

A Multi-Objective Formulation for Facility Layout Problem

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Abstract— This paper extends a mixed integer programming formulation for facility layout problem. We consider two common conflicting objectives in facility layout: minimizing departmental material handling cost and maximizing closeness rating. We also modify our formulation to involve simultaneously design layout and determine location of Input/Output (I/O) points simultaneously.

Index Terms — Facility layout, I/O points locations, Mixed integer programming, Multi-objective.

I. INTRODUCTION

One of the oldest activities done by industrial engineers is facilities planning. The term facilities planning can be divided into two parts: facility location and facility layout. The latter is one of the foremost problems of modern manufacturing systems and has three sections: layout design, material handling system design and facility system design [15]. Determining the most efficient arrangement of physical departments within a facility is defined as a facility layout problem (FLP) [5]. Layout problems are known to be complex and are generally NP-Hard [5]. For more detailed studying in facility layout problem, readers are referred to these references: [3], [11] and [12].

In a typical layout design, each cell is represented by a rectilinear, but not necessarily a convex polygon. The set of the fully packed adjacent polygons is known as a block layout [4]. The two most general mechanisms in the literature for constructing such layouts are the flexible bay and the slicing tree [2].

Classical approach to facility layout is to minimize material handling cost. However, in real world cases, the designer interfaces with many multiple conflicting objectives to facility design. There are some works in literature which deal with multi-objective facility layout problems that are described here.

Lee et al. [10] propose a genetic algorithm (GA) for multi-floor design considering inner walls and passage. Their objectives are minimizing departmental material handling cost and maximizing closeness rating. They use weighted sum method to solve problem. With similar objectives, Ye and

Zhou [17], develop a hybrid GA-Tabu search (TS) algorithm. They also use weighted sum method. Aiello et al. [1] consider two additional objectives, to maximize the satisfaction of distance requests and to maximize the satisfaction of aspect ratio requests. GA determines pareto optimal solution and by means of electre procedure optimal is defined. Kulturel-Konak et al. [9] propose Multi- objective tabu search for combinatorial optimization problems. Suman and Kumar [14] and Konak et al. [7] present a survey for multi-objective simulated annealing (SA) optimization and a tutorial for multi-objective genetic algorithm (MOGA) respectively. Sujono and Lashkari [13] develop a multi-objective mathematical model in flexible manufacturing system (FMS) environment.

In more research in literature, to reduce complexity, it assumes that Input/Output points are located in center of departments. But in real work, I/O points are located in perimeter of departmental boundaries especially in intersections between each pair of departments. For more detail reviewing I/O point's locations problem, [2] can be helpful.

Some researchers propose integrated approaches for determination of block layout and locations of I/O points. Arapoglu et al. [2] use a GA to determine block layout and I/O points in a flexible bay environment. Kim and Goetschalckx [6] present an SA algorithm wherein a mixed integer programming (MIP) formulation to determine layout and three heuristics to find I/O points are located.

In this paper, we extend a mixed integer programming formulation for facility layout problem that was presented by Konak et al, [8]. They focus on flexible bay layout in which departments are located in vertical or horizontal columns (see Fig. 1). They consider single objective, minimizing departmental material handling cost. According to literature, there is no formulation for multi-objective facility layout problem. Our formulation involves both minimizing departmental material handling cost and maximizing closeness rating. To input closeness rating in the objective function, some constraints are necessary added to the previous model. We also modify our formulation to involve concurrent design layout and determine location of Input/Output (I/O) points with multi-objective approach. Paper is organized as follow: In section II mathematical model is presented, the approach of generating test problems is discussed in section III, computational results are stated in section IV, mathematical model to integrate determination of facility layout and locations of I/O points is presented in section and finally in

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section VI, Conclusions are showed.

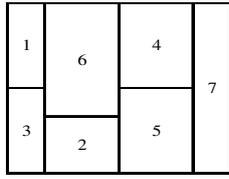


Fig 1. Flexible bay layout

II. MATHEMATICAL FORMULATION

All notations and constraints of Konak et al. [8] formulation are used in our mathematical model, however to preserve vagueness, notations are stated here:

A. Parameter

- N Number of departments,
- W Width of the facility along the x-axis,
- H Length of the facility along the y-axis,
- B Maximum number of parallel bays,
- a_i Area requirement of department i ,
- α_i Aspect ratio of department i ,
- l_i^{max} Maximum permissible side length of department i
- l_i^{min} Minimum permissible side length of department i
- f_{ij} Amount of material flow between departments i and j ,

B. Variables

- $z_{ik} = \begin{cases} 1, & \text{If department } i \text{ is assigned to bay } k \\ 0, & \text{Otherwise} \end{cases}$
- $r_{ij} = \begin{cases} 1, & \text{If department } i \text{ is above department } j \text{ in the same bay} \\ 0, & \text{Otherwise} \end{cases}$
- $\delta_k = \begin{cases} 1, & \text{If bay } k \text{ is occupied} \\ 0, & \text{Otherwise} \end{cases}$

- w_k Width (the length in the x-axis direction) of bay k ,
- l_i^y Height (the length in the y-axis direction) of department i ,
- h_{ik} Height of department i in bay k ,
- (o_i^x, o_i^y) Coordinates of the centered of department i ,
- d_{ij}^x Distance between the centered of departments i and j in the x-axis direction,
- d_{ij}^y Distance between the centered of departments i and j in the y-axis direction.

In addition to these parameters and variables, we introduce some others as follow:

C. additional Parameter

- adj_{ij} Adjacency ratio between departments
- p Weighted of objective functions ($0 \leq p \leq 1$)

D. additional variables

- $y_{ij} = \begin{cases} 1, & \text{If departments } i \text{ and } j \text{ have common boundary} \\ 0, & \text{Otherwise} \end{cases}$
- (Up_i^x, Up_i^y) Coordinates of the north east corner of

- department i ,
- (Low_i^x, Low_i^y) Coordinates of the south west corner of department i ,
- $w'_{i,k}$ Width (the length in the x-axis direction) of department i in each bay k ,
- Co_{ij} The length of common boundary between departments i and j .
- Co'_{ij} Is equal to product of Co_{ij} and y_{ij}

E. Problem formulation

$$w'_{ik} = w_k z_{ik} \quad \forall i, k \quad (1)$$

Constraint (1) determines width of each department. It is linearized as follow:

$$w'_{ik} \leq W z_{ik} \quad \forall i, k \quad (1.1)$$

$$w'_{ik} \leq w_k + W(1 - z_{ik}) \quad \forall i, k \quad (1.2)$$

$$w'_{ik} \geq w_k - W(1 - z_{ik}) \quad \forall i, k \quad (1.3)$$

Proposition 1. Constraint (1) can be lineared as constraints (1.1) – (1.3).

Proof: If $z_{ik} = 1$, then according to constraints (1.2) and (1.3) w'_{ik} is equal to w_k . if $z_{ik} = 0$, then according to constraint (1.1) $w'_{ik} = 0$. If $w'_{ik} > 0$ then, constraint (1.1) causes to $z_{ik} = 1$ and due to constraints (1.2) and (1.3) w'_{ik} is equal to w_k .

$$\left| \begin{matrix} \left(o_i^x - 0.5 \sum_k w'_{ik} \right) \\ - \left(o_j^x + 0.5 \sum_k w'_{jk} \right) \end{matrix} \right| \leq W(1 - y_{ij}) \quad \forall i < j \quad (2)$$

$$|(o_i^y - 0.5l_i^y) - (o_j^y + 0.5l_j^y)| \leq H(1 - y_{ij}) \quad \forall i < j \quad (3)$$

Constraint (2) and (3) are used to determine whether two departments have a common boundary. They state that if lower edge of a department is above upper edge of other department either in x-axis or y-axis, these departments don't have common boundary (see Figure 2.). They are linearized below:

$$\begin{matrix} \left(o_i^x - 0.5 \sum_k w'_{ik} \right) \\ - \left(o_j^x + 0.5 \sum_k w'_{jk} \right) \end{matrix} \leq W(1 - y_{ij}) \quad \forall i < j \quad (2.1)$$

$$\begin{matrix} \left(o_j^x - 0.5 \sum_k w'_{jk} \right) \\ - \left(o_i^x + 0.5 \sum_k w'_{ik} \right) \end{matrix} \leq W(1 - y_{ij}) \quad \forall i < j \quad (2.2)$$

$$(o_i^y - 0.5l_i^y) - (o_j^y + 0.5l_j^y) \leq H(1 - y_{ij}) \quad \forall i < j \quad (3.1)$$

$$(o_j^y - 0.5l_j^y) - (o_i^y + 0.5l_i^y) \leq H(1 - y_{ij}) \quad \forall i < j \quad (3.2)$$

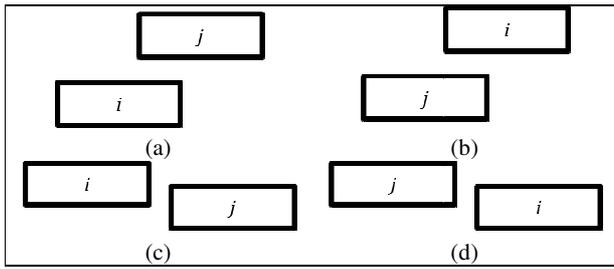


Fig 2. (a) Lower edge of department j is above upper edge of department i in y -axis, (b) Lower edge of department i is above upper edge of department j in y -axis, (c) Lower edge of department j is above upper edge of department i in x -axis, (d) Lower edge of department i is above upper edge of department j in x -axis.

$$Co_{ij} = \begin{pmatrix} \min(o_i^y + 0.5l_i^y, o_j^y + 0.5l_j^y) \\ -\max(o_i^y - 0.5l_i^y, o_j^y - 0.5l_j^y) \end{pmatrix} + \begin{pmatrix} \min\left(o_i^x - 0.5 \sum_k w'_{ik}, o_j^x - 0.5 \sum_k w'_{jk}\right) \\ -\max\left(o_i^x + 0.5 \sum_k w'_{ik}, o_j^x + 0.5 \sum_k w'_{jk}\right) \end{pmatrix} \quad \forall i < j \quad (4)$$

Constraint (4) calculates common boundary of each two departments if they have common boundary and if not it would be a negative number. The effect of this negative number can be eliminated easily as discussed later in describing the objective function. Figure 3 shows types of common boundaries between two departments.

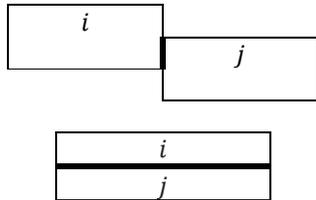


Fig 3. Common boundary of two departments

Constraint (4) is linearized as follow:

$$Up_i^y = o_i^y + 0.5l_i^y \quad \forall i \quad (4.1)$$

$$Up_i^x = o_i^x + \sum_k w'_{ik} \quad \forall i \quad (4.2)$$

$$Low_i^y = o_i^y - 0.5l_i^y \quad \forall i \quad (4.3)$$

$$Low_i^x = o_i^x - 0.5 \sum_k w'_{ik} \quad \forall i \quad (4.4)$$

$$E_{ij}^x \leq Up_i^x \quad \forall i < j \quad (4.5)$$

$$E_{ij}^x \leq Up_j^x \quad \forall i < j \quad (4.6)$$

$$E_{ij}^y \leq Up_i^y \quad \forall i < j \quad (4.7)$$

$$E_{ij}^y \leq Up_j^y \quad \forall i < j \quad (4.8)$$

$$F_{ij}^x \leq low_i^x \quad \forall i < j \quad (4.9)$$

$$F_{ij}^x \leq low_j^x \quad \forall i < j \quad (4.10)$$

$$F_{ij}^y \leq low_i^y \quad \forall i < j \quad (4.11)$$

$$F_{ij}^y \leq low_j^y \quad \forall i < j \quad (4.12)$$

$$Co_{ij} = (E_{ij}^x - F_{ij}^x) + (E_{ij}^y - F_{ij}^y) \quad \forall i < j \quad (4.13)$$

$$\text{Min } p \sum_i \sum_{i < j} f_{ij}(d_{ij}^x + d_{ij}^y) - (1-p) \sum_i \sum_{i < j} Co_{ij} y_{ij} \quad 0 \leq p \leq 1 \quad (5)$$

The objective function is given by Equation (6). The Objective function is sum weighted total of the departmental material handling costs and the maximum closeness rating. If Co_{ij} becomes a negative number it will cause the objective function to become worse. So y_{ij} which product in Co_{ij} become zero automatically in solving model and its effect is eliminated readily. Objective is linearized as follows:

$$Co'_{ij} \leq (W + H)y_{ij} \quad \forall i < j \quad (5.1)$$

$$Co'_{ij} \leq Co_{ij} + (W + H)y_{ij} \quad \forall i < j \quad (5.2)$$

$$Co'_{ij} \geq Co_{ij} - (W + H)y_{ij} \quad \forall i < j \quad (5.3)$$

$$\text{Min } p \sum_i \sum_{i < j} f_{ij}(d_{ij}^x + d_{ij}^y) - (1-p) \sum_i \sum_{i < j} Co'_{ij} \quad 0 \leq p \leq 1 \quad (5.4)$$

III. GENERATING TEST PROBLEMS

Based on our knowledge, there is not any benchmark for multi-objective facility bay layout problem. So we have modified test problem in literature and customized it for our problem. We use test problem Van camp et al. [16] that is for single objective bay layout problem with ten departments, then we generate matrix of closeness rating between departments as Ye and Zhou [17] who quantify closeness ratings that are qualitative Relationships according to Table 1.

We generate matrix of closeness rating matrix with random integer numbers in the range of [0,5] which has 20% density. Also, we assume aspect ratio is four ($\alpha_i = 4$). Areas of departments, matrix of material handling cost and matrix with closeness rating of ten-sized test problem are seen in Appendix.

Symbol	Assigned number	Relationship
A	5	Absolutely necessary
E	4	Especially important
I	3	Important
O	2	Ordinary
U	1	Unimportant
X	0	Undesirable

After that, we generate test problems with sizing 5 to 9 with random selection of departments which are in ten-sized test problem. Numbers of departments that are used to generate test problems with sizing 5 to 9 are noted in Table 2.

Table 2. Representation of test problems

Size	Number of departments
5	1,2,6,7,9
6	1,4,6,7,8,9
7	1,3,4,5,6,8,9
8	2,3,4,5,6,7,8,10
9	2,3,4,5,6,7,8,9,10
10	1,2,3,4,5,6,7,8,9,10

IV. COMPUTATIONAL RESULTS

In flexible-bay layout problem, departments are located in vertical or horizontal columns and each department is located just in one bay. Number of bays is an integer number in the range of [1, n] and optimal number of bays is determined by solving the problem. In this paper, we use fixed-bay layout approach wherein number of bays is predetermined. We fixed the number of bays with considering $\delta_k = 1$ that k is ranging from 1 to n. merely; we solve the mathematical model n times with different number of bays that is ranging from 1 to n instead of single running flexible-bay layout problem. We solve the problem for $p = 0.1, 0.3, 0.5, 0.7, 0.9$ to show the change of objective function in respect to the change of weighted of objective. We run mathematical model by use CPLEX10.1 software in a PC with 2.4 core Duo GHz CPU and 1GB RAM. The value of Objective function of each test problem is indicated in Table 3.

The summation of the computational time result of fixed-bay layout approach always is shorter than running problem with flexible bay as Table 4 illustrate. It is reasonable, because for some fixed-bay layout approach, there is no feasible solution. Also, in order to square shape of facility, having single bay is similar to having n bays with counterclockwise rotation (see fig 2.).

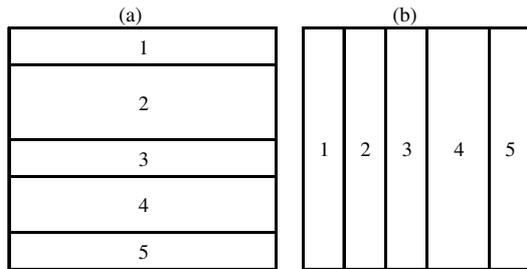


Fig 3. (a) Single bay layout, (b) Layout with n bays.

We use a measure to compare computational result of fixed-bay layout approach versus to flexible-bay layout approach as follow:

$$RPD = \frac{(\text{Time of flexible-bay} - \text{Time of fixed-bay})}{\text{Time of flexible bay}} \times 100$$

Figure 4 shows that increasing size of problem causes to increase RPD.

V. BAY-LAYOUT PROBLEM CONSIDERING I/O POINTS

In previous section, we assume I/O points are located in center of departments, whereas in industrial work, I/O points are located in perimeter of department specially, in intersections

points (see figure 6.) We assume departments have single I/O points.

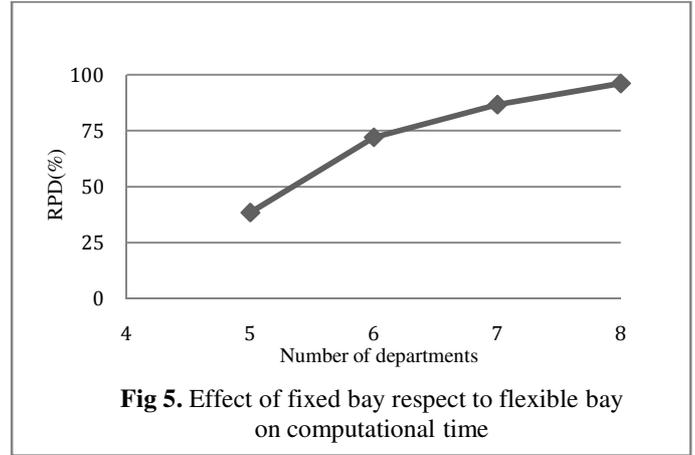


Fig 5. Effect of fixed bay respect to flexible bay on computational time

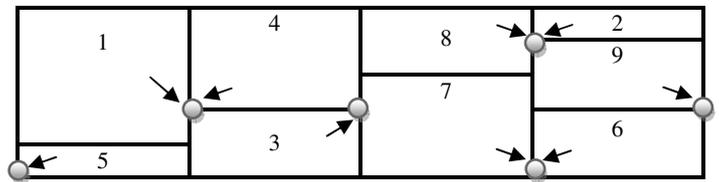
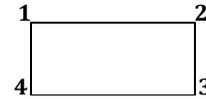


Fig 6. Layout and I/O points (circle and arrow show I/O points)

To model bay layout considering I/O points we introduce some notation as follow:

(x_i^t, y_i^t) coordination of t th corner of department $i \quad \forall i, t$
 Corner of each department are numbered as follow:



$$\alpha'_{imt} = \begin{cases} 1, & \text{If } t\text{th point of department of } m \text{ is I/O} \\ & \text{points of department } t \\ x, & \text{otherwise} \end{cases}$$

$dx_{ij}^{tt'}$ Distance between t th corner of department i to t' th corner of department j in x-axis $\forall i, j, t, t'$

$dy_{ij}^{tt'}$ Distance between t th corner of department i to t' th corner of department j in y-axis $\forall i, j, t, t'$

$\alpha\alpha'_{jpt'imt}$ is an integer variable that is equal to $\alpha\alpha'_{jpt'} \times \alpha'_{imt}$

$dx_{ij}^{tt'}$ Is equal to $|x_i^t - x_j^{t'}| \times$

$$\begin{aligned} \text{Min } p \sum_i \sum_{i < j} \sum_{p=1}^n \sum_{m=1}^n \sum_{t'=1}^4 \sum_{t=1}^4 f_{ij} (|x_i^t - x_j^{t'}| & \\ + |y_i^t - y_j^{t'}|) \alpha'_{imt} \alpha'_{jpt'} & \\ - (1-p) \sum_i \sum_{i < j} Co'_{ij} & \end{aligned} \quad (6)$$

We linearize constraint (7) as follow:

$$x_i^t - x_j^{t'} \leq dx_{ij}^{tt'} \quad \forall i < j, t, t' \quad (6.1)$$

$$x_j^{t'} - x_i^t \leq dx_{ij}^{tt'} \quad \forall i < j, t, t' \quad (6.2)$$

$$y_i^t - y_j^{t'} \leq dy_{ij}^{tt'} \quad \forall i < j, t, t' \quad (6.3)$$

$$y_j^t - y_i^{t'} \leq dy_{ij}^{tt'} \quad \forall i < j, t, t' \quad (6.4)$$

$$\alpha'_{jpt'imt} \leq \alpha'_{imt} \quad \forall i < j, t, t', p, m \quad (6.5)$$

$$\alpha'_{jpt'imt} \leq \alpha'_{jpt'} \quad \forall i < j, t, t', p, m \quad (6.6)$$

$$\alpha'_{jpt'imt} \geq \alpha'_{jpt'} + \alpha'_{imt} - 1 \quad \forall i < j, t, t', p, m \quad (6.7)$$

$$\sum_{t=1}^4 \sum_{m=1}^n \alpha'_{imt} = 1 \quad \forall i, t, m \quad (6.8)$$

$$\left(o_i^x - 0.5 \sum_k w'_{ik} \right) - x_m^t \leq W(1 - \alpha'_{imt}) \quad \forall i, t, m \quad (6.9)$$

$$x_m^t - \left(o_i^x + 0.5 \sum_k w'_{ik} \right) \leq W(1 - \alpha'_{imt}) \quad \forall i, t, m \quad (6.10)$$

$$(o_i^y - 0.5l_i^y) - y_m^t \leq H(1 - \alpha'_{imt}) \quad \forall i, t, m \quad (6.11)$$

$$y_m^t - (o_i^y + 0.5l_i^y) \leq H(1 - \alpha'_{imt}) \quad \forall i, t, m \quad (6.12)$$

Product of an integer variable by continues variable can be linearize has proposition 1.

$$\begin{aligned} \text{Min } p \sum_i \sum_{i < j} \sum_{p=1}^n \sum_{m=1}^n \sum_{t=1}^4 \sum_{t'=1}^4 f_{ij} dx_{ij}^{tt'} \\ - (1-p) \sum_i \sum_{i < j} Co'_{ij}; 0 \leq p \leq 1 \end{aligned} \quad (6.13)$$

VI. CONCLUSIONS

In this paper, we extended a mixed integer programming formulation for facility layout problem that was presented by Konak et al. [8]. According to literature, there is no formulation for multi-objective facility layout problem. Our formulation involved both minimizing departmental material handling cost and maximizing closeness rating. Computational results show the effect of fixed-bay layout approach to flexible-bay layout approach. We also modified our formulation to involve concurrent design layout and determine location of I/O points with multi-objective approach. In spite of enormous complexity of layout problem considering I/O points, it can be used to find good lower bound for heuristic and metaheuristic approach. For future research we propose to develop a metaheuristic to consider concurrent determination of layout and locations of I/O points with multi-objective view.

APPENDIX

Material flow for the ten-department problem (van Camp et al.[16]).

Department	1	2	3	4	5	6	7	8	9	10
1	-	0	0	0	0	218	0	0	0	0
2		-	0	0	0	148	0	0	296	0
3			-	28	70	0	0	0	0	0
4				-	0	28	70	140	0	0
5					-	0	0	210	0	0
6						-	0	0	0	0
7							-	0	0	28
8								-	0	882
9									-	59.2
10										-

Aspect ratio (α) = 4

Departmental areas for the ten-department problem (van Camp et al.[16])

Department	1	2	3	4	5	6	7	8	9	10
Area a_i	238	112	160	80	120	80	60	85	221	119

Closeness rating for the ten-department problem.

Department	1	2	3	4	5	6	7	8	9	10
1	-	3	0	0	0	0	4	0	5	0
2		-	0	0	2	0	4	1	1	
3			-	0	5	0	5	0	0	1
4				-	0	0	3	0	1	2
5					-	0	0	1	0	5
6						-	0	0	0	2
7							-	0	0	0
8								-	2	0
9									-	0
10										-

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Table 3. Objective function

n	5	6	7	8	9	10
0.1	1184.6	312.5	460.1	1091.6	1584.1	2272.4
0.3	3744.4	1115.72	1722.2	3601.3	5179.4	7168.5
0.5	6304.3	1918.9	2984.2	6111.1	8769.8	12078.2
0.7	8864.1	2722.2	4246.2	8620.9	12360.2	16987.9
0.9	11424	3525.4	5508.2	11130.6	15950.6	21897.4

Table 4. Computational time (Sec)

n	5		6		7		8		9		10	
	Flexible layout	Fixed layout										
0.1	0.8	0.5	9.6	2.8	51.4	5.2	2115.1	88.8	>10000	282.5	>15000	1611.4
0.3	0.7	0.5	11.6	2.8	45.2	6.1	2421.3	89.5	>10000	209.3	>15000	1788.4
0.5	1.1	0.5	10.5	3.5	34.0	6.1	2089.1	85.2	>10000	299.0	>15000	1407.8
0.7	0.9	0.5	10.2	2.9	49.9	6.2	2315.1	91.2	>10000	236.3	>15000	1320.3
0.9	0.8	0.5	10.8	2.6	44.0	5.7	2416.5	84.2	>10000	234.2	>15000	1540.4