Inventory and Production Planning in A Supply Chain System with Fixed-Interval Deliveries of Manufactured Products to Multiple Customers with Scenario Based Probabilistic Demand

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Abstract— **In this paper we developed a weighted probabilistic total cost function in a manufacturing center which supplies manufactured products to multiple customers, with a fixed-quantity at a fixed time-interval to each of the customers while total demand of the customers have a probabilistic behavior and maybe changes during planning horizon. A closed-form solution for the minimal total cost for the entire inventory-production system formulated. The algorithm considered as the solution finding procedure for multiple customer systems with probabilistic future demand rate.**

Index Terms— **Probabilistic demand, Scenario based system, Single vendor-single buyer supply chain.**

I. Introduction

Inventory plays a significant role in many of manufacturing systems. A large number of manufacturing facilities uses for carry large inventories of manufactured products at the supply docks. Newman [1] illustrated that for eliminating any delays in the delivery process when the buyers receive manufactured products based on JIT system. Yilmaz[2], Parlar and Rempala[3], Pan and Liao[4] and Ramasesh[5] developed optimal ordering policy and quantitative production models in the single-stage production system. Lu[6] formulated a one-vendor multi-buyer integrated inventory model. Goyal[7], Goyal and Gupa[8], and Aderhunmu et al.[9] have developed some quantitative models for integrated vendor-buyer policy in a just-in-time manufacturing process. Golhar and Sarker[10], Jamal and Sarker[11], and Sarker and Parija[12,13] documented some single-product models in a just-in-time production-delivery system. Banerjee[14] illustrated a lot sizing model with the concentration to the work-in-process in response to periodic and not probabilistic demands. Park and Yun[15] suggested a stepwise partial enumeration algorithm for solving fluctuating demand problems. Sarker and Parija[12,13] developed the model of Golhar and Sarker[10]. To take into account cyclic scheduling for a multi-product manufacturing system, Nori and Sarker[16] revised and improved the model of Sarker and Parija[13]. Robert et al.[18] considered a two-echelon supply chain. Liang-Yuh et al.[19] presented a single vendor-single buyer integrated system in which lead time for demand is deterministic and stochastic with permitted shortage. ManMohan S. et al. [20] analyzed supply chain system under demand uncertainty with using stochastic programming. M.E. Seliaman et al.[21] considered the case of a three-stage non-serial supply chain system. With the review of the literature this is obvious that limited researches related to optimal ordering and production policies for the manufacturing systems with multiple customers, fixed time-intervals and probabilistic customer's demands, has been taken.

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A. Logistic System

In this paper we focused to a scenario based production-inventory system. A vendor supplies parts to manufactures and plays a significant role in the industries in the world, which, in turn, are delivered to several outside customers. While the total demand of the customer's maybe changes according to their planning updates. To satisfy buyers demand in the different time-intervals, the manufacturing company has to regularly maintain its production rate for procuring parts at regular time intervals. Because each of the customer's demand is not fix during production time horizon, and also these changes has a predictable rule or probabilistic function, therefore we should consider all of the alternatives and possibilities according to its probabilistic existence weights.

A. Problem Definition

A manufacturing company acquire its raw materials via outsourcing process, takes them under process to produce a manufactured product, stores them in a manufactured products inventory, and at last delivers manufactured products to several customers with a fixed quantities and intervals. The annual demands of these customers are not known precisely and have a scenario based behavior but for each of the customers at the beginning of the planning process for each time horizon we can assume that it is a constant parameter. The raw material is non-perishable, and therefore it should be supplied instantaneously to the manufacturing facility. Shortage of manufactured products due to insufficient manufactured products production is not allowed. Demand is base on probabilistic function and according to company policy, instead of demand, average demand for planning time horizon takes into calculations. The supply chain system is described in section 2. A total cost model for the manufacturing system is developed in section 3. A solution algorithm for this model explained in section 4. Finally, conclusions stated in the last section.

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II. The Supply Chain System

To find an Economic Order Quantity (EOQ) for the raw materials and an Economic Manufacturing Quantity (EMQ) for the production run, two types of inventory holding costs are considered [17]: raw materials holding cost, H_0, and manufactured products holding cost, H_F. Order costs includes the ordering cost of the raw materials, K_0 and the manufacturing setup cost for each batch, K_S. Opportunity cost, OC_F and extra sales cost, EC_F considered for illustrating upper and lower demand situations costs. α, β, γ considered as demand probabilities for three different states of the customer's demand. Following notation is used to model the system:

A. *Definitions and Notation*

Raw material related: D_R, H_0, K_0, n, Q_0 and Q_R.

Manufactured products related: D_F, $D_{i,F}$, f, H_F, K_S, m_i, P, Q_{avg}, Q_F, Q_M, Q_S, x_i, Y, OC_F, EC_F, $E(D_F)$, α, β, γ, $F(D_F)$. Cycle time related: L_i, T, T_0, T_1 and T_2.

Definitions:

D_F = total demand for manufactured products by all customers, units/year;

$D_{i,F}$ = total demand for manufactured products by customer i, units/year;

D_R = total demand for raw materials by the production facility, units/year;

f = conversion factor of the raw materials to manufactured products, $f = D_F/D_R$;

H_F = holding cost of manufactured products, \$/unit/year;

H_0 = holding cost of raw materials, \$/unit/year;

K_0 = ordering cost of raw materials, \$/order;

K_S = manufacturing setup cost per batch, \$/batch;

L_i = given time between successive shipments of manufactured products to customer i $(i = 1, \dots, \mathrm{N})$;

m_i = number of full shipments of manufactured products to customer i per cycle time T;

n = number of orders of raw materials during the uptime T_1;

P = production rate, units/year;

Q_{avg} = average inventory of manufactured products per cycle, in units;

Q_F = quantity of manufactured products manufactured per setup, units/batch;

Q_H = quantity of manufactured products inventory held at the end of uptime T_1, in units;

$Q_F(t)$ = manufactured products inventory on hand at time t; $Q_F(t) = Q_M(t) - Q_S(t)$

$Q_M(t)$ = quantity of manufactured products in the inventory at time t, in units;

Q_0 = quantity of raw materials ordered each time, units/order; $Q_0 = Q_R/n$;

Q_R = quantity of raw materials required for each batch; $Q_R = \frac{Q_F}{f} = nQ_0$;

Q_S = total quantity of manufactured products shipped, in units/cycle;

$Q_S(t)$ = total quantity of manufactured products shipped by time t;

T = cycle time; $T = \frac{Q_F}{D_F} = m_i L_i$, While $i \in \{1,\dots,\mathrm{N}\}$;

T_0 = production start time;

T_1 = manufacturing period (uptime); $T_1 = \frac{Q_F}{P}$;

T_2 = downtime; $T_2 = T - T_1 = Q_F\left(\frac{1}{D_F} - \frac{1}{\mathrm{P}}\right)$;

x_i = quantity of manufactured products shipped to customer i at a fixed interval of time L_i, units/shipment;

$$x_i = \frac{Q_F}{m_i} = L_i D_F, \text{While } i \in \{1, \dots, \mathrm{N}\};$$

Y = quantity produced during L_i period; $Y = L_i P = \frac{x_i P}{D_F}$;

$Y - x_i = (\frac{P}{D_F} - 1)x_i$

OC_F = opportunity cost for sale manufactured products when demand for manufactured products (D_F) is more than it's calculated average ($E(D_F)$), \$/unit;

EC_F = extra sale costs for sale manufactured products when demand for manufactured products (D_F) is less than it's calculated average $(E(D_F))$, \$/unit;

$E(D_F)$ = average total demand for manufactured products by all customers, units/year;

$F(D_F)$ = customer's total demand probabilistic function;

α = total customer's demand probability when manufactured products demand (D_F) is equal to average total demand for manufactured products $E(D_F)$, $(D_F = E(D_F))$, $0 \le \alpha \le 1$;

β = total customer's demand probability when manufactured products demand (D_F) is less than average total demand for manufactured products $E(D_F)$, $(D_F \le E(D_F))$, $0 \le \beta \le 1$;

γ = total customer's demand probability when manufactured products demand (D_F) is more than average total demand for manufactured products $E(D_F)$, $(D_F \ge E(D_F))$, $0 \le \gamma \le 1$;

$\alpha + \beta + \gamma = 1$.

B. *The Supply Chain System*

Manufactured products inventory in this model, doesn't have equal behavior comparison with traditional economic batch quantity model with continuous demand [13]. As depicted in Fig. 1, to discourage the undesirable inventory buildup, raw materials in this model are ordered n times during the uptime. Because production rate, P, is considered to be higher than the consumption rate, the inventory will keep on building while the production (or uptime) continues. A fluctuating-demand (fixed quantity) of x_i units of manufactured products at the end of every L_i time units to customer i, $(i = 1,2, \dots, \mathrm{N})$, is imposed to the manufacturer. This fluctuating demand decreases the manufactured products inventory buildup instantaneously by x_i that makes as a result, an inventory buildup in an increasing triangular fashion during the production period T_1. x_i units of manufactured products, to satisfy the demand of customer i at an interval of L_i time units, are delivered instantaneously that as a result remains $Y - X_i$ units at hand, where $Y = L_i P$, the quantity that produced during L_i time units at the rate of P units per unit-time. The delivery schedules of manufactured products and quantities shipped to customer i ,$(i = 1,2,\dots,\mathrm{N})$, are also shown in Fig. 1. For N customers $L_1 \le L_2 \le \cdots \le L_{N-1} \le L_N$ and all $x_\mathrm{i}, (i = 1,2,\dots,\mathrm{N})$, may not be equal and $m_\mathrm{i}, (i = 1,2,\dots,\mathrm{N})$, the number of full shipments to customer i , is a non increasing set of integer numbers $(m_1 \ge m_2 \ge \cdots \ge m_{\mathrm{N}-1} \ge m_\mathrm{N})$ such that $m_1 L_1 = m_2 L_2 = \cdots = m_{\mathrm{N}-1} L_{\mathrm{N}-1} = m_\mathrm{N} L_\mathrm{N} = T$. The on-hand manufactured products inventory at any time t is the manufactured products produced by that time minus the total inventory delivered to all customers by time t . the on-hand manufactured products at the end of uptime period T_1, Q_H, decrease instantaneously by x_i units at a regular interval of L_i time units (after the production run) till the end of the last shipment in a cycle are carried over to the next cycle, resulting in a shifted production schedule as reflected in Fig. 1.[17]

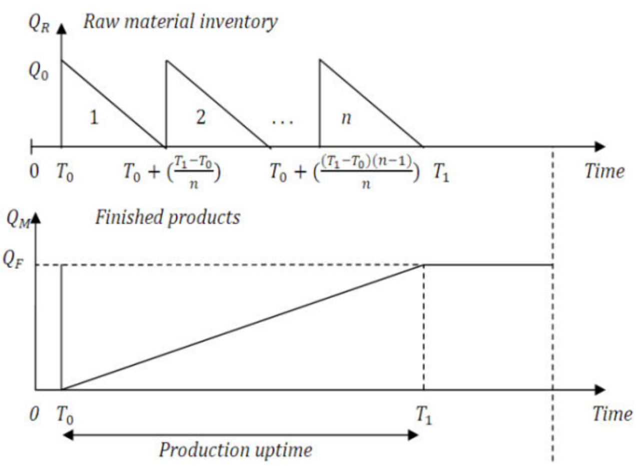

Fig.1. Raw materials and manufactured products inventory [17]

III. PRODUCTION COST MODEL

We assume that $L_1 \le L_2 \le \cdots \le L_{N-1} \le L_N$. and to be more confident in case of materials availability and proper delivery, we further assume that $Q_F = P\ (T_1 - T_0) = D_F = \sum_{i=1}^{N} m_\mathrm{i} x_\mathrm{i}$ and $T = m_\mathrm{i} L_\mathrm{i}$, While $i \in \{1,2,\dots,\mathrm{N}\}$.

A. Total Cost Functions

In the our model we assume that total future demand for manufactured products is equal to demand probability function expected value $E(D_F)$. System has costs as below:

A.1. Raw Material Costs

In an inventory state where $Q_F = P\ (T_1 - T_o) = D_F = \sum_{i=1}^{N} m_\mathrm{i} x_\mathrm{i}$, there might be some manufactured products hands over to the next cycle after $\sum_{i=1}^{N} m_\mathrm{i}$ shipments to all customers. A series of fluctuating deliveries of manufactured products inventory continues after T_1 until the cycle repeats same as what that done before. Thus the total cost of raw materials, as stated with Sarker and Parija[13], is given by

$$TC_R = \left(\frac{D_R}{Q_0}\right) K_0 + \frac{Q_0(T_1 - T_o)}{2\mathrm{T}} H_0. \quad (1)$$

Now, if there are n replenishments of raw materials during the uptime period $[T_0, T_1]$, and m_i transports of manufactured products of size x_i to customer i ,$(i = 1,2,\dots,\mathrm{N})$, during the cycle time T , then, for Q_R, each batch required raw materials, we can write $Q_0 = Q_R/n$ and $Q_F = (T_1 - T_o)P = T\ D_F = T\ \sum_{i=1}^{N} m_\mathrm{i}\, x_\mathrm{i}$. therefore equation (1) could rewrite as below:

$$TC_R = \left(\frac{nD_R}{Q_R}\right) K_0 + \frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right) H_0. \quad (2)$$

A.2 Manufactured Products Costs

Because of probabilistic changes that exist in the demand of manufactured products, three different demand states maybe arise during the planning time horizon.

A.2.1 $D_F = E(D_F)$

When manufactured products demand (D_F) is equal to expected value for manufactured products demand $(E(D_F))$, we have this state for system costs (when $D_F = E(D_F)$). Since $\frac{D_R}{Q_R} = \frac{D_F}{Q_F}$ and $Q_F = fQ_R$ for a conversion factor (or production process efficiency) of raw materials to manufactured products, f, the total cost for manufactured products inventory for average inventory Q_{avg} at a holding cost H_F \$/unit/year can also be written as[17]

$$TC_F = \frac{D_F}{Q_F} K_S + Q_{avg} H_F. \quad (3)$$

and the entire cost of the system may be expressed as

$$TC_1(Q_F, n) = \left(\frac{nD_R}{Q_R}\right) K_0 + \frac{D_F}{Q_F} K_S + \frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right) H_0 + Q_{avg} H_F. \quad (4)$$

Therefore, with replacing Q_{avg} in equation (4) with the results from what that stated by Sarker and Parija [17]:

$$TC_1(Q_F, n) = \left(\frac{nD_R}{Q_R}\right) K_0 + \frac{D_F}{Q_F} K_S + \frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right) H_0 + \left(\frac{\left(1 - \frac{D_F}{P}\right) Q_F}{2} + \frac{\sum_{i=1}^{N} x_\mathrm{i}}{2} + T_0 D_F\right) H_F. \quad (5)$$

A.2.2 $D_F \le E(D_F)$

In this state manufactured products costs could formulate as below:

$$TC_F = \frac{D_F}{Q_F} K_S + Q_{avg} H_F + EC_F(E(D_F) - D_F). \quad (6)$$

Therefore, replacing Q_{avg} in equation (6) with the results from Sarker's study[17] and with regarding raw material costs from equation (2) in section 3.1.1 we have

$$TC_2(Q_F, n) = \left(\frac{nD_R}{Q_R}\right) K_0 + \frac{D_F}{Q_F} K_S + \frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right) H_0 + \left(\frac{\left(1 - \frac{D_F}{P}\right) Q_F}{2} + \frac{\sum_{i=1}^{N} x_\mathrm{i}}{2} + T_0 D_F\right) H_F + EC_F(E(D_F) - D_F). \quad (7)$$

A.2.3 $D_F \ge E(D_F)$

In this state manufactured products costs could formulate as below:

$$TC_F = \frac{D_F}{Q_F} K_S + Q_{avg} H_F + EC_F(E(D_F) - D_F). \quad (8)$$

Therefore, replacing Q_{avg} in equation (8) with the results from Sarker's results[17] and with regarding raw material costs from equation (2) in section 3.1.1 we have

$$TC_3(Q_F,n)=\left(\frac{nD_R}{Q_R}\right)K_0+\frac{D_F}{Q_F}K_S+\frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right)H_0+\left(\frac{\left(1-\frac{D_F}{P}\right)Q_F}{2}+\frac{\sum_{i=1}^{N}x_i}{2}+T_0D_F\right)H_F+OC_F(D_F-E(D_F)). \quad (9)$$

As mentioned, three different total cost functions as three different scenarios exists for different states of the entire system base on its total manufactured products demands. Now, if T' is a time such that $T_0 \le T'$, then T_0 must satisfy[17]

$(T'-T_0)P \quad \ge \{\sum_{i=1}^{N} [T'/L_i]x_i\}$ While$T' \ge T_0$. (10)

That is, the quantity of manufactured products produced in the time interval $[T_0, T']$ is sufficient to meet the demands (shipments) for each individual customer until the time T', at all times T', after the product begins at T_0.

Theorem 1. [17]

If $T_0=\text{Min}\left\{T'-\sum_{i=1}^{N} m_i\left(\frac{x_i}{P}\right)\right\}$

Subject to $T' \ge L_1$, $T'/L_i \ge m_i \ge T'/L_i - 1$ for $i = 1,2,\dots,N$, and T': real, m_i: integer, then T_0 satisfies the inequality (10). (11)

The lower bound on the integer variable m_i will be very useful in actual computations. In theorem 1, illustrated that if T_0 is chosen as stated, then meeting demands for all the products will guarantee.

B. Start Time Determination[17]

The production start time, T_0, can be determined by solving the problem:

(PST): $Max\ T_0$,

Subject to: $(T'-T_0)P \quad \ge \{\sum_{i=1}^{N} [T'/L_i]x_i\}$, While$T' \ge T_0 \ge 0$. (12)

This means that we would like to delay starting the procurement and the production until it is absolutely necessary. The constraint in problem (PST) implies that, if

i) The quantity of manufactured products produced in the interval $[T_0, T']$ is $(T'-T_0)P$;
ii) $\left[\frac{T'}{L_i}\right]$ is the number of shipments for customer i until time, T'; *and*
iii) $\left[\frac{T'}{L_i}\right]x_i$ is the total shipment quantity to customer i until time T',

Then the quantity produced in the interval $[T_0, T']$ is enough to meet the demands for all the customers until timeT'. Note that the constraint in (PST) can be written as : $T_0 \le \left\{T'-\left(\frac{\sum_{i=1}^{N} m_ix_i}{P}\right)\right\}$, $T' \ge 0$, m_i=$\left[\frac{T'}{L_i}\right] \ge 0$, (13) which is equivalent to the inequality

$T_0 \le \text{Min}\left\{T'-\left(\frac{\sum_{i=1}^{N} m_ix_i}{P}\right) \mid \frac{T'}{L_i} \ge m_i \ge \frac{T'}{L_i}-1, T' \ge 0, m_i \ge 0 \text{ and integer.}\right.$ (14)

The right-hand side of inequality (13) is a Mixed-Integer Programming (MIP) problem. Therefore, the upper bound on T_0 is the solution of the MIP in inequality (13). Hence, the optimal production start time T^*_0 is given clearly by [17]

$T^*_0=\text{Min}\left\{T'-\left(\frac{\sum_{i=1}^{N} m_ix_i}{P}\right) \mid \frac{T'}{L_i} \ge m_i \ge \frac{T'}{L_i}-1, T' \ge 0, m_i \ge 0 \text{ and integer.}\right.$ (15)

IV. Solution Algorithm

Algorithm 1 as a strategy to arrive at a feasible solution for this problem could be use.

Algorithm 1: solution algorithm

$Step$ 1. Compute T_0 by solving the MIP in (15)

$Step$ 2. Compute$TC_1(Q_F,n)$, $TC_2(Q_F,n)$ and $TC_3(Q_F,n)$.

$Step$ 3. Obtain probabilistic weighted total cost function $(PWTC\ \ (Q_F,n))$:

$PWTC\ \ (Q_F,n)=$
$(\alpha TC_1(Q_F,n)+\beta TC_2(Q_F,n)+\gamma TC_3(Q_F,n))/$
$(TC_1(Q_F,n)$+ $TC_2(Q_F,n)$+ $TC_3(Q_F,n))$ (15)

$Step$ 4. Minimize $PWTC\ (Q_F,n)$ over $\mathbf{R}^2$ to obtain (Q^*_F, n^*)

$Step$ 5. Obtain a pragmatic solution:

(a) Generate two feasible integer solutions in the neighborhood of (Q^*_F, n^*).

$Q^0{}_F=\max\{\sum_{i=1}^{N} m_ix_i \mid m_iL_i = \text{constant for all } i,$ $\sum_{i=1}^{N} m_ix_i \le Q^*_F$, m_i:integer,

$Q^1{}_F=\max\{\sum_{i=1}^{N} m_ix_i \mid m_iL_i = \text{constant for all } i,$ $\sum_{i=1}^{N} m_ix_i \ge Q^*_F$, m_i:integer,

n^0 = Integer *n* such that *n* minimizes $\mid (Q^0{}_F/n)-(Q^*_F/n^*) \mid$, and

n^1 = Integer *n* such that *n* minimizes $\mid (Q^1{}_F/n)-(Q^*_F/n^*) \mid$.

(b) Choose the better of the two candidate solutions:

Optimum (Q_F,n) =($Q^k{}_F/n^k$) where k = arg min {$PWTC$ $(Q^i{}_F/n^i)$, i = {0,1}}

V. Conclusion

The situation that developed in this paper was about demand behavior. In the previous research demand considered as a fix parameter that we manufacturer efforts to balance its raw materials and manufactured products inventory to achieve minimum total cost for the entire system but in this paper we focused on the behavior of demand and assumed that demand has a probabilistic scenario based behavior and for each alternative we have an specified total cost function that the goal of the model is to minimize the probabilistic weighted total cost function for the entire system Future researches can be directed to considering more probabilistic sub-systems and factors in the different combinations of inventory-production-sale or value chain systems.

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