Modeling Knowledge of Mathematics Teaching for Students with Mathematical Learning Difficulties: A Knowledge Management Perspective

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Abstract — E-learning that integrates digital knowledge content, network and information technology has become an emerging learning method. E-learning is a knowledge intensive process, it is therefore necessary to manage the acquisition, storage, sharing, and innovation of knowledge to enhance the effectiveness of learning. As knowledge is the core of e-learning, knowledge identification, analysis, and modeling become the primary tasks in implementing e-learning. This paper presents a systematic approach for modeling teachers’ knowledge for an e-learning platform based on the theory and technology of system engineering, knowledge management and knowledge engineering. This paper focuses on modeling knowledge of mathematics teaching for students with mathematical learning difficulties to support a KM-based e-learning platform.

Index Terms — e-learning, knowledge management, knowledge engineering, students with mathematical learning difficulties, mathematics teachers¹ professional knowledge

I. INTRODUCTION

Improving the educational system for children with mathematical learning difficulties has been a key focus of educational reforms. The subject area of mathematics is complex with multiple domains. Difficulties may result from deficits in one or several cognitive skills [1]. It is evident that the group of children with mathematical learning difficulties is very heterogeneous [2], and mathematical difficulties are persistent and evident from the early elementary grades through secondary levels [3]. In recent years, researchers have been paying increased attention to students who demonstrate challenges in learning and applying mathematics skills and concepts [4][5][6][7]. However, teachers have often found themselves lack of knowledge and skills required for mathematics teaching. It is imperative for teachers to improve their professional knowledge and competence through continuous learning. Moreover, factors such as geographic dispersion, resources disparity, and constraints of time and space have all made it pressing for the education community to provide teachers with opportunities for continuous learning and adequate support.

With the booming advances in information technologies and the ever-increasing popularity of Internet, e-Learning that features an integration of the Internet and information technologies has become a newly rising trend in learning as well as an important strategy and direction for upgrading and reforming education in all major countries of the world [8][9][10]. To improve the mathematics teaching for students with mathematical learning difficulties with the considerations of teachers’ knowledge development and problem-solving capabilities in real-life teaching situations, a national research project was conducted in developing a Problem-Based e-Learning (PBel) model by adopting the situated learning as a theoretical basis along with social constructivism, case-based learning, and problem-based learning approaches. The model was then used as the basis for the planning and design of an e-Learning platform.

Since e-learning is a knowledge intensive process, the effectiveness of e-learning is highly dependent on the quality of its content knowledge. It is therefore necessary to manage the acquisition, storage, sharing, and innovation of knowledge to enhance the effectiveness of learning. Consequently, identification, exploration, modeling and building of knowledge become the primary tasks in implementing e-learning.

In this paper, a Problem-based e-Learning model, which was developed as the basis for the planning and design of a Problem-based e-Learning platform is first introduced. The focus of this paper is then a systematic approach for modeling teachers’ knowledge for the proposed Problem-based e-Learning platform. This paper focuses on modeling knowledge of mathematics teaching for students with mathematical learning difficulties to provide formal knowledge, practical knowledge and empirical knowledge through knowledge identification, exploration, modeling, and building to support pre-service and in-service teachers in developing knowledge of mathematics teaching for students with mathematical learning difficulties to effectively improve mathematics teaching.

II. PROBLEM-BASED E-LEARNING MODEL (PBEL)

This study designed an e-Learning model with problem-based learning as its core and social constructivism and situated learning as its auxiliary theories. This model includes the stages of analysis, design, development, and practice as shown in Fig. 1.

The analysis stage involves assessing a learner’s (i.e. teacher’s) knowledge of the students with mathematical learning difficulties, pedagogical content knowledge of mathematics, knowledge of modifications of curriculum, teaching methods, materials, techniques, and learning environments for teaching students with mathematical learning difficulties, mathematical content knowledge, and then diagnosing the students’ learning problems. The learning goal for the learner is then translated into “solving students’ learning problems.” The design stage identifies the learner’s background information and teaching objectives in order to outline a personalized learning plan. The development stage develops contents, such as concepts and cases, for the personalized learning plan. Finally, the practice stage guides the learner to initiate learning activities, such as concept learning, case studies, practical teaching, feedback on teaching experience and knowledge sharing. After the learner has completed the concept learning and case studies, he/she is required to begin realistic teaching, by applying learned knowledge to realistic teaching context. Lastly, the system knowledge content can continue to expand and update as the learners would share their knowledge and thoughts.

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A learner undertaking case studies may select either “individual learning” or “group learning”. The “group learning” takes the learner to a learning mode based on social constructivism, where the learner may initiate a group discussion and direct questions to experts or learners with related experience in any phase of the case study. During Q&A sessions or online discussions in this forum, an experienced teacher or expert plays the role of an e-consultant to guide the learners to complete their learning processes.

III. KNOWLEDGE MANAGEMENT IN PBEL

Knowledge management (KM) refers to the identification, acquisition, selection, storage, management, application, and sharing of knowledge, by an organization or individuals [11]. KM represents a systematic process of knowledge accumulation as well as the effective use of knowledge to help realize greater benefits when both individuals and the whole organization improve as a result of the popularization of knowledge.

As e-learning is a knowledge intensive process, the effectiveness of e-learning is highly dependent on the quality of its content knowledge, which in turn counts on the success of knowledge capture, storage, sharing and innovation. Therefore, the concepts and methods of knowledge management were employed in the approach of this research, which includes steps of knowledge identification, knowledge exploration, knowledge capture, storage, sharing and innovation. Therefore, the knowledge required in this application can be identified from the knowledge model of expert teachers.

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The objective of knowledge identification is to identify the knowledge requirements as well as the relationships among the areas of involved knowledge. The output of this phase is a knowledge framework that indicates the areas of the domain knowledge and their relationships.

The phase of knowledge exploration explores the elements of each area of knowledge and their relationships to present domain knowledge in a structured format. The output of this phase is a domain knowledge map illustrating the elements of domain knowledge and their relationships.

Knowledge modeling aims to analyze the type, characteristics, attributes, and structure of each area of knowledge and formalize them using modeling methods and conceptual representation schemes. The outputs of this step are the conceptual models of knowledge and their specifications.

Knowledge content building conducts the building of domain ontology (i.e. concept map), domain corpus, cases and rule base based on the results of knowledge exploration and modeling. Each layer of knowledge was built through steps of identification, validation, acquiring and implementation.

IV. KNOWLEDGE IDENTIFICATION

The requirements of knowledge can be identified from the scope of applications, which in turn can be specified based on the objective of the e-learning platform. In this research, the application objective is to help pre-service and in-service teachers enhance knowledge to help elementary school students with mathematical learning difficulties improve mathematics ability. Therefore, the knowledge required in this application can be identified from the knowledge model of expert teachers.

Basically, an expert teacher possesses formal knowledge and practical know. Formal knowledge is acquired through a teacher’s formal training program and teaching principle that are categorized according to different scenarios. They include, as enumerated by Shulman [12], content knowledge, general content knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, and knowledge of educational contexts and objectives.

Practical knowledge is derived from the application of formal knowledge to real-life teaching situations and the resolution and rumination of teaching difficulties [13]. Teachers, through real-life teaching experience, are able to develop various principles, conventions, and effective practices and experiences based on formal knowledge through personal exposure, experiences, and observations made in different situations.

Practical knowledge can be further identified into procedural knowledge and strategic knowledge. The former is related to the procedure to carry an action out, which is the instruction-oriented knowledge about “how” to do something. For example, the methodology to solve students’ learning problems would be considered procedural knowledge. The latter is the principles or rules to carry out an action. Empirical knowledge was obtained from practical knowledge by capturing and generalizing the experiential principles, procedures and methods to solving problems without presenting the background, intent, logic, reasoning, principle, and thought that were implicit in the particular method.

In light of aforementioned knowledge of teachers, this study has analyzed and modeled the learning resources as discussed below (See Fig. 2).

Concept is the fundamental elements of knowledge, therefore the formal knowledge of the domain in this research can be...
viewed as an aggregation of domain concepts associated with in-depth knowledge of the concepts. The former is the conceptual knowledge of educational theory, learning theory, mathematics education theory, mathematics teaching, physical and psychological characteristics of students, students’ cognition, and educational contextual knowledge. The latter is about the know-why as well as strategic and procedural knowledge (i.e. know-how) of the aforementioned concepts.

Studies found that the case method which involves the narration of teaching practices based on real classroom cases helped the teachers link formal and practical knowledge [14][15] and stimulated reflections and effective development of teaching concepts [16]. Therefore, the present study employed cases for practical knowledge presentation, each of which included parts of “teaching narration” and “teaching narration explanation.” The former consisted of student profile, teaching objective, teaching procedure, teaching activity and teaching content, and general assessments. The teaching activity and content contained students’ error patterns, teaching objectives, teaching strategies, teaching methods, teaching aids and unit assessments. The latter was the explanation of the teaching narration which included interpretation of disability assessment and problem identification, the reasons for proposed teaching objective, procedure, strategy, content, and assessment. A case presents the “know-what” of the encountered student’s learning problems and related concepts, the problem-solving “know-how” experienced by teachers, and the “know-why” involved in the “know-how”. To make practical and formal knowledge interoperable, the concepts appear in the cases can link back to formal knowledge layer for concept explanations. On the other hand, some of the concepts and theories defined in the formal knowledge layer may also link to related cases, giving learners relevant practical materials to study and verify. Besides, questions in forms of 5W1H (what, why, when, who, where, & how) and reference answers as well as explanations are embedded in each section of the cases to allow learners to repeatedly study the case content to enhance learning efficiency.

V. KNOWLEDGE EXPLORATION AND MODELING

This section presents the exploration and modeling of formal, practical and empirical knowledge.

5.1 Formal knowledge

To explore domain concepts, this study developed an “inside-out concept exploration approach” using domain core concepts as the basis to explore other domain concepts. Since this approach only defines the concepts that are directly related to the core concepts, one can control the scope of concept definition and decrease the amount of unneeded concept enumeration to speed up the development of domain concept model. Moreover, the further exploration of core concepts may enhance the completeness of the domain concept map.

This study took “knowledge of mathematics teaching for students with mathematical learning difficulties” as its domain root concept and it included core concepts such as mathematics knowledge, general content knowledge, mathematics teaching knowledge, physical and psychological characteristics of students with mathematics difficulties, students’ cognition, and educational contextual knowledge by referencing teachers’ formal knowledge enumerated by Shulman [12]. These seven concepts were further explored and organized into a seven-layer core concept map. In order to reify the meanings of the concepts, the definitions, attributes and axioms of each knowledge concept and the relationships among concepts were defined. Moreover, the terms appear in the definition of a core concept were defined as the auxiliary concepts of the core concept.

Ontology is believed as an effective method for defining entity, property and relationship of knowledge concepts of specific domain [17][18]. Therefore, to model and present the concept knowledge as well as to provide a basis for integration of formal knowledge and practical knowledge, an ontology for “mathematics teaching for students with mild disabilities” was developed as a domain knowledge map as shown in Fig. 3.

To enhance ordinary ontology models that simply use “hierarchy” to define relationships among concepts [17][18][19], this study proposed an ontology model with more concept relationships as shown in Fig. 4. The main constructs of the ontology model are concept and concept relation. A concept is defined in terms of Concept name, Classification code, Concept definition, Concept properties and Property constraints, and the concept relations can be hierarchical relations or non-hierarchical relations. The hierarchical relations can be further classified into types of inclusion and attachment, while non-hierarchical relations could be types of synonyms or association. “Inclusion” indicates a parent concept with a set of child concepts. “Attachment” which defines a child concept is a “sub-class” of its parent concept. “Synonyms” defines the synonyms of a concept. Finally, “association” defines the “auxiliary concepts” of a core concept.

The in-depth knowledge defines the detailed explanation, including the “know-what”, “know-why”, “know-how”, “know-with” and examples of each core concept defined in the concept map. The present study viewed in-depth knowledge as “learning object”, which is defined in terms of description, example, external links and content type.

Fig. 3. Part of the domain concept map.

Fig. 4. The proposed ontology-based concept map model.
5.2 Practical knowledge

The procedure for practical knowledge model development includes steps of (1) teaching narration analysis and modeling, (2) common practical knowledge map development, and (3) case model development and verification. In the first step, teaching narrations developed by certified teachers from their own real teaching experience, observation, discussion, and expert assessments were analyzed and modeled into teaching narration knowledge maps. The teaching narration knowledge maps were used and generalized into a common knowledge map, which was the preliminary case model. The common knowledge map was then verified by a group of expert teachers to get a consensus of the learning case model.

The learning case model was composed of two parts: student profile and teaching plan. The student profile contains the statements of student problem identifications and analyses, as well as assessments of student’s learning characteristics, which included mathematics capability, intelligence, cognitive abilities, strengths and weaknesses, types of disability, and preferences. The teaching plan contains general teaching objective, teaching procedures, the teaching activities and general learning assessments. The teaching procedure consists of one or more teaching activities, which are basically teaching cases containing student’s error patterns, unit teaching objective, teaching strategy, teaching material and teaching aids.

Fig. 5 shows the learning case model defined in terms of UML [20] notations, where a box represents a class of learning objects and a diamond indicates a composite class which is composed of its component classes. As indicated in the figure, each class includes portions of what, why, how and Q&A, and together they explain what has been done in the teaching content and why and how it was done, as well as a series of related questions for discussion and the learners’ ruminations.

In order to effectively store, organize, manage, and use the case contents, this study defined the instances of each class in the case model as learning objects by employing object technology [21]. Besides the data (i.e content) of a learning object, methods were encapsulated in the objects as functions for manipulation of those object data to improve its adaptivity. For example, the presentation formats and display sequence of the content may vary with the learner’s learning style and cognitive trait, which were manipulated by the methods of learning objects.

The learning objects are stored in an object database, and the links among the learning objects provide a basis for case configuration and object clustering. The learning contents can be retrieved through individual learning object retrieval, entire learning case retrieval or grouping of certain learning objects. For instance, besides retrieving a whole learning case for study, a learner may retrieve student profiles to investigate the relationships between students’ learning problems and their personal and learning characteristics. The learner may also search the teaching activity as teaching cases for reference.

5.3 Empirical knowledge

Empirical knowledge aims to provide the learners with concepts, experiential principles, procedures and methods to solve students learning problems without presenting the background, intent, logic, reasoning, principle, and thought that were implicit in the particular method.

Using problem solving process as reference, the empirical knowledge model on student’s learning problem solving was obtained through interviews with expert teachers. It includes stages of problem identification, problem analysis, solution development, solution implementation, and performance evaluation. Problem identification is to identify the student’s learning problem, clearly define it and establish a precise problem statement, and decide what teaching objective to achieve. The problem analysis is to identify root cause of student’s learning problem and collect and analyze data related to the problem. Possible solution that may address the root cause of the student’s learning problem is then proposed in the stage of solution development. Solution implementation is to put the possible solution into action based on a planning on when and how to do it. Re-planning and redesign on the solution may be required during the implementation. Performance evaluation is conducted to identify how effective the solution is, to assess if the teaching goal has been achieved, and to validate the consequences it has on the situation. Each stage of the problem solving process was further identified into detailed steps or activities as shown in Fig. 6.

This study presented the knowledge nodes defined in the empirical knowledge structure as “learning objects” containing knowledge of “know-what”, “know-why”, “know-how” and the “know-why of know-how” to structurally present the layers of empirical knowledge. Know-what which defines the concept of the knowledge is basically declarative knowledge. Know-why is causal knowledge to express the applications and the needs of the knowledge. The know-how knowledge which includes strategic how and procedural how is to express the experiential principles and detailed procedures or methods of student’s learning problem solving activities.
VI. KNOWLEDGE CONTENT BUILDING

This section presents the building of concept map, case bases, and empirical knowledge bases based on the results of knowledge exploration and modeling. Each layer of knowledge was built through steps of identification, validation, acquisition, and implementation.

6.1 Concept map and concept learning object development

The process of concept map building included steps of (1) defining the knowledge hierarchy of the ontology via a proposed “Inside-out” ontology development method as discussed in Section 5.2.1, (2) distributing the knowledge hierarchy to three experts on special education and two expert teachers for validation and revision, (3) developing the content of knowledge hierarchy by referencing textbooks, (4) expanding the associated concepts by identifying the concepts appearing in the definition of concepts in the knowledge hierarchy, (5) distributing the preliminary ontology to domain experts and expert teachers for validation and revision, and (6) implementing the ontology in Protégé, a free, open source ontology editor and knowledge-base framework [17].

The concept learning objects (i.e. in-depth knowledge) were developed through the following steps: (1) identifying the concepts required for learning object development from the concept map, (2) acquiring the content of each concept from relevant textbooks, (3) distributing the preliminary pre-developed concept learning objects to the domain experts and expert teachers for validation and revision, (4) implementing the concept learning objects in Micro Soft SQL XML database, and (5) making the links between nodes in the concept map and the concept learning objects.

6.2 Learning case development

Learning cases were developed in two stages of teaching narration development and implicit knowledge acquisition. Teaching narrations were developed by certified teachers based on their own teaching records, and were analyzed through actual teaching, observation, discussion, and assessed by expert teachers. Implicit knowledge acquisition was to extract the “know-how”, “know-why” and “know-why” of the “know-how” in teaching narrations. Through implicit knowledge acquisition, practical knowledge of teaching not only recorded solutions, but also the motives, background, and insights of teaching expert for a particular problem.

To conduct implicit knowledge acquisition, this research developed a case-based map for implicit knowledge acquisition as shown in Fig. 7. The knowledge elicitation starts with inquiry of the cause of student’s learning problem indicated in the statement and the reason why making such statement, which can be explained using student’s personal characteristics. Since these characteristics are identified through the assessments results of mathematic abilities, cognitive abilities etc., the inquiries can be made on the interpretation of the assessment results (i.e. why) and the methods for characteristic assessment (i.e. how). For teaching objective, inquiry is made on the reason to set the objective and the “strategic how” to achieve the teaching objective. The former explains why the teaching objective is decided, while the latter indicates the approach to solving student’s learning problem as well as the relationship between teaching objective and teaching procedure. The reason for taking the teaching procedure is identified as “why”, and the teaching activities are the “how” of the procedure. Similarly, each teaching activity has reasons (i.e. why) for taking it, and teaching objective determinations, student error pattern assessments, teaching strategy selections, and teaching aid selections are the “how” of each teaching activity. The student error pattern can partially explain why the teaching objective is determined, while teaching strategy can be seen as the “how” to achieve the objective. Generally speaking, teaching aids can be used as method to accomplish the teaching strategy, it can be therefore seen as the “how” of teaching strategy.

To summarize, as shown in the figure, the “know-how” and “know-why” at each node can be obtained through knowledge acquisition. For declarative statement such as “problem statement”, “teaching objective” and “student error pattern”, the “why” explains “the reason why to make such statements” and the “how” depicts “the methods or approaches to achieve or obtain the facts in the statement”. Similarly, for procedural nodes such as “teaching procedure”, “teaching activity”, “teaching aids” and “assessments”, the “why” explains “the reason why to take such actions” and the “how” depicts “the methods or steps to accomplish the actions”. Most of the “why” and “how” are the implicit knowledge of expert teachers, and some may appear as casual relationships between nodes, which are indicated in terms of “why (with)” and “how (with)” respectively. For example, the teaching procedural is the “how (with)” of teaching objective and student characteristics is the “why (with)” of problem statement.

6.3 Empirical knowledge elicitation

In order to effectively acquire empirical knowledge from expert teachers, this study developed a knowledge elicitation method using Analysis Hierarchy Process (AHP) developed by Saaty [22]. The AHP method was developed to provide an analysis framework, using hierarchical approaches to divide complex and unstructured problems, and thereafter proceeding with the analysis. The analysis results of each layer were further summarized, drawn up an effective solution. It was often applied in solution creation, solution selection, and solution optimization, and implemented using survey questionnaires or interviews.

The framework of the knowledge elicitation method includes a sequence of know-what, know-why, know-how, and know-with analyses (see Fig. 8). The know-how is layered into strategic how and procedural how, where the procedural how itself may include a sequence of activities as the next layer’s what and how. The strategic how, procedural how, and the detailed activities are further analyzed into what, why, how and with.

By applying the method, a map for empirical knowledge elicitation was obtained as shown in Fig. 9. At the first layer, the know-what, know-why, know-how (including strategic how and procedural how) and know-why of the know-how are identified. The procedural how includes activities of learning difficulty identification, student awareness, teaching planning, teaching practice, and performance evaluation, each of which becomes the second layer’s what for further knowledge elicitation. The elicitation process continues until no further activity is identified.

Through the elicitation, implicit and unstructured empirical knowledge of student’s learning problem solving were obtained. Meanwhile, the results at each layer were classified according to teaching subjects and disability types to facilitate knowledge access. They were then generalized into empirical rules for further applications such as development of expert system for mathematics teaching for students with mathematical difficulties.
Besides problem-solving context and methods, relevant theories map of theoretical knowledge and topic maps of practical ontology schema was developed for building domain concept thinking of domain experts. Moreover, an object-oriented content repository was presented in a structured manner closed to the thinking of human. Effectively stored through the transformation of concepts and their relationships, problem resolution of expert teachers can be extracted and used for problem-based e-learning [23].

This paper presented a systematic approach for modeling knowledge of mathematics teaching for students with mathematical learning difficulties. Through the proposed approach, a digital knowledge content repository was developed to support a KM-based e-learning platform for knowledge management and the semantic web. With examples from the areas of knowledge management, e-commerce and the semantic web, the alignment of two layers of knowledge can be obtained to enhance the effectiveness of learning.

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