Comparative Evaluation of Production Efficiency: A DEA Approach

Pamini Thangarajah and Kalinga Jagoda *

Abstract

The rapid changes in oil prices in last five years forced companies to rethink about their manufacturing process. This paper examines application of DEA and revenue management models for measuring and improving of productivity of a Canadian packaging company. The critical success failure factors are also highlighted.

Keywords: Data Envelopment Analysis, Packaging Industry, Productivity improvements

1 Introduction

With the current economic downturn and theatrical improvements in information and communication technologies, maintaining a company's competitiveness has become increasingly difficult. Productivity has been a vital part of firm's competitiveness and is one of the widely used performance measures of business [1]. In last four decades, productivity measurements and improvements have been discussed and analyzed by researchers, practitioners and educators. However, absence of clear practical definition of productivity lead managers to implement inefficient initiatives and those even resulted in decease in productivity [2]. Productivity is often used interchangeably with efficiency and effectiveness [3]. Therefore, understanding what is mean by term productivity and factors affecting productivity at firm level will be a good starting point for any productivity improvement exercise. The term productivity is generally defined as the relation between output and input [3]. In the cost engineering perspective it is defined as quantities produced per unit of input while in legal perspective it is the units of inputs required to produce one unit of output [4]. Determination of input and output to produce or provide a service is not an easy task. While direct inputs such as labor, material or energy are easy to determine, indirect inputs such as overhead and environmental costs are difficult to measure at the product or process level. This is further complicated by the finding ways to include opportunity costs, waste and under utilization of equipment. Productivity measurement and improvements are key building blocks of improving operating performances. In last two decades, focus on these have shifted from attempts to characterize performance in terms of a simple indicator) to a multi-dimensional systems perspective. In contemporary manufacturing settings, managers are required to implement suite of initiatives simultaneously rather than single initiative. In this scenario managers must consider the relative effects of one initiative on the other as each initiative may link to the outcome of the other. This paper discusses the evaluation of productivity measures by employing the method of Data Envelopment Analysis (DEA). Empirical data obtained from a packaging company in Canada used to illustrate the model. The proposed model evaluate relative-to-best performance efficiency of productivity improvement alternatives related to the packaging industry using multiple inputs and outputs and it also evaluate the relative efficiency of implementing multiple alternatives simultaneously.

2 Literature Review

More than three decades ago DEA was introduced by Charnes, Cooper, & Rhodes [5] as an evaluation tool for decision-making units (DMUs). The model developed by them is commonly referred to as CCR model. Since then it has been applied to various problems in variety of industries including the semiconductor [6], fast food [7], hotels [8], airlines [9], power generation [10], banks [11], and retail [12]. It is used to assess efficiency of DMU and evaluate the performance of the DMUs within a given sample without imposing any functional form on the data. The units are referred to as the literature identifies three types of DEA programming, radial efficiency measure, non-radial efficiency measure, and efficiency achievement measure. In their study of 30 ERP projects, Stensrud and Myrtveit [13] deployed Data Envelopment Analysis Variable Returns to Scale (DEA VRS) for comparisons of individual software projects. They argues that that when combined with methods for hypothesis testing, DEA VRS is a better tool useful for assessing the effect of alleged process improvements by software companies. Wu and Bai [14] adopted DEA to benchmark and monitor the productivity of retailing units. Using empirical data of convenience stores in Taiwan they mea-

^{*}Pamini Thangarajah Ph.D, Associate Professor, Dept. of Mathematics, Physics and Engineering, Mount Royal College, Calgary, Alberta, Canada, Email: pthangarajah@mtroyal.ca, Kalinga Jagoda Ph.D, Assistant Professor, Bisset School of Business, Mount Royal College, Calgary, Alberta, Canada, Email: kjagoda@mtroyal.ca

sured relative-to-best performance efficiency of retail and examine the competitiveness of a particular retail chain. Odeck [15] combined DEA and a Malmquist index with a bootstrap method to measure performance of grain producers in Eastern Norway. In similar approach Chang et al [16] used DEA to calculate Malmquist indexes of productivity and efficiency changes of 62 US accounting firms before and after enactment Sarbanes-Oxley Act. Their results indicate that those firms have shown considerable post SOX growth in productive efficiency. Packaging is an integral part of contemporary supply chain. As the products travel much longer distances from producers to consumers, High-quality packaging is becoming a key to successful competition. It aids the producers to transport products over short and long distances; to store products for much longer periods; and to sell products efficiently in wholesale and retail stores [17]. The polymers are the most popular choice of material in the packaging industry. Apart from basic role of protecting the product it also assist to combat counterfeiting and misuse. Increased awareness of the environment and sustainability put packaging industry under the microscope. Many legislations and directives have been introduced to reduce the packaging waste. Many directives were aimed at introducing measures to reduce waste by redesigning of products and increase the durability and reducing the amount of waste [18, 19] by the packaging companies. In this study authors attempt to incorporate these operating factors in their decision making model. Although DEA is widely used in many industries, its applications in the packaging industry are rather limited. Liu and Wang [20] used to measure the Malmquist productivity of semiconductor packaging and testing firms in Taiwan. However their study is focused on Liability ratio as inputs and growth rate, net profit after tax, profitability ratio, and output value by employee as outputs. Yang and Kuo [21] deployed DEA with analytic hierarchy process to solve a plant layout design problem. They used flow distance, adjacency score, shape ratio, flexibility, accessibility, and maintenance to determine the best layout for an IC packaging company.

3 Methodology

This study proposes the developing a model to evaluate the alternatives in the packaging industry. An explanatory study using focus groups was carried out in Western Canada to identify the common alternatives used to improve productivity in the packaging industry. The CEOs, operations managers, consultants, line supervisors' as well lower level workers were interviewed. Based on the interviews three alternatives regrind, reduce waste and running machine faster were identified. The dramatic increase in petroleum based products in last five years, cost of raw materials of the packaging industry polymers skyrocketed. Therefore, the waste produced in the manufacturing process became a challenge and introducing

methods to reduce waste was essential for survival. In most packaging methods the most amount of waste was produced in the starting of a new batch and out-of-line production. However, implementing waste reduction programs has not been cheap. It requires considerable investment of time and money and any decision should incorporate these cost factors. Regrind is collecting waste material (polymer) and breaking them down into small pellets and use suing them as a part of input. Many companies interviewed indicated that this is an expensive process that requires significant investment. Running machines faster (extruder in particular) is a common practice among many companies as it would reduce unit fixed costs. Although this provides short-term high productivity gains, with higher raw material costs this could lead to an increased production cost. In addition, as there will be more finished products inventory costs would also increase.

4 Model Formulation

In the packaging industry, common methods companies implement to improve productivity are Regrinding, Reducing waste, and Running machines faster.

To study the relative efficiency of regrinding, reducing waste and running machines faster (3R) with the same goals and objective, by using Data Envelopment Analysis, we shall consider the following operating units:

- regrinding material
- reducing waste
- running machine faster
- operating normally
- regrinding, reducing waste and running machines faster (doing all three)

To do this we have identified the following three input measures

- Cost of material
- Extrusion cost
- $\bullet\,$ Waste rate

and products as output measure.

To summarize the the input and output measures for each of the options, let us consider the following notations:

4.1 Notations

$$\begin{split} X_1 &:= \text{new Material} \\ a &:= \text{cost of new material per unit} \\ X_2 &:= \text{regrinding material}(\text{from scrap}) \\ g &:= \text{cost of regrinding material per unit} \\ X_3 &:= \text{extrusion Cost} \\ d &:= \text{less extrusion Cost when running machine faster} \\ X_4 &:= \text{waste percentage} \\ h &:= \text{reduced waste percentage} \\ Y_1 &:= \text{units of pieces}(\text{good product}) \\ b &:= \text{cost of the product per unit} \\ c &:= \text{cost of the scrap per unit} \\ f &:= \text{increase in good product when waste reduced} \end{split}$$

With the notations for the constants we have the table 1 for input measures for each option:

Table 1: Input Measures

	Regrind	Reduce	Run	Normal	Do
		waste	machine		all
			faster		three
Cost of	$a(X_1$	aX_1	aX_1	aX_1	$a(X_1$
Materials	$-X_{2}$)				$-X_{2}$
	$+g\bar{X}_2$				$+g\bar{X}_2$
Extrusion	X_3	X_3	$X_3 - d$	X_3	$X_3 - d$
Cost	-	-	-	-	-
Waste	X_4	$X_4 - h$	X_4	X_4	$X_4 - h$

and the table 2 for the output measure for each option:

Table 2: Output Measures

	Regrind	Reduce waste	Run machine faster	Normal	Do all three
products	$bY_1 \\ + c(Y_2 \\ - X_2)$	$\begin{array}{c} b(Y_1+f)\\ +cY_2 \end{array}$	$bY_1 + cY_2$	$bY_1 + cY_2$	

To determine the weight of each option will have in computing the outputs and inputs for the virtual production, we use the following decision variables: $W_1 := \text{regrind}$

 $W_2 :=$ reduce waste

 $W_3 :=$ run machine faster

 $W_4 :=$ Normal

 $W_5 := \text{ do all three}$

The DEA approach requires that the sum of these weights equal 1. Hence the first contraint is

$$W_1 + W_2 + W_3 + W_4 + W_5 = 1.$$

Other constraints will be obtained from

1. The output measure. That is,

the output for the virtual production (which is determined by computing the weighted average of the corresponding output for all the options) to be greater than or equal to the output for the option that we are measuring the relative efficiency.

2. Each of the three input measures. These constraints formulated from the relation

Input for the virtul production $\leq E_i$ (input for the option i),

where E_i is the efficiency index for the option *i*.

Thus we have the DEA progarmming model for the efficiency evaluation of each option.

4.2 Research Problem

Let

$$W = \left(\begin{array}{c} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \end{array} \right),$$

Input measures:

$$X = \left(\begin{array}{cccc} a(X_1 - X_2) + gX_2 & X_3 & X_4 \\ aX_1 & X_3 & X_4 - h \\ aX_1 & X_3 - d & X_4 \\ aX_1 & X_3 & X_4 \\ a(X_1 - X_2) + gX_2 & X_3 - d & X_4 - h \end{array} \right)^T,$$

and Output measures:

$$\mathbf{Y} = \begin{pmatrix} bY_1 + c(Y_2 - X_2) \\ b(Y_1 + f) + cY_2 \\ bY_1 + cY_2 \\ bY_1 + cY_2 \\ b(Y_1 + f) + c(Y_2 - X_2) \end{pmatrix}^T$$

Then, for each i = 1, 2, 3, 4, 5 we have the following five models,

the objective function for the DEA Model is min ${\cal E}_i$

subject to the following constaints

where Y^i and X^i are i^{th} column of the matrices Y and X respectively.

Thus we have the following models:

4.3 Examples of Models

Model 1: Efficiency evaluation of Regrinding

Objective function is

 $\min E_1$

Subject to

(b

Model 2: Efficiency evaluation of Reducing waste

Objective function is

 $\min E_2$

Subject to

Model 3: Efficiency evaluation of Running machine faster

Objective function is

$$\min E_3$$

Subject to

Model 4: Efficiency evaluation of doing normal operation

Objective function is

 $\min E_4$

Subject to

$$\begin{split} & W_1 + W_2 + W_3 + W_4 + W_5 &= 1 \\ & (a(X_1 - X_2) + bX_2) W_1 &= 1 \\ & + \sum_{i=2}^4 aX_1 W_i + (a(X_1 - X_2) + bX_2) W_5 &\leq E_4 aX_1 \\ & X_3 W_1 + X_3 W_2 + (X_3 - d) W_3 + X_3 W_4 + (X_3 - d) W_5 &\leq E_4 X_3 \\ & W_1 X_4 + (X_4 - h) W_2 + X_4 W_3 + X_4 W_4 + (X_4 - h) W_5 &\leq E_4 X_4 \\ & (bY_1 + c(Y_2 - X_2)) W_1 + (b(Y_1 + f) + cY_2) W_2 \\ & + (bY_1 + cY_2) W_3 + (bY_1 + cY_2) W_4 \\ & + (b(Y_1 + f) + c(Y_2 - X_2)) W_5 \geq (bY_1 + cY_2) \\ & E_4, W_1, W_2, W_3, W_4, h, d, f &\geq 0 \\ & h &\leq 1 \end{split}$$

Model 5: Efficiency evaluation of Regrinding, reducing waste and running machine faster

 $\min E_5$

Objective function is

Subject to

4.4 Application of the Model

To test the validity of the models, a set of data was obtained from a Canadian Company in the packaging industry in Calgary Alberta, which is given in the table 3.

Table 3: Data

X_1	100000
X_2	4500
a	.9
g	0
X_3	14750
d	1341
X_4	.07
h	.02
Y_1	93000
b	1.4
Y_2	7000
C	0

For the data, the table 4 illustrates the input and output measures:

Table 4:	Input	and	output	measurements
----------	-------	-----	--------	--------------

	Regrind	Reduce	Run	Normal	Do all
		waste	machine		three
			faster		
Cost of	85950	90000	90000	90000	85950
Materials					
Extrusion Cost	14750	14750	13409	14750	13409
Waste	.7	.5	.7	.7	.5
products	130200	133000	130200	130200	133000

From that we have the following model for the efficiency evaluation of regrinding:

Objective function is

 $\min E_1$

Subject to

```
W_1 + W_2 + W_3 + W_4 + W_5
                                                                             1
                                                                      =
   85950W_1 + 9000W_2 + 9000W_3 + 9000W_4 + 85950W_5
                                                                             85950E_{1}
                                                                      \leq
14750W_1 + 14750W_2 + 13409W_3 + 14750W_4 + 13409W_5
                                                                             14750E_{1}
                                                                      \leq
              .07W_1 + .05W_2 + .07W_3 + .07W_4 + .05W_5
                                                                             .07E_{1}
                                                                      <
    1.302W_1 + 1.33W_2 + 1.302W_3 + 1.302W_4 + 1.33W_5
                                                                      \geq
                                                                             1.302
                                   {}^{}_{E_1}, {}^{}_{W_1}, {}^{}_{W_2}, {}^{}_{W_3}, {}^{}_{W_4}, {}^{}_{W_5}
                                                                     > 0
```

The solution to the model is

This solution shows there is no evidence that regrinding is inefficient compare to others, for the company's data.

The solutions for other models are listed in the table 5.

Table 5: Solutions for other models

	W_1	W_2	W_3	W_4	W_5
$E_2 = 1$	0	1	0	0	0
$E_3 = 1$	0	0	1	0	0
$E_4 = .95$	0	0	0	0	1
$E_5 = 1$	0	0	0	0	1

Hence there is evidence that doing normal operation is relatively ineffecient compare to others, for the company's data.

5 Conclusions and Future Work

5.1 Conclusion #1:

For packaging company, we have the following relations:

- 1. Since the good product is the amount of material used less of the waste, we have $Y_1 = X_1 X_1 X_4$.
- 2. Since f is the increase in good product when waste reduced by h percent, we have $f = hX_1$.

- 3. Since Y_2 is the scrap, $Y_2 = X_1 X_4$.
- 4. Since X_2 is the part of Y_2 , $X_2 = \alpha Y_2$, where α is the percentage of scrap reused.

With this relation our Model 1 becomes,

Objective function is

 $\min E_1$

Subject to

$$\begin{split} & W_1 + W_2 + W_3 + W_4 + W_5 &= 1, \\ & \sum_{i=1,5} \left(b \left(1 - X_4 \right) + c \left(X_4 - \alpha X_4 \right) \right) W_i \\ & + \left(c X_4 + b \left(1 - X_4 + h \right) \right) W_2 \\ & + \sum_{k=3,4} \left(b \left(1 - X_4 \right) + c X_4 \right) W_k &\geq b \left(1 - X_4 \right) \\ & + c \left(X_4 - \alpha X_4 \right), \\ & \left(a \left(1 - \alpha X_4 \right) + g \alpha X_4 \right) W_1 \\ & + \sum_{i=2}^4 a W_i + \left(a \left(1 - \alpha X_4 \right) + g \alpha X_4 \right) W_5 &\leq E_1 \left(a \left(1 - \alpha X_4 \right) + g \alpha X_4 \right), \\ & \sum_{k=1,2,4} X_3 W_k + \left(X_3 - d \right) W_3 + \left(X_3 - d \right) W_5 &\leq E_1 X_3, \\ & \sum_{k=1,3,4} X_4 W_k + \left(X_4 - h \right) W_2 + \left(X_4 - h \right) W_5 &\leq E_1 X_4. \end{split}$$

This Model is independent of the amount of new material used. Thus the efficiency evaluation is independent of the amount of material used.

5.2 Conclusion #2:

In the company's data there was no cost attached to the waste. That is c = 0. If there is a cost attached to getting rid of the waste (i.e c < 0), then the following model shows that running machine faster is relatively inefficient.

Consider the Model 3,

Objective function is

 $\min E_3$

Subject to

$W_1 + W_2 + W_3 + W_4 + W_5$	=	1,
$89212.5W_1 + 9000W_2 + 9000W_3 + 9000W_4 + 89212.5W_5$	\leq	$9000E_{3}$
$14750W_1 + 14750W_2 + 13275W_3 + 14750W_4 + 133275W_5$	\leq	$13275E_{3}$
$.07W_1 + .05W_2 + .07W_3 + .07W_4 + .05W_5$	\leq	$.07E_{3}$
$1.302W_1 + 1.331W_2 + 1.302W_3 + 1.302W_4 + 1.331W_5$	\geq	1.302
$E_1, W_1, W_2, W_3, W_4, W_5$	≥ 0	

The solution to this model is

E_3	=	.99
W_1	=	0
W_2	=	0
W_3	=	0
W_4	=	0
W_5	=	1

Hence there is evidence that running machine faster is relatively ineffecient compare to others.

5.3 Future work

What we have done so far is for only one product. We would like create a model and study the model for multiple products.

References

- Rao, M.P., Miller, D.M., Expert systems applications for productivity analysis, Industrial management & Data Systems, V104, N9, pp 776-785, 2004.
- [2] Neely,A., The evolution of performance measurement research, International Journal of Operations & Production Management, V25, N12,pp. 1228-1263, 2005.
- [3] Tangen, S., An overview of frequently used performance measures, Work Study, V52, N7, pp 347-354, 2003.
- [4] Kelly, R.D., Labor productivity impacts, AACE International Transactions, ESTA 12.1-6, 2000.
- [5] Charnes, A., Cooper, W.W., Rhodes, E., European Journal of Operations Research, V2, N6, pp. 429-444, 1978.
- [6] Kozmetsky, G., Yue, P., Comparative performance of global semiconductor companies, Omega, V26, pp. 153-175, 1998.
- [7] Donth, N., Hershberger, E. H., Osmonbekov, T., Benchmarking marketing productivity using data envelopment analysis, Journal of Business Research, V58, N11, pp. 1472-1482, 2005.
- [8] Keh, H. T., Chu, S., Xu, J., Efficiency, effectiveness and pMeasuring efficiency of decision-making unitsroductivity of marketing in services, European Journal of Operational Research, V170, N1, pp.265-276, 2006.
- Schefczyk, M., Operational performance of airlines: an extension of traditional measurement paradigms, Strategic Management Journal, V 14, N 4, pp. 301-317, 1993.
- [10] Korhonen, P.J., Luptacik, M., Eco-efficiency analysis of power plants: an extension of data envelopment analysis, European Journal of Operations Research, V154, pp. 437-446, 2004.

- [11] Camanho, A.S.,Dyson, R.G., Cost efficiency measurement with price uncertainty: a DEA application to bank branch assessments, European Journal of Operational Research, V161, pp. 432-46, 2005.
- [12] Barros, C.P., Alves, C., Hypermarket retail store efficiency in Portugal, International Journal of Retail & Distribution Management, Vol. 31 No.11, pp.549-60, 2003.
- [13] Stensrud, E., Myrtveit. I, Identifying high performance ERP projects, IEEE Transactions on Software Engineering, V29, N5, 398-411, 2003.
- [14] Wu, W, Bai, C., Benchmarking and monitoring performance of retailing: an empirical study of convenience stores in Taiwan, The Business Review, V10, pp. 289-294, 2008.
- [15] Odeck, J., Statistical precision of DEA and Malmquist indices: A bootstrap application to Norwegian grain producers, Omega, V37, N5, pp. 1007-1014, 2009.
- [16] Chang, H, Choy, H.L., Cooper, W.W., Ruefli, T.W. Using Malmquist Indexes to measure changes in the productivity and efficiency of US accounting firms before and after the Sarbanes-Oxley Act, Omega,V37, N5, pp. 951-960, 2009.
- [17] Lewis, H., Defining product stewardship and sustainability in the Australian packaging industry, Environmental Science & Policy, V8, N1, pp. 45-55, 2005.
- [18] Fullerton, D., Wu, W., Policies for Green Design, Journal of Environmental Economics and Management, V36, N2, pp.131-148, 1998.
- [19] van Schaik, Reuter, M.A., The optimization of endof-life vehicle recycling in the European Union, Journal of the Minerals, Metals and Materials Society, V 56, N 8, pp. 39-47, 2004.
- [20] Liu, F.F., Wang, P., DEA Malmquist productivity measure: Taiwanese semiconductor companies, International Journal of Production Economics, V112, N1, pp. 367-379, 2008.
- [21] Yang, T., Kuo, C., A hierarchical AHP/DEA methodology for the facilities layout design problem, European Journal of Operational Research, V147, V1, pp. 128-136, 2003.