problem called the Class-Constrained Bin Packing where the bins have a capacity *v* and *c* compartments. In their problem every item has the same size and a color. The items must be deposited on bin, subject to capacity constrains such that items of different colors are placed in different compartments. The goal is to minimize the number of used bins.

Fresen and Langston [3] define the variable sized bin packing problem, where the supply of containers is not only of a single bin type, but some fixed (finite) number of given sizes is available. The bin using cost is simply its size. The goal of the problem is to pack the items into bins witch sum of sizes is minimal.

Eptsein and Levin [4] propose a problem called Generalized Cost Variable Sized Bin Packing. They are given an infinite supply of bins of *r* types whose sizes are denoted by $b_r < \ldots < b_1 = 1$. Items of sizes in $0,1$ are to be partitioned into subsets. A bin type *i* is associated with a cost $c_i$, they assume $c_1 = 1$. The goal is to find a feasible solution whose total cost is minimized.

Langston [5] investigates the problem of maximizing the number of item packed into *m* available bins, where the bin sizes can be different.

Menakerman and Rom [6] investigate a bin packing problem variant in which items may be fragmented into smaller size pieces called fragments. Their model is derived from a scheduling problem present in data over CATV network. Mandal et al. [7] show that the decision problem for *N* fragmentable object bin packing when $N \geq 2$ is NP-hard.

Xing [8] introduces the problem called Bin packing with Oversized Items, where items have a size large than the largest bin size. The bins cannot be overpacked; the oversized item is free to be divided up such that the part is no larger than the largest bin size.

## V. Algorithm

Several papers study bin packing problem in offline and online environments. An online algorithm assigns items to the bins in the order they are given in the original list, without using any knowledge about subsequent items in the list; see for example [9] and [10]. If offline, the entire list is available to compute the packing [1].

There are: *R* bins, *N* sets of pieces, matrix *O* of utilization of *R* bins for *N* sets of pieces, matrix *K* of capacities of *R* bins on *G* container types.

The following heuristic offline algorithm provides a solution to the problem (1):

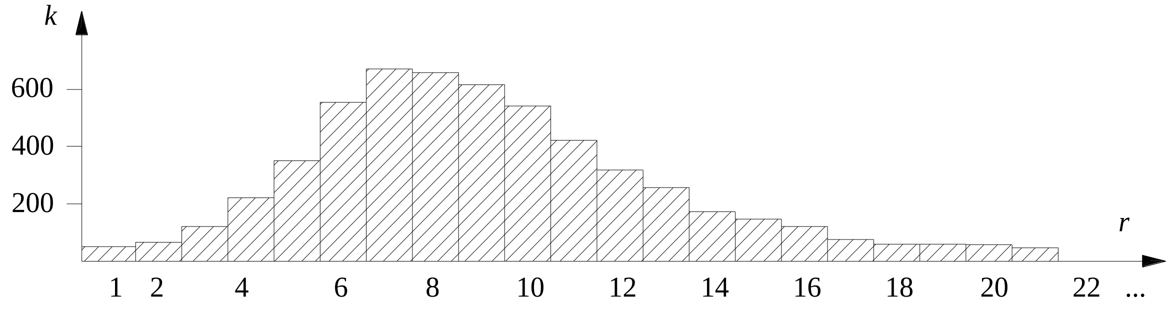


Fig. 6. A container formed by *R* bins, $r = 1,\ldots,R$, of capacities $k_{G2,r}$ associated with the type glass G2 of lot.

1. Create a table with $N+1$ rows and $R+1$ columns, where each of $N$ rows is used for assignation of pieces of set $j$, $j=1,...,N$ to $R$ bins. Initially all cells are zero; $l_g = 0$, $g=1,...,G$. In column $R+1$ is written the number of pieces in each set. The cell in intersection of row $j$ and column $r$ is available if $o_{p_j r} = 1$. The unavailable cells are blocked for each row.
2. Sort the rows in the table using a weight rule (more pieces, fewer pieces, larger number of available bins, etc).
3. Create $G$ copies of the table.
   3.1. On the copy $g$, $g=1,...,G$, the cell $r$ of row $N+1$, is summed with $k_{gr}$, $r=1,...,R$.
   3.2. The not blocked cells of the table are processed per rows, starting in the upper left corner while the corresponding value on column $R+1$ is different from zero.
   The cell values of row $N+1$ are assigned to the corresponding cells of row $r$ so that the assigned values do not overflow the value on cell $(j, R+1)$.The assigned value is subtracted from the cells $(N+1, r)$ and $(j, R+1)$.
   It continues until the values on all cells of row $N+1$ are assigned to the available cells or the value of cell $(j, R+1)$ is zero.
4. Calculate the totals for each table as the sum of values on the column $R+1$.
5. Select the table $g_i$, $i \in \{1,...,G\}$, which total is minimum.
   - If two or more minimal totals values exist, select the table which total of row $N+1$ is minimal.
   - If there is a tie in the two previous rules, select the first table.
   $l_{g_i} = l_{g_i} + 1$.

   It keeps the packing history (log): the table state and the selected indexes $g_i$.
   Clear the available cells for allocation on selected table.
6. If exist values on column $R+1$ different from zero, go to 2.
7. End

To illustrate the algorithm execution the next example is presented:

Let 3 sets of size: 70(2), 100(1, 2), 75(2, 3), where values on parenthesis are the bins allowed to use. There are 2 containers type (G1, G2) with 3 bins each one. The bin sizes on container G1 are correspondingly: 10, 20 and 60; the bin sizes on container G2 are: 20, 30 and 50. Fig. 7 shows the first iteration of the algorithm.

In four iterations the algorithm receives next result: $N=\{2,2\}$ and 110 excess pieces (WIP) are generated.

If only G1 container type is used, the result is: $N_{G1}=\{5\}$ and 255 excess pieces are generated. If only is used G2 the number of containers is $N_{G2}=\{4\}$ and 155 excess pieces are generated. It coincides with containers quantity received from algorithm with two container types, but generates 45 excess pieces more.

| | | | |
|---|---|---|---|
| 0 | 0 | X | 100 |
| X | 0 | 0 | 75 |
| X | 0 | X | 70 |
| 0 | 0 | 0 | 0 |

| | | | |
|---|---|---|---|
| 0 | 0 | X | 100 |
| X | 0 | 0 | 75 |
| X | 0 | X | 70 |
| 0 | 0 | 0 | 0 |

| | | | |
|---|---|---|---|
| | | X | 100 |
| X | | | 75 |
| X | | X | 70 |
| 10 | 30 | 60 | |

| | | | |
|---|---|---|---|
| | | X | 100 |
| X | | | 75 |
| X | | X | 70 |
| 20 | 30 | 50 | |

| | | | |
|---|---|---|---|
| 10 | 30 | X | 60 |
| X | - | 60 | 15 |
| X | - | X | 70 |
| 0 | 0 | 0 | 145 |

| | | | |
|---|---|---|---|
| 20 | 30 | X | 50 |
| X | - | 50 | 25 |
| X | - | X | 70 |
| 0 | 0 | 0 | 145 |

| | | | |
|---|---|---|---|
| | | X | 60 |
| X | | | 15 |
| X | | X | 70 |
| | | | |

N={ 1 0 }

Fig. 7. Example of the first iteration of algorithm

## VI. Conclusions and Future Work

The nonfulfilment of demands at delivery date produces economic losses and the production process disorganization. The classification process results affect directly the demands fulfillment capacity.

The pieces that compose one lot are classified into bins, according to the distribution type characteristic for the glass type of lot.

The prior knowledge of the classification results allows the production process modeling.

The stochastic problem is replaced by the deterministic through the use of empirical distributions of pieces for each glass type obtained from real data of the company.

The model for minimization of lots necessary to carry out all demands on delivery date is created based on empirical distributions.

The problem of lots quantity minimization is interpreted as a bin packing problem.

This problem occurs in electronics companies when products to be classified use materials that have some variation type.

A heuristic offline algorithm that minimizes the lots quantity necessary to carry out a set of demands, taking into account the material type in the lot, is presented. The distribution parameters do not affect the algorithm. The log of the algorithm indicates the lots processing turn, and pieces number of lot assigned to each demand.

To compensate the classification results variation respect to the performance of the real process, it is necessary: update the samples of the company, investigate the behavior of scrap, maintain a certain level of WIP; they will be exanimate on future research.

## ACKNOWLEDGMENT

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