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Inventory Risk Mitigation by Financial Hedging

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Abstract—In this paper, we study the opportunities of financial hedging to mitigate inventory risks when the demand process is correlated with the price of a financial asset. Firstly, we develop a continuously reviewed inventory model with uncertain demand and mean-variance criterion to address the financial hedging problem. Then, we propose a simple but effective strategy of financial hedging that allows a manager to exploit various financial securities as the instrument for mitigation of inventory risks. Finally, we provide a numerical experiment using Monte Carlo simulation to assess the effectiveness of the financial hedging approach.

***Index Terms*—inventory risks, stochastic demand, financial hedging.**

I. INTRODUCTION

The effective management of inventory and procurement is crucial to the effective management of an entire supply chain. Inventory serves as a buffer to meet the uncertainties in customer demand while its timely replenishment is through procuring from suppliers. The minimisation of inventory operation cost used to be manager's main concern. However, today's supply chain operations are complicated by uncertain global demand and by the highly volatile commodities markets. The inventory risk due to the financial exposure incurred by both uncertain demand and fluctuate price has greatly increased as a result.

A variety of approaches have been proposed to mitigate the inventory and procurement risks. Most work on inventory risks derives from the classical newsvendor model and relies on the adjustment of procurement policy for inventory replenishment to mitigate risks, at the expense of expected profit. Recent practices reveal another innovative approach which may simultaneously mitigate the risk and improve expected profit. Inventory and procurement managers are now exploring the use of financial assets to mitigate inventory risks. For example, commodities futures are applied to hedge against undesirable risks due to excessive price movements in the market. Very often, such undertakings are either taken as purely investment (i.e. one that without considering actual inventory operations) or their methods are inadequate to simultaneously track both demand and fluctuate price movement, and then hedge accordingly.

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Recently, more systematic and holistic approaches on inventory risk mitigation based on the techniques of financial hedging have been developed. Caldentey and Haugh (2006) develop a general framework to dynamically hedging the operation profits for a risk-averse company when these profits are correlated with returns in the financial market. They view the operations and facilities of the corporation as an asset generating continuous cash flows in the company's portfolio. So, the problem can be posed as one of financial hedging in incomplete markets, a problem that has been studied extensively in mathematical finance. Chen et al. (2007) explore the financial hedging opportunities in inventory management in a multi-period inventory model with exponential utility functions as the objective. Goel and Gutierrez (2006) investigate inventory risk caused by fluctuating procurement price, and suggest that it is possible to reduce inventory related costs by trading appropriate amount of futures/forwards in corresponding commodity market. Xu (2006) extends the above work by developing a multistage stochastic program to obtain optimal decisions for both procurement planning and risk hedging. His model specifically investigates the problem copper procurement. The stochastic behaviour of commodities prices of copper is represented using Gibson-Schwartz's two-factor model.

However, the effective mitigation of inventory risk needs to address risks arising both from the fluctuating price and uncertain demand. While the effects of price dynamics on inventory risks above have generally been addressed (e.g. by the above approaches), the problem of demand in relation to financial hedging remains difficult to resolve. Then, a related problem is that the hedging of both types of risk should best be performed simultaneously, and in a holistic framework. Our study starts from Gaur and Seshadri (2005), who suggest that the S&P 500 index have high correlation with the demand process of discretionary products. Based on such a correlation, they show how to construct hedging transactions that minimise the variance of profit and increase the expected utility for a risk-averse decision maker.

In this study, a continuously reviewed inventory model with uncertain demand will be developed. Based on this model, a financial hedging approach will be established for mitigating the inventory risks caused by stochastic demand. The mean-variance criterion is applied to balance the potential profits and the associated risks. Under appropriate assumptions, an approximating or sub-optimal solution on financial hedging of the problem is obtained. Then, a financial hedging policy can be established to minimise the variance of the profit and enhance the expected profit. A numerical experiment is conducted to quantify the findings of the analytical study and show the effectiveness of the hedging policy.

II. Model development

This study is concerned with the mitigation of risk incurred by stochastic inventory demand, which was rarely studied in the past. For accurate analysis of the problem, the whole system should be properly described in a mathematic framework. Then, a stochastic model is developed to facilitate the practical use of financial hedging of demand risks in inventory. In order to adequately model the inventory operations and financial hedging, some assumptions will first be made.

We consider a one product inventory problem which is monitored continuously by a company. Suppose that the company has the capital at time $t = 0$ to invest in two projects, namely in inventory replenishment and trading in the financial market. The latter undertaking will generate a stochastic cash flow (return on investment) that is assumed to be significantly correlated with the inventory demand (Gaur and Seshadri, 2005). This relationship indicates a potential to "smooth" the overall cash flow by an elaborated strategy of financial hedging which generate a cash flow offsetting the exposure of the stochastic operations revenue. Intuitively, the higher the correlation is, the greater the potential will be. Therefore, the company will desire to find out both the best financial security and an optimal strategy for hedging in order to balance the profits and relevant risks. The two undertakings, namely the inventory replenishment and financial investment, could respectively generate two cash flows R_t (operations revenue) and G_t (cash flow from financial hedging).

Firstly, R_t is comprised of (i) the cumulative demand of product D_t ($t \in [0,t]$), cumulative purchasing quality Q_t ($t \in [0,t]$) and (iii) inventory level Y_t at time t.

(i) D_t is assumed to follow a diffusion process $dD_t = gdt + \sigma_D dW_D(t)$. This is justified given that there are a large number of demands from independent customers (Girlich, 1996; Axsater, 1993; Liu and Cheung, 1997).

(ii) Q_t is due to the company's purchasing decisions and will be modelled as a stochastic control process. The purchasing decision is assumed to follow a (s,S) policy: an order is triggered to bring the stock up to an upper level S when the inventory level decreases below a critical low level s. The lead time is assumed to be zero.

(iii) Positive Y_t denotes holding unsold inventory and negative value denotes shortage/backlogging. The dynamics of Y_t satisfies $dY_t = -gdt + \sigma_D dW_D(t) + dQ_t$. To obtain the cost associated with Y_t, let the cost rate function be $h(y) = \max\{hy, -ly\}$, where h (or h) is the inventory holding (or backlogging) cost rate. The cost of purchasing during time interval dt can be represented as $h(Y_t)dt$.

Let T_n be a stochastic stopping time representing the time point of the $n-\text{th}$ inventory replenishment, $n = 1,2,3...$, and $T_0 = 0$. Without lose generalization the inventory level at the beginning is assumed to be the upper level S. It is apparently that: $T_n - T_{n-1} = T$, where $T = \inf\{\tau \mid D(0) = 0; D(\tau) \geq S - s\}$ is a stochastic stopping time. The inventory operations system has a generalized period T under (s,S) policy. In other words, the operations cash flows in the time interval $[T_n, T_{n+1}]$ shares the same stochastic dynamics as in the time interval $[T_0, T_1]$. This property is easy to understand: The inventory operations system shares a same state at every time point $T_n = nT$ ($n = 0,1,2,3...$). And then it will also share a same state at every $nT + t$ ($n = 0,1,2,3...$). In each time interval $[T_n, T_{n+1}]$, the revenue process of pure inventory operation is $dR_t = -h(Y_t)dt$.

Secondly, to develop G_t, suppose that there is a money market, in which the price per share is S_0, and a security market, in which the price S_1 is highly correlated to the stochastic demand D_t. Both markets are available for the company. We assume that at time t, the price processes S_0 and S_1 satisfy the stochastic differential equation:

$$\begin{cases} dS_0 = rS_0 dt \\ dS_1 = \mu S_1 dt + \sigma S_1 dW(t) \end{cases}$$

where r is the risk-free interest rate, μ and σ are constants. Let π_0 and π_1 be the money invest in the money market (or equivalently, deposit in bank) and security market, respectively, we get:

$$dG_t = \pi_0(t) r dt + \pi_1(t)\left(\mu dt + \sigma dW(t)\right)$$

It is natural to assume the company will immediately deposit its received income in bank, which is equivalent to invest in the money market. So:

$$X_t \triangleq R_t + G_t = \pi_0(t) + \pi_1(t)$$

Then the following formula can be got through calculation:

$$d(e^{-rt} X_t) = e^{-rt}\left(\left(-h(Y_t) + (\mu - r)\pi_1\right)dt + \sigma \pi_1 dW(t)\right)$$

As stated before, the demand process is correlated with the price process of the selected financial security. According to studies in mathematical finance, this correlation can be represented by the following equation

$$dW_D(t) dW(t) = \rho dt$$

where ρ is the correlation between the demand and the selected finance security.

The high correlation between the demand process and the security price process reveals the potential of an effective financial hedging. But proper assumptions should be made to prevent our approach to hedge risks from falling into an undesirable speculation. Today, a key issue in the application of financial hedging is the abuse of the concept "hedging" in a financial contract which is essentially a speculation. The difference between hedging and speculation lies in what is the transaction used for. Behind a hedging there is an adequate transaction generating a cash flow which has the opposite exposure can be offset by the hedging, while behind a speculation there is no such an transaction need to be hedged. In view of this, our approach of hedging must be strictly confined in the dimension of "smoothing" the cash flow. So the inventory and procurement operations of the company are required to be not disturbed by the financial hedging transactions. And to balance risks and gains, the well-known Mean-variance criterion is used in the evaluation of the overall cash flow $R_t + G_t$.

Ideally, through this study, a close form solution of the financial hedging problem would be produced to facilitate the tasks of operations manager. Unfortunately, it seems that no simple close-form solution could be obtained. We aim instead at developing an approximate close-form solution, which is easy for the company to use but is also powerful enough for mitigating inventory risks. Suppose that the approximate optimal hedging policy takes the form: $\pi_1(t) = a + bt$. Later we shall show that this kind of simple policies can create a significantly effective financial hedging. Also, since it is common practice to keep a certain amount of safety stock and to maintain a satisfactory customer service level, stockout is usually not allowed. Assuming $s \geq 0$, X_t can be formulated as:

$$X_t = e^{rt}\int_0^t e^{-rz}(-h(S - gz - \sigma_D W_D(z))$$
$$+(\mu - r)\pi_1(z))dz + e^{rt}\int_0^t e^{-rz}\sigma\pi_1(z)dW_z$$

The expected value of X_t can be obtained by direct cauculation:

$$E(X_t) = e^{rt}\int_0^t e^{-rz}(-h(S - gz) + (\mu - r)\pi_1(z))dz$$

Using the theory integration by parts, we have:

$$\int_0^t e^{-rz}h\sigma_D W_D(z)dz = \int_0^t \frac{e^{-rt} - e^{-rz}}{-r}h\sigma_D dW_D(z)$$

Then by the Ito's isometry, the variance of X_t can be calculated:

$$Var(X_t)$$
$$= Var\left(e^{rt}\int_0^t e^{-rz}h\sigma_D W_D(z)dz + e^{rt}\int_0^t e^{-rz}\sigma\pi_1(z)dW_z\right)$$
$$= e^{2rt}\int_0^t\left(\frac{e^{-rt} - e^{-rz}}{-r}h\sigma_D\right)^2$$
$$+\left(e^{-rz}\sigma\pi_1(z)\right)^2$$
$$+2\rho\left(\frac{e^{-rt} - e^{-rz}}{-r}h\sigma_D\right)\cdot\left(e^{-rz}\sigma\pi_1(z)\right)dz$$

Since interest rate is small, rt will also be small due to the myopic nature of inventory planning. Substituting $\pi_1(t)$ by $a + bt$, the following approximation can be obtained:

$$E(X_t) = \frac{1}{2}(hg + (\mu - r)b)t^2 - hSt + (\mu - r)at$$
$$Var(X_t) = \frac{1}{3}(h^2\sigma_D{}^2 + \sigma^2 b^2 + \rho h\sigma_D\sigma b)t^3$$
$$+(\sigma^2 ab + \rho h\sigma_D\sigma a)t^2 + \sigma^2 a^2 t$$

The mean-variance criterion is applied to mitigate the variance of the cash flow:

$$\min_{(a,b)}\left(EMV = E\left(\frac{1}{T}\int_0^T E(X_t) - \theta\cdot Var(X_t)dt\right)\right)$$

Here $\theta > 0$ represent the importance of the variance. When θ becomes bigger, the cash flow after financial hedge will be smoother.

The unique solution to the above optimal solution for a given θ is:

$$\pi_1^*(t) = a^* + b^* t$$
$$a^* = \frac{\beta}{2\theta\sigma} - \frac{\rho h\sigma_D E_2 E_3}{2\sigma\left(3E_1E_3 - 2(E_2)^2\right)}$$
$$b^* = -\frac{\rho h\sigma_D\left(3E_1E_3 - 4(E_2)^2\right)}{\sigma\left(3E_1E_3 - 2(E_2)^2\right)}$$

where,

$$E_1 = E(T) = \frac{S - s}{g}$$
$$E_2 = E(T^2) = \frac{(S - s)^2}{g^2 - \sigma_D{}^2}$$
$$E_3 = E(T^3)$$
$$= \frac{g^3(S - s)^5 + 2g\sigma_D{}^2(S - s)^3 + 3\sigma_D{}^4(S - s)^2}{g^4(g^2 - \sigma_D{}^2)}$$

The above form of approximating policy is of high efficiency. This is because analysis shows in most of the time the risk reduced (measured by variance) is nearly ρ^2 , which is the upper bound for an arbitrary financial policy can achieve. Besides, searching for securities whose prices are highly correlated with the demand process is crucial to the success of financial hedging. In fact, there are already some known results. For discretionary purchase items, the S&P 500 is a good choice for a high ρ^2 from 0.6 to 0.9 (Gaur and Seshadri, 2005). For goods traded in Commodity Exchanges, the relevant futures are also candidate choice.

III. NUMERICAL EXPERIMENTS

A numerical experiment has been conducted to assess the

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performance and properties of the financial hedging policy. For this purpose, Monte Carlo simulation is applied to compute both the expected value and the variance of the total cash flow generated by the inventory systems with or without financial hedging. Both the demand process and the price process of security are represented by 3,000 sample paths. The value of the experiment parameters involved in this experiment are given in Table 1:

Table 1 Experiment Parameters

parameter	g	σ_D	S	s
value	0.5	0.4	3	0.5
parameter	h	r	μ	σ
value	0.6	0.01	0.03	0.04

To study the effects of θ and ρ on the hedging policy, their values are varied. The numerical results of the Monte Carlo simulation are summarised in Table 2.

Table 2 Results of the Monte Carlo simulation

ρ	θ	Hedged average income	Hedged average variance
-0.8	10	1.2257	2.5343
-0.8	1	1.5089	2.7891
-0.8	0.1	4.3315	17.9834
0.8	10	-3.2317	2.5524
0.8	1	-2.9354	2.7828
0.8	0.1	-0.1157	18.1572
-0.3	1	0.1213	11.4629
-0.5	1	0.6736	8.8401
-0.7	1	1.2292	4.8927
-0.75	1	1.3680	3.8801
-0.85	1	1.6458	1.3849
-0.9	1	1.7847	0.0054
Un-hedged average income		Un-hedged average variance	
-1.1261		12.6712	

Firstly, the effectiveness of hedging when changing θ is studied. When $\theta = 0.1$, the resulting policy will behave like a speculation, which means high income with high risk. When $\theta = 10$, the reduction of variance will result in a significant decrease of average income, compared with the case of $\theta = 1$. Thus, a medium $\theta \geq 1$ is suggested in the formulation of financial hedging policy in practice. When $\theta = 1$, the efficiency of the hedging (represented by the variance reduced) has a significant positive relationship with the absolute value of ρ. Although the sign of ρ does not seem to affect the efficiency of risk mitigation, it has great influence on the expected income. A negative ρ can increase the income while a positive one will reduce it. The hedged variance goes down gradually and approximately linearly with the increase of ρ^2, which is consistent with the analytical results. When the correlation is significantly negative (usually $\rho \leq -0.75$), the efficiency of the hedging policy can be extremely high, with both average income and corresponding risks significantly improved.

IV. Conclusion

This paper describes a study of the financial hedging policy to mitigate inventory demand risks. A stochastic model with mean-variance criterion has been built to get an efficient form of approximating solution. The contribution of this paper is to develop an effective financial hedging policy for inventory managers to mitigate risks caused by fluctuate demand.

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